Digital Communication Systems

Spread spectrum and Code Division Multiple Access (CDMA) communications
Spread Spectrum Communications - Agenda Today

• Basic principles and block diagrams of spread spectrum communication systems
• Characterizing concepts
• Types of SS modulation: principles and circuits
  – direct sequence (DS)
  – frequency hopping (FH)
• Error rates
• Spreading code sequences; generation and properties
  – Maximal Length (a linear, cyclic code)
  – Gold
  – Walsh
• Asynchronous CDMA systems
How Tele-operators* Market CDMA

**Coverage**
For Coverage, CDMA saves wireless carriers from deploying the 400% more cell site that are required by GSM.

**Capacity**
CDMA’s capacity supports at least 400% more revenue-producing subscribers in the same spectrum when compared to GSM.

**Cost**
A carrier who deploys CDMA instead of GSM will have a lower capital cost.

**Clarity**
CDMA with PureVoice provides wireline clarity.

**Choice**
CDMA offers the choice of simultaneous voice, async and packet data, FAX, and SMS.

**Customer satisfaction**
The Most solid foundation for attracting and retaining subscriber is based on CDMA.
Direct Sequence Spread Spectrum (DS-SS)

- This figure shows BPSK-DS transmitter and receiver
  (multiplication can be realized by RF-mixers)

\[
\sqrt{2P} c(t - T_d) \cos(\omega_t + \theta_d(t - T_d) + \phi)
\]

+ interference

\[
c(t - \hat{T}_d)
\]

\[
P_{av} = \frac{A^2}{2} \Rightarrow A = \sqrt{2P_{av}}
\]
Characteristics of Spread Spectrum

- Bandwidth of the transmitted signal \( W \) is much greater than the original message bandwidth (or the signaling rate \( R \))
- Transmission bandwidth is independent of the message. Applied code is known both to the transmitter and receiver

- Interference and noise immunity of SS system is larger, the larger the processing gain
- Multiple SS systems can co-exist in the same band (=CDMA). Increased user independence (decreased interference) for (1) higher processing gain and higher (2) code orthogonality
- Spreading sequence can be very long -> enables low transmitted PSD-> low probability of interception (especially in military communications)
Characteristics of Spread Spectrum (cont.)

- **Processing gain**, in general
  - Large $L_c$ improves noise immunity, but requires a larger transmission bandwidth
  - Note that DS-spread spectrum is a repetition FEC-coded systems

- **Jamming margin**
  - Tells $L_{c} = 30$ dB, available processing gain
  - $L_{sys} = 2$ dB, margin for system losses
  - $SNR_{desp} = 10$ dB, required SNR after despreading (at the RX)
  - $M_{j} = 18$ dB, additional interference and noise can deteriorate received SNR by this amount
Characteristics of Spread Spectrum (cont.)

- **Spectral efficiency** $E_{eff}$: Describes how compactly TX signal fits into the transmission band. For instance for BPSK with some pre-filtering:

  $$\begin{align*}
  B_{RF} & \approx \frac{B_{RF, filt}}{k} \approx \frac{1}{T_c} \approx \frac{L_c}{T_b \log_2 M} \\
  \Rightarrow E_{eff} &= \frac{R_b}{B_{RF}} \approx \frac{1}{T_b} \frac{\log_2 M}{L_c} = \frac{\log_2 M}{L_c} \\
  \quad \left\{ \begin{array}{l}
  B_{RF, filt} \text{: bandwidth for polar mod.} \\
  M \text{: number of levels} \\
  k \text{: number of bits} \\
  \end{array} \right. \\
  \quad \left( M = 2^k \Rightarrow k = \log_2 M \right)
  \end{align*}$$

- **Energy efficiency (reception sensitivity)**: The value of $\gamma_b = E_b / N_0$ to obtain a specified error rate (often $10^{-9}$). For BPSK the error rate is

  $$p_e = Q(\sqrt{2\gamma_b}), Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^\infty \exp(-\lambda^2/2) d\lambda$$

- QPSK-modulation can fit twice the data rate of BPSK in the same bandwidth. Therefore it is more energy efficient than BPSK.
A QPSK-DS Modulator

- After serial-parallel conversion (S/P) data modulates the orthogonal carriers
- Modulation on orthogonal carriers spreaded by codes $c_1$ and $c_2$
- Spreading codes $c_1$ and $c_2$ may or may not be orthogonal (System performance is independent of their orthogonality, why?)
- What kind of circuit can make the demodulation (despreading)?
DS-CDMA (BPSK) Spectra (Tone Jamming)

• Assume DS - BPSK transmission, with a single tone jamming (jamming power $J [\text{W}]$ ). The received signal is

• The respective PSD of the received chip-rate signal is

• At the receiver $r(t)$ is multiplied with the local code $c(t)$ (=despreading)
Tone Jamming (cont.)

