UNIT I – BASIC CIRCUIT CONCEPTS

– Circuit elements
– Kirchhoff's Law
– V-I Relationship of R,L and C
– Independent and Dependent sources
– Simple Resistive circuits
– Networks reduction
– Voltage division
– current source transformation.
- Analysis of circuit using mesh current and nodal voltage methods.

Resistance

- All materials resist the flow of current
- Resistance is usually represented by the variable R
- Depends on geometry and resistivity of the material
  - Copper 1.673e-8 ohm-meters
  - Lead 20.648e-8 ohm-meters
- Ohms per square for sheet.

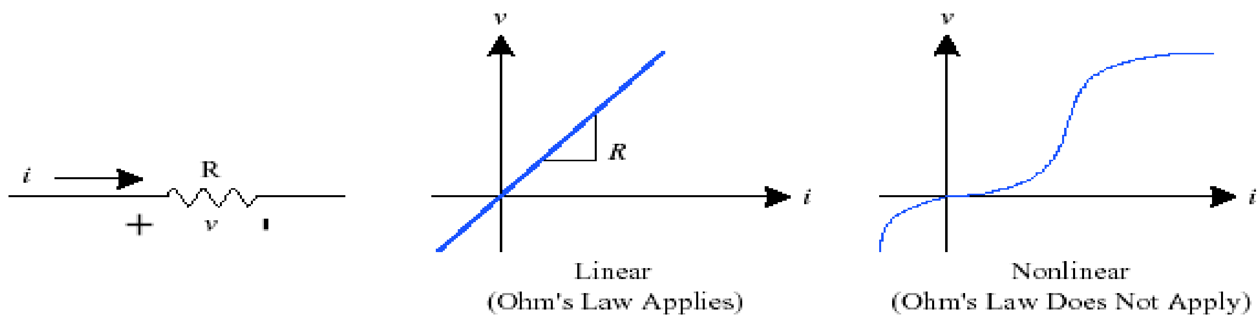
A cylinder of length ℓ and cross-sectional area A has a resistance:

$$R = \rho \frac{\ell}{A}$$

where

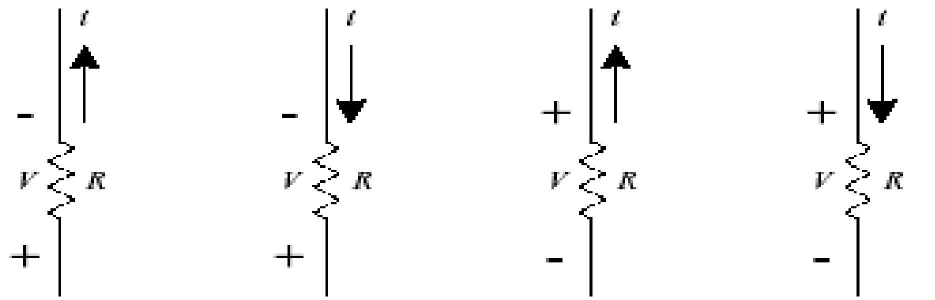
$R =$ resistance of an element in ohms (Ω)
$\rho =$ resistivity of the material in ohm-meters
$\ell =$ length of cylindrical material in meters
$A =$ Cross sectional area of material in meters^2

Ohm's Law



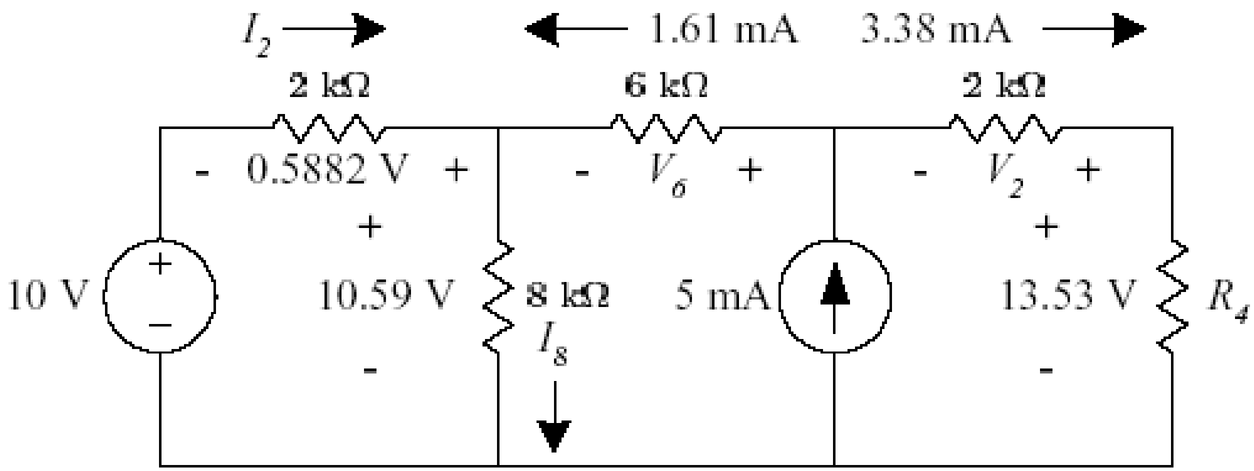
- As with all circuit elements, we need to know how the current through and voltage across the device are related
- Many materials have a complicated nonlinear relationship (including light bulbs): $v = \pm f(i)$
- Materials with a linear relationship satisfy Ohm's law: $v = \pm mi$
- The slope, m, is equal to the resistance of the element
- Ohm's Law: $v = \pm iR$
- Sign, $\pm$, is determined by the passive sign convention (PSC)

Resistors & Passive Sign Convention



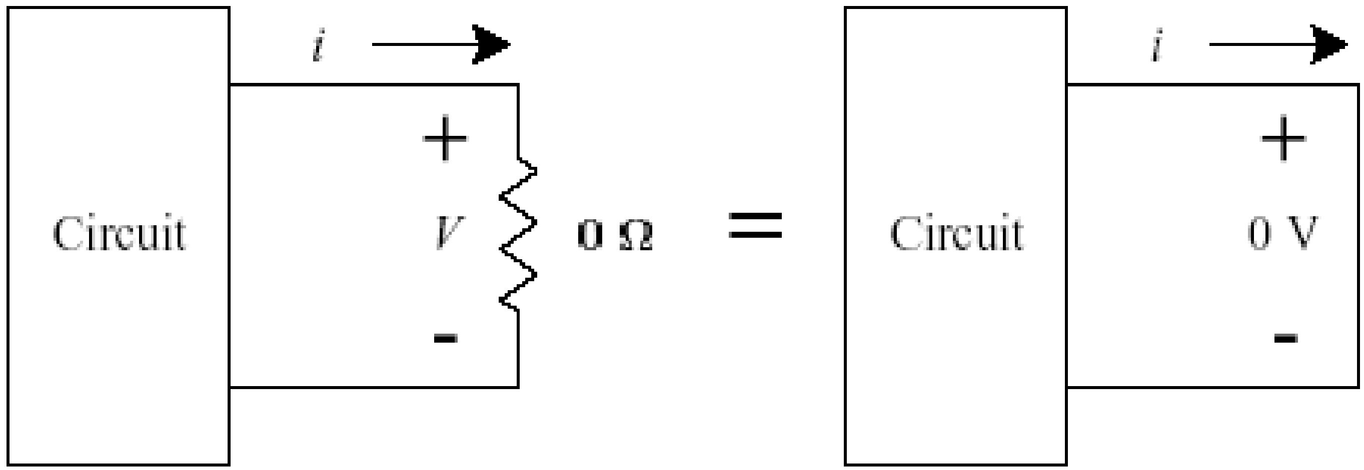
- Recall that relationships between current and voltage are sign sensitive
- Passive Sign Convention: Current enters the positive terminal of an element
  - If PSC satisfied: $v = iR$
  - If PSC not satisfied: $v = -iR$

Example: Ohm's Law



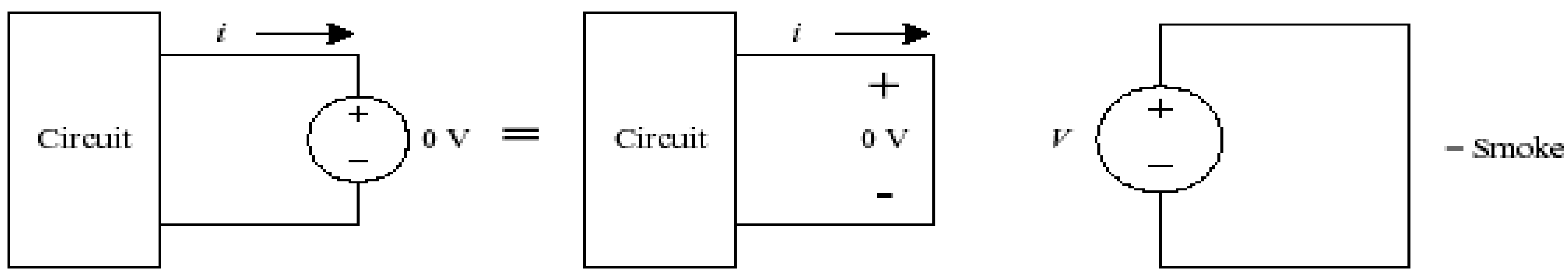
$$
\begin{aligned}
I_2 &= \\
V_6 &= \\
R_4 &= \\
V_2 &= \\
I_8 &=
\end{aligned}
$$