Despreading spreads the jammer power and despreads the signal power:

\[ \frac{1}{2} PT_c \]

(a) Before despreading

\[ S_s(f) \]

\[ \text{Jammer area} = \frac{1}{2} J \]

Signal

(b) After despreading

\[ S_p(f) \]

\[ \frac{1}{2} PT \]

Signal

\[ \frac{1}{2} JT_c \]

Jammer

\[ \sqrt{2P} c(t - T_d) \cos(\omega_0 t + \text{interference}) \]

\[ c(t - \hat{T}_d) \]

Despreadi

Banc fil
Tone Jamming (cont.)

- Filtering (at the BW of the phase modulator) after despreading suppresses the jammer power.
Error Rate of BPSK-DS System*

- DS system is a form of coding, therefore number chips, e.g., code weight determines, from its own part, error rate (code gain)

- Assuming that the chips are uncorrelated, prob. of code word error for a binary-block coded BPSK-DS system with code weight \( w \) is therefore

\[
P_e = Q\left( \sqrt{\frac{2E_b R_c W_m}{N_0}} \right), R_c = k / n \quad (= \text{code rate})
\]

This can be expressed in terms of processing gain \( L_c \) by denoting the average signal and noise power by \( P_{av}, N_{av} \), respectively, yielding

\[
P_{av}, N_{av}
\]

\[
E_b = P_{av} T_b, N_0 = N_{av} T_c \Rightarrow
\]

- Note that the symbol error rate is upper bounded due to repetition code nature of the DS by

\[
P_e = Q\left( \sqrt{\frac{2P_{av} T_b R_c W_m}{N_{av} T_c}} \right) = Q\left( \sqrt{\frac{2P_{av} L_c R_c W_m}{N_{av}}} \right)
\]

where \( t \) denotes the number of erroneous bits that can be corrected in the coded word

\[
P_{es} \leq \sum_{m=t+1}^{n} \binom{n}{m} p^m (1 - p)^{n-m}, t = \left\lfloor \frac{1}{2}(d_{min} - 1) \right\rfloor
\]

*For further background, see J.G. Proakis: Digital Communications (IV Ed), Section 13.2
Example: Error Rate of Uncoded Binary BPSK-DS

- For uncoded DS \( w=n \), thus \( R_c = (1/n)n = 1 \) and

\[
P_e = Q\left(\sqrt{\frac{2E_b}{N_0}} \cdot R_c \cdot w_m\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\]

- We note that \( E_b = P_{av} T_b = P_{av} / R_b \) and \( J_0 = J_{av} / W \) yielding

\[
\frac{E_b}{J_0} = \frac{P_{av} / R}{J_{av} / W} = \frac{W / R}{J_{av} / P_{av}}
\]

\[
\Rightarrow P_e = Q\left(\sqrt{\frac{2W / R}{J_{av} / P_{av}}}\right)
\]

- Therefore, we note that increasing system processing gain \( W/R \), error rate can be improved
Code Generation in DS-SS

- Binary message
- Binary adder
- Balanced modulator
- Transmitted signal
- Carrier $f_0$
- PN code generator
- Clock
- DS modulator
- Maximal length (ML) spreading code
- $N = 2^n - 1$

ML code generator

- Feedback taps
- Clock
- $n - 2$
- $n - 1$
- $n$
- Mod 2
- $1$
- $2$
- $n$
Some Cyclic Block Codes