Short Circuit as Zero Resistance



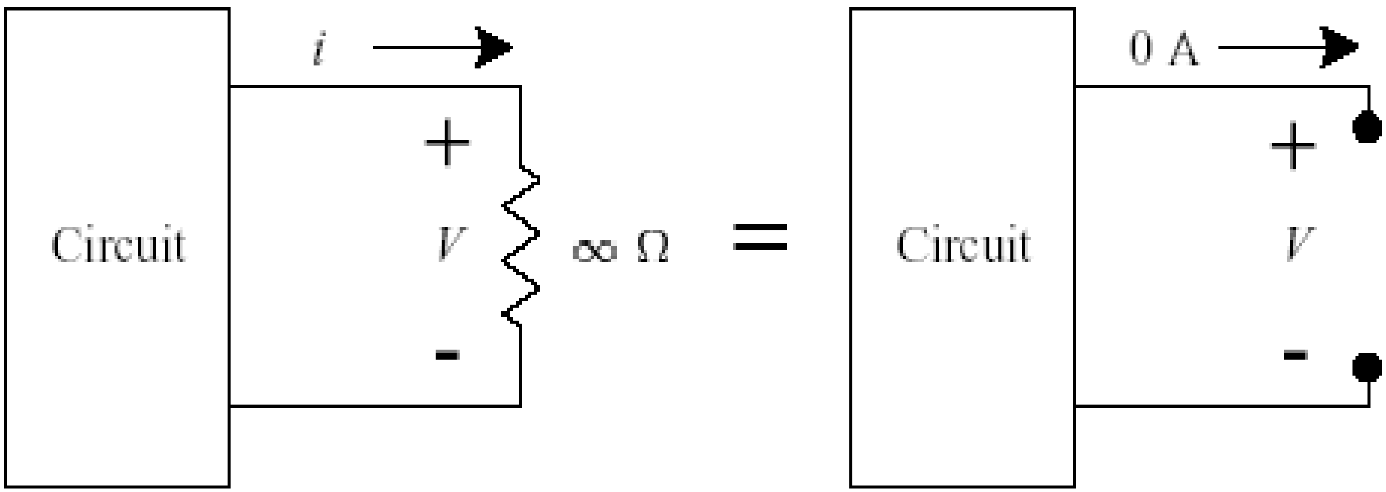
- An element (or wire) with $R = 0$ is called a short circuit
- Just drawn as a wire (line)

Short Circuit as Voltage Source (0V)



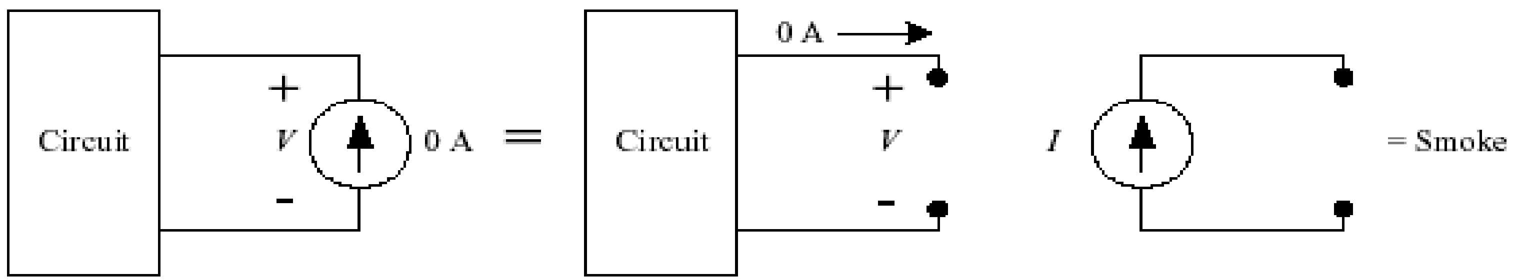
- An ideal voltage source $V_s = 0$ V is also equivalent to a short circuit
- Since $v = iR$ and $R = 0$, $v = 0$ regardless of i
- Could draw a source with $V_s = 0$ V, but is not done in practice
- Can not connect a voltage source to a short circuit
- Irresistible force meets immovable object
- In practice, the wire usually wins and the voltage source melts (if not protected)

Open Circuit



- An element (or wire) with $R = \infty$ is called a open circuit
- Just omitted

Open Circuit as Current Source (0 A)



- An ideal current source $I = 0$ A is also equivalent to an open circuit
- Could draw a source with $I = 0$ A, but is not done in practice
- Cannot connect a current source to an open circuit (spark coil)
- Irresistible force meets immovable object
- In practice, you blow the current source (if not protected)
- The insulator (air) usually wins. Else, sparks fly.

Conductance

- Sometimes conductance is specified instead of resistance
- Inverse of resistance
- $G = \frac{1}{R} = \frac{i}{v}$
- Units: siemens (S) or mhos (℧)
- 1 S = 1 ℧ = 1 A/V
- In words: the ability of an element to conduct electric current

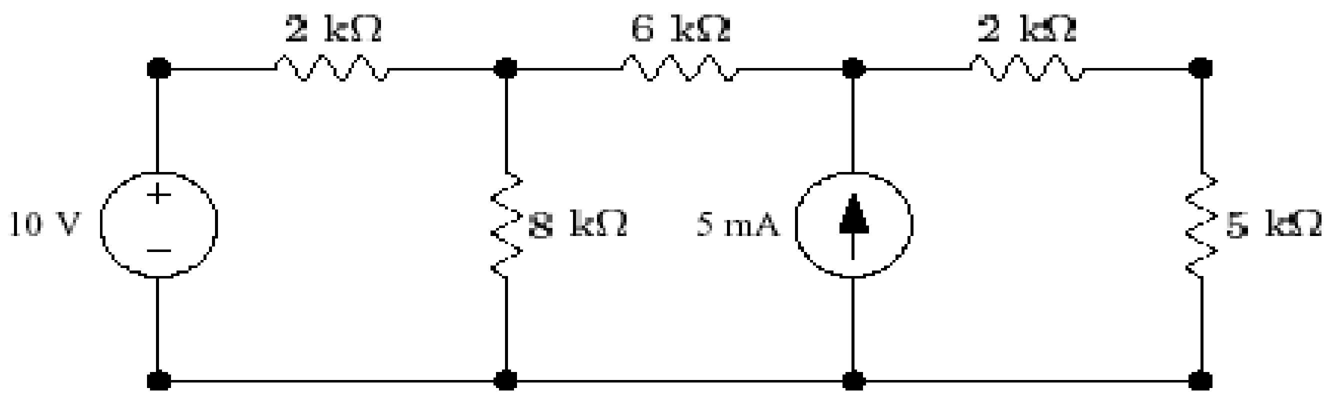
$$v = Ri \qquad p = vi = i^2 R = \frac{v^2}{R}$$

$$i = Gv \qquad p = vi = v^2 G = \frac{i^2}{G}$$

Circuit Building Blocks

- Before we can begin analysis, we need a common language and framework for describing circuits
- For this course, networks and circuits are the same
- Networks are composed of nodes, branches, and loops

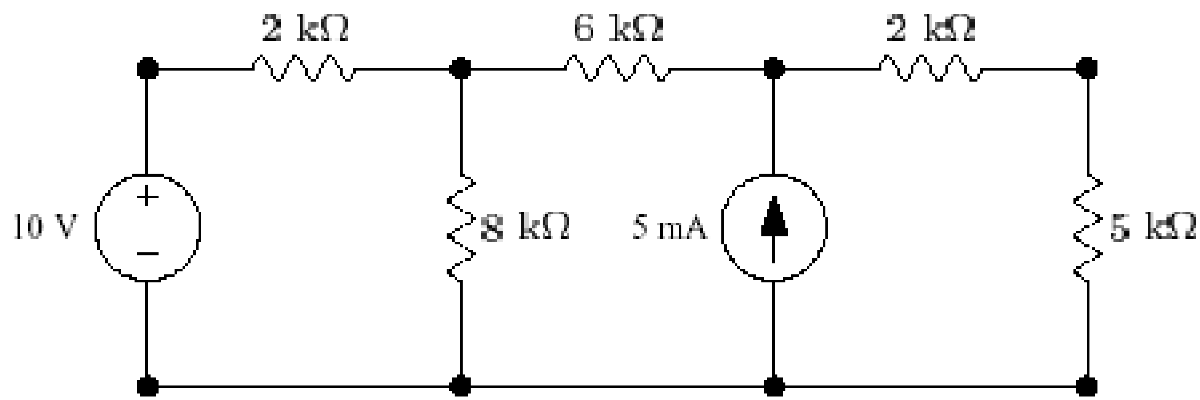
Branches



Example: How many branches?

- Branch: a single two-terminal element in a circuit
- Segments of wire are not counted as elements
- Examples: voltage source, resistor, current source

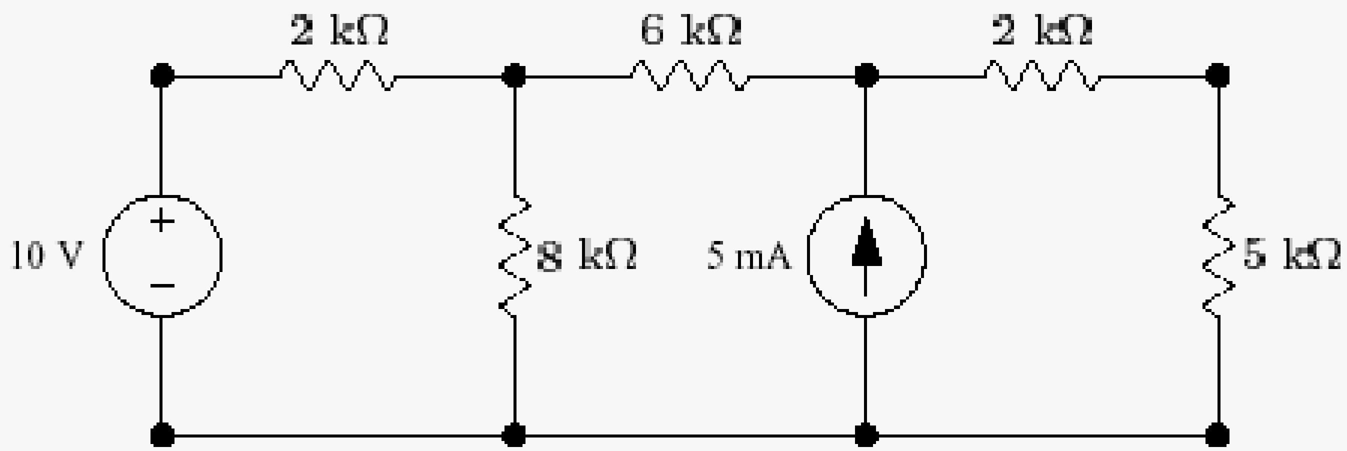
Nodes



Example: How many nodes? How many essential nodes?