- **Repetition codes.** High coding gain, but low rate
- **Hamming codes.** Minimum distance always 3. Thus can detect 2 errors and correct one error. $n=2^m-1$, $k = n - m$,
- **Maximum-length codes.** For every integer $m$ there exists a maximum length code $(n,k)$ with $n = 2^k - 1, d_{\text{min}} = 2^{k-1}$. Hamming codes are dual of maximal codes.
- **BCH-codes.** For every integer $m$ there exist a code with $n = 2^m-1$, and where $t$ is the error correction capability
- **Reed-Solomon (RS) codes.** Works with $k$ symbols that consist of $m$ bits that are encoded to yield code words of $n$ symbols. For these codes and
- Nowadays BCH and RS are very popular due to large $d_{\text{min}}$, large number of codes, and easy generation
- For further code references have a look on self-study material!
Maximal Length Codes

\[ R_c(\tau) \]

\[ S_c(f) \]

\[ \frac{N+1}{N^2} \cdot \text{sinc}^2 \left( fT_c \right) \]
Design of Maximal Length Generators by a Table Entry

\[
\begin{array}{cccc}
3 & 6 & 7 & \text{octal} \\
0 & 1 & 1 & \text{binary} \\
\downarrow & \downarrow & \downarrow & \text{coefficient} \\
g_7 & g_6 & g_5 & g_4 & g_3 & g_2 & g_1 & g_0
\end{array}
\]
Other Spreading Codes

- **Walsh codes**: Orthogonal, used in *synchronous systems*, also in WCDMA downlink
  - Generation recursively: \( H_0 = [0] \), \( H_n = \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & H_{n-1} \end{bmatrix} \)
  - All rows and columns of the matrix are orthogonal:
    \[ (-1)(-1) + (-1)1 + 1(-1) + 1 \cdot 1 = 0 \]

- **Gold codes**: Generated by summing *preferred pairs* of maximal length codes. Have a guarantee 3-level crosscorrelation: \( \{ -t(n)/N, 1/N, (t(n) - 2)/N \} \)
  - For \( N \)-length code there exists \( N + 2 \) codes in a code family and
    \[ N = 2^n - 1 \quad \text{and} \quad t(n) = \begin{cases} 1 + 2^{(n+1)/2}, & \text{for } n \text{ odd} \\ 1 + 2^{(n+2)/2}, & \text{for } n \text{ even} \end{cases} \]
  - Walsh and Gold codes are used especially in multiple access systems
  - Gold codes are used in *asynchronous communications* because their crosscorrelation is quite good as formulated above
Frequency Hopping Transmitter and Receiver

- In FH-SS hopping frequencies are determined by the code and the message (bits) are usually non-coherently FSK-modulated.

\[ BW = W_d \]
\[ BW = W_s \]
\[ BW = W_s \]
\[ BW = W_d \]

![Diagram of Frequency Hopping Transmitter and Receiver](image)

- This method is applied in BlueTooth®
Frequency Hopping Spread Spectrum (FH-SS)

- Example: transmission of two symbols/chip

- 2 levels
- 2 slots
- $k$: chip duration
- $b$: bit duration
- $s$: symbol duration
- $T$: chip duration
- $T_b$: bit duration
- $T_s$: symbol duration

- $W_d = 2^L f_d$ (≈ data modulator BW)
- $W_s = 2^k W_d$ (≈ total FH spectral width)

- $2^L$ levels
- $2^k$ slots
Error Rate in Frequency Hopping

- If there are multiple hops/symbol we have a fast-hopping system. If there is a single hop/symbol (or below), we have a slow-hopping system.

- For slow-hopping non-coherent FSK-system, binary error rate is
  \[ P_e = \frac{1}{2} \exp(-\gamma_b / 2), \gamma_b = E_b / N_0 \]
  and the respective symbol error rate is (hard-decisions)
  \[ P_{es} = \frac{1}{2} \exp(-\gamma_b R_c / 2), R_c = k / n < 1 \]