- Node: the point of connection between two or more branches
- May include a portion of the circuit (more than a single point)
- Essential Node: the point of connection between three or more branches

Loops



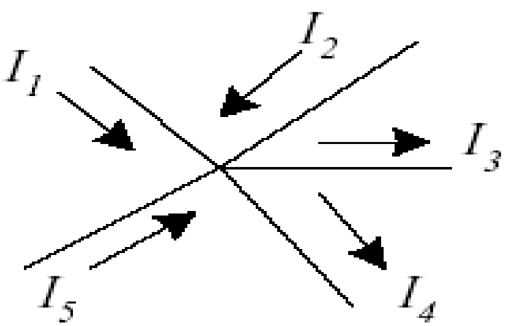
Example: How many loops?

- Loop: any closed path in a circuit

Overview of Kirchhoff’s Laws

- The foundation of circuit analysis is
  - The defining equations for circuit elements (e.g. Ohm’s law)
  - Kirchhoff’s current law (KCL)
  - Kirchhoff’s voltage law (KVL)
- The defining equations tell us how the voltage and current within a circuit element are related
- Kirchhoff’s laws tell us how the voltages and currents in different branches are related

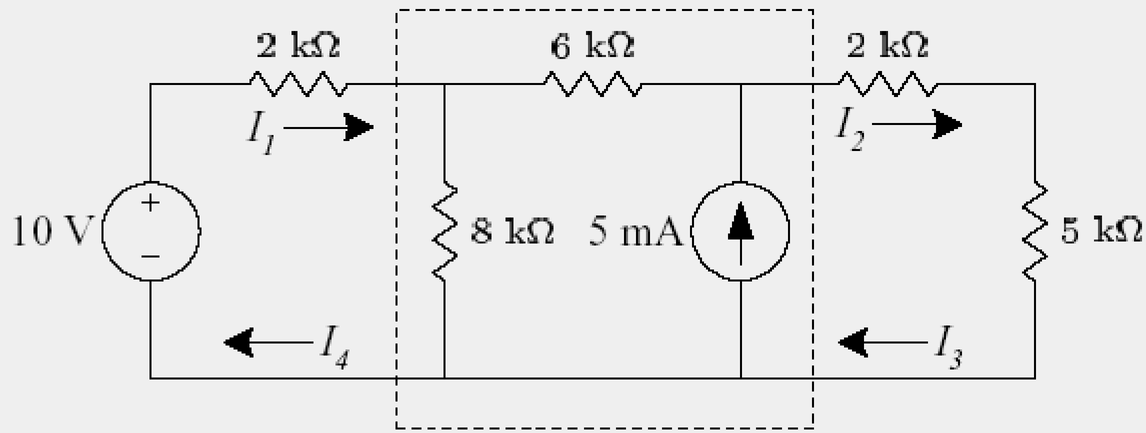
Kirchhoff's Current Law



$$\begin{aligned} I_1 + I_2 - I_3 - I_4 + I_5 &= 0 \\ I_1 + I_2 + I_5 &= I_3 + I_4 \end{aligned}$$

- Kirchhoff's Current Law (KCL): the algebraic sum of currents entering a node (or a closed boundary) is zero
- The sum of currents entering a node is equal to the sum of the currents leaving a node
- Common sense:
  - All of the electrons have to go somewhere
  - The current that goes in, has to come out some place
- Based on law of conservation of charge and $\nabla j = \frac{\partial \rho}{\partial t} = 0$

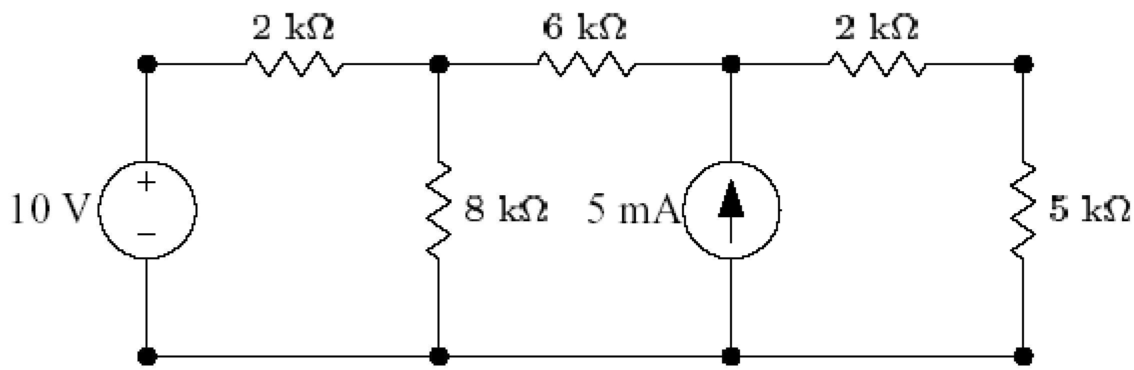
Kirchhoff's Current Law for Boundaries



$$
\begin{aligned}
I_1 - I_2 + I_3 - I_4 &= 0 \\
I_1 + I_3 &= I_2 + I_4
\end{aligned}
$$

- KCL also applies to closed boundaries for *all* circuits

KCL - Example



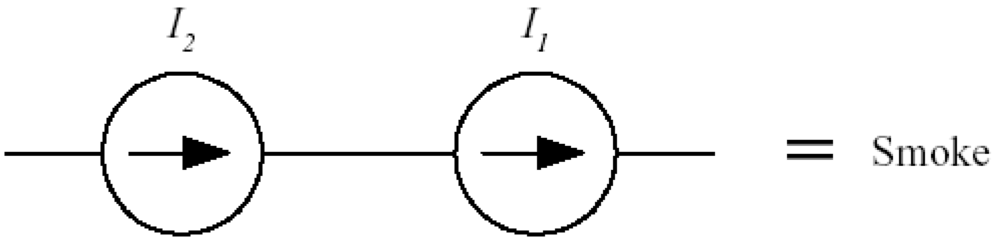
Apply KCL to each essential node in the circuit.

Essential Node 1:

Essential Node 2:

Essential Node 3:

Ideal Current Sources: Series



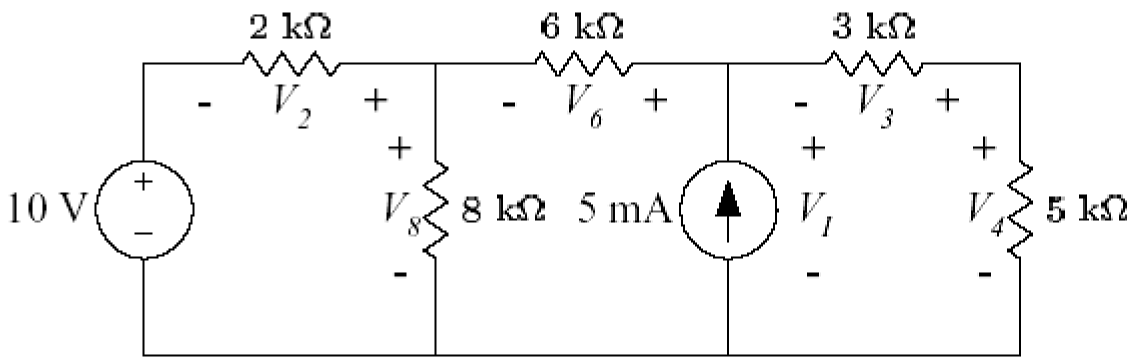
- Ideal current sources *cannot* be connected in series
- Recall: ideal current sources guarantee the current flowing through source is at specified value
- Recall: the current entering a circuit element must equal the current leaving a circuit element, $I_{in} = I_{out}$
- Could easily cause component failure (smoke)
- Ideal sources do not exist
- Technically allowed if $I_1 = I_2$, but is a bad idea

Kirchhoff's Voltage Law - KVL

$$\sum_{m=1}^{M} V_m = 0$$

- Kirchhoff's Voltage Law (KVL): the algebraic sum of voltages around a closed path (ie loop) is zero

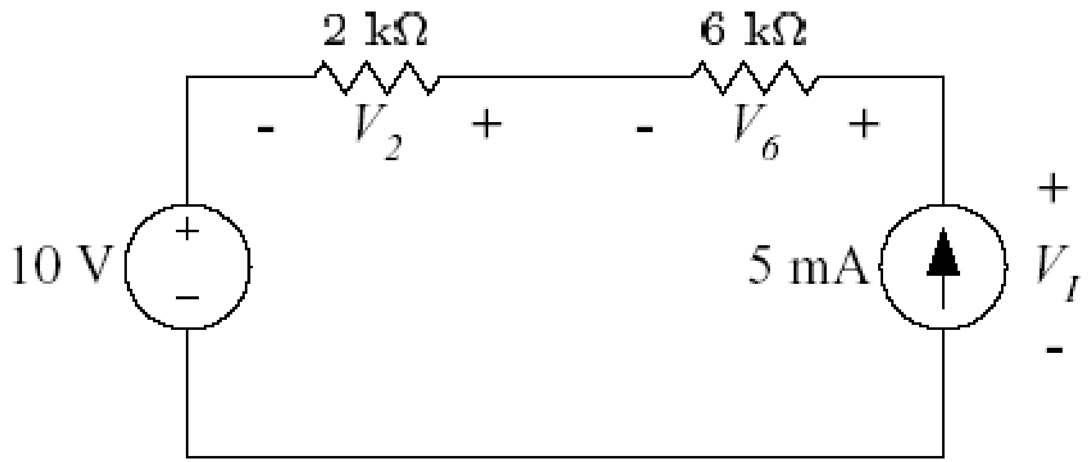
KVL - Example



Apply KVL to each loop in the circuit.