- A fast-hopping FSK system is a diversity-gain system. Assuming non-coherent, square-law combining of respective output signals from matched filters yields the binary error rate (with \( L \) hops/symbol)

\[
P_e = \frac{1}{2^{2L-1}} \exp(-\gamma_b / 2) \sum_{i=0}^{L-1} K_i \left( \frac{\gamma_b}{2} \right)^i, \gamma_b = L \gamma_c
\]

\[
K_i = \frac{1}{i!} \sum_{r=0}^{L-1-i} \binom{2L-1}{r}
\]
DS and FH compared

- FH is applicable in environments where there exist **tone jammers** that can be overcame by avoiding hopping on those frequencies
- DS is applicable for **multiple access** because it allows **statistical multiplexing (resource reallocation)** to other users (power control)
- FH applies usually **non-coherent modulation** due to carrier synchronization difficulties -> modulation method degrades performance
- Both methods were first used in **military communications**,– FH can be advantageous because the **hopping span** can be very large (makes eavesdropping difficult)
  – DS can be advantageous because **spectral density** can be much smaller than background noise density (transmission is unnoticed)
- FH is an **avoidance system**: does not suffer on **near-far effect**!
- By using **hybrid systems** some benefits can be combined: The system can have a low probability of interception and negligible near-far effect at the same time. (**Differentially coherent modulation** is applicable)
Multiple access: FDMA, TDMA and CDMA

- FDMA, TDMA and CDMA yield conceptually the same capacity.
- However, in wireless communications, CDMA has improved capacity due to statistical multiplexing and graceful degradation.
- Performance can still be improved by adaptive antennas, multiuser detection, FEC, and multi-rate encoding.
Example of DS multiple access
FDMA, TDMA and CDMA compared (cont.)

• TDMA and FDMA principle:
  – TDMA allocates a time instant for a user
  – FDMA allocates a frequency band for a user
  – CDMA allocates a code for user

• CDMA-system can be synchronous or asynchronous:
  – Synchronous CDMA can not be used in multipath channels that destroy code orthogonality
  – Therefore, in wireless CDMA-systems as in IS-95, cdma2000, WCDMA and IEEE 802.11 user are asynchronous

• Code classification:
  – Orthogonal, as Walsh-codes for orthogonal or near-orthogonal systems
  – Near-orthogonal and non-orthogonal codes:
    • Gold-codes, for asynchronous systems
    • Maximal length codes for asynchronous systems
Capacity of a cellular CDMA system

- Consider uplink (MS->BS)
- Each user transmits Gaussian noise (SS-signal) whose deterministic characteristics are stored in RX and TX
- Reception and transmission are simple multiplications
- Perfect power control: each user’s power at the BS the same
- Each user receives multiple copies of power $P_r$ that is other user’s interference power, therefore each user receives the interference power

$$I_k = (U - 1)P_r$$

where $U$ is the number of equal power users
Capacity of a cellular CDMA system (cont.)

• Each user applies a demodulator/decoder characterized by a certain *reception sensitivity* $E_b/I_0 \ (3 \text{-} 9 \text{ dB depending on channel coding, channel, modulation method etc.})$

• Each user is exposed to the *interference power density* (assumed to be produced by other users only) where $B_T$ is the spreading (and RX) bandwidth

• Received signal energy / bit at the signaling rate $R$ is
  \[ E_b = \frac{P_r}{R} \]
  \[ [J] = [W][s] \]

• Combining (1)-(3) yields the number of users
  \[ I_k = (U - 1)P_r \Rightarrow \]
  \[ U - 1 = \frac{I_k}{P_r} = \frac{I_0B_T}{E_bR} = \frac{(1/R)B_T}{E_b(1/I_0)} = E_b/I_0 \]

• This can still be increased by using *voice activity coefficient* $G_v = 2.67$ (only about 37% of speech time effectively used), *directional antennas*, for instance for a 3-way antenna $G_A = 2.5$. 
Capacity of a cellular CDMA system (cont.)

• In cellular system neighboring cells introduce interference that decreases capacity. It has been found out experimentally that this reduces the number of users by the factor

$$1 + f \approx 1.6$$

• Hence asynchronous CDMA system capacity can be approximated by

$$U = \frac{W/R}{E_b/I_o} \frac{G_v G_A}{1 + f}$$

yielding with the given values \(G_v = 2.67\), \(G_A = 2.4\), \(1 + f = 1.6\),

$$U = \frac{4W/R}{E_b/I_o}$$

• Assuming efficient error correction algorithms, dual diversity antennas, and RAKE receiver, it is possible to obtain \(E_b/I_o = 6 \text{ dB} = 4\), and then

$$U \approx \frac{W}{R}$$