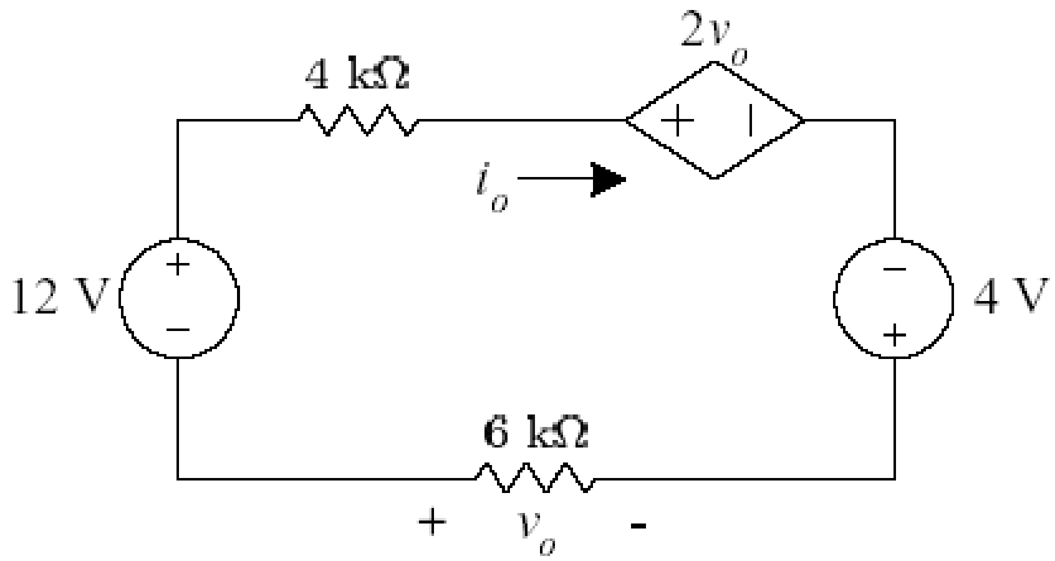
Loop 1:
Loop 2:
Loop 3:
Loop 4:
Loop 5:
Loop 6:

Example – Applying the Basic Laws



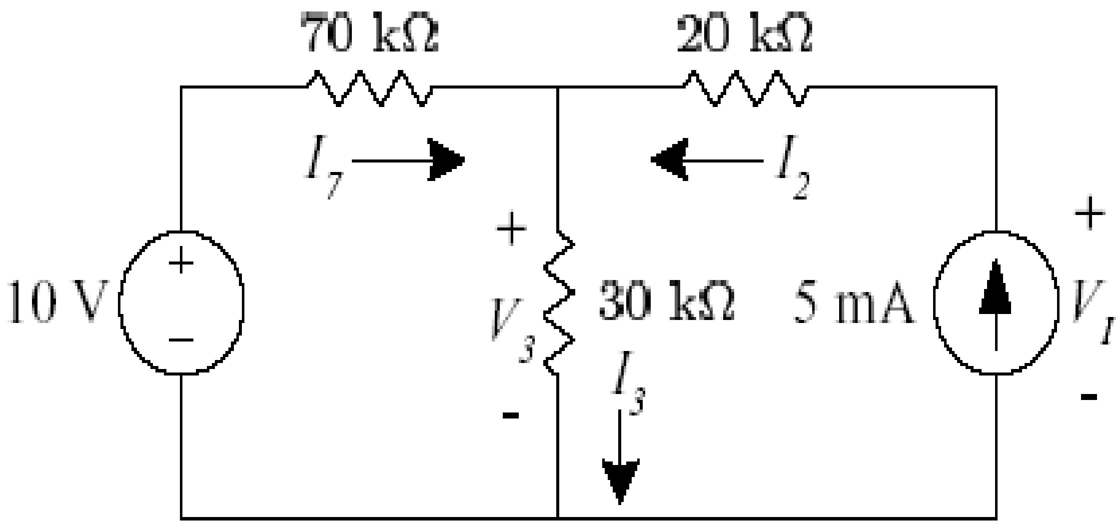
Find V_2, V_6 and V_I.

Example – Applying the Basic Laws



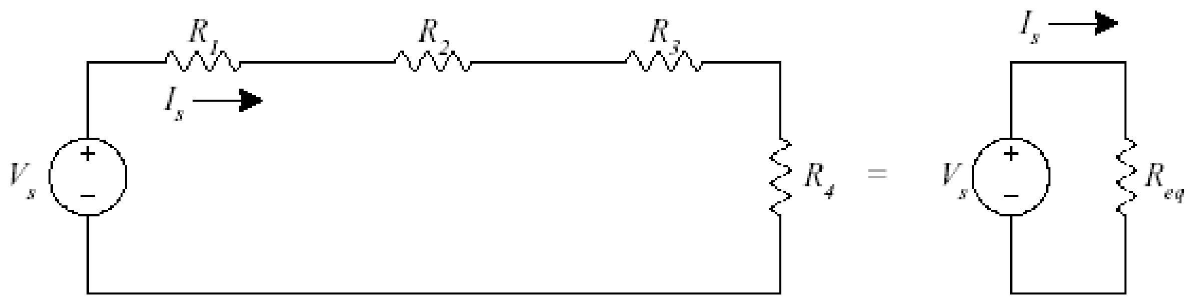
Find i_o and v_o.

Example – Applying the Basic Laws



Find I_7, I_3, I_2, V_3, and V_I.

Resistors in Series

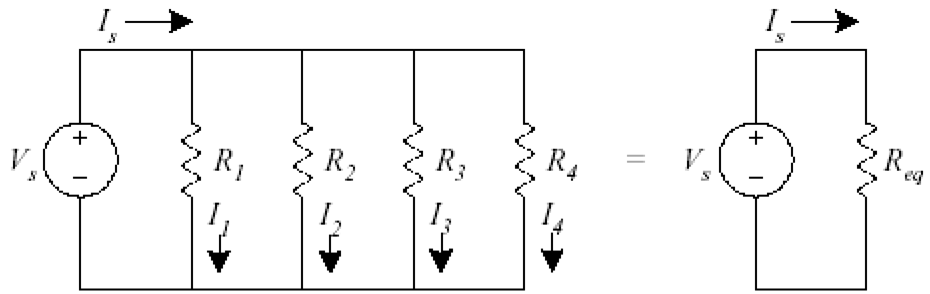


By KVL

$$\begin{aligned} V_s &= R_1 I_s + R_2 I_s + R_3 I_s + R_4 I_s \\ &= I_s (R_1 + R_2 + R_3 + R_4) \\ &= R_{eq} I_s \\ R_{eq} &= R_1 + R_2 + R_2 + R_4 \end{aligned}$$

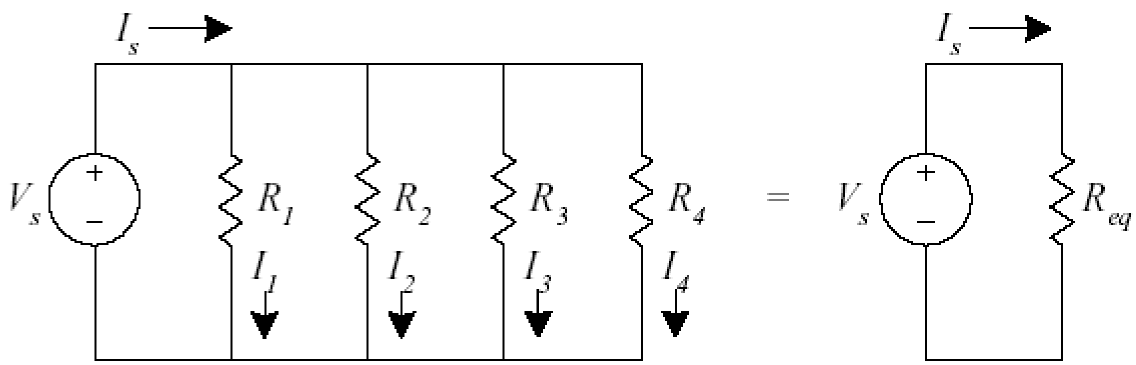
- Resistors in series add

Resistors in Parallel



$$
\begin{aligned}
I_s &= I_1 + I_2 + I_3 + I_4 \\
&= V_s/R_1 + V_s/R_2 + V_s/R_3 + V_s/R_4 \\
&= V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) \\
&= \frac{V_s}{R_{eq}} \\
\frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\
R_{eq} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}
\end{aligned}
$$

Resistors in Parallel

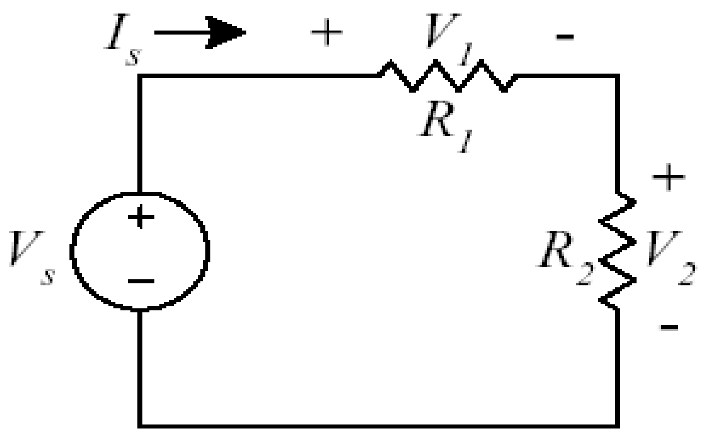


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$G_{eq} = G_1 + G_2 + G_3 + G_4$$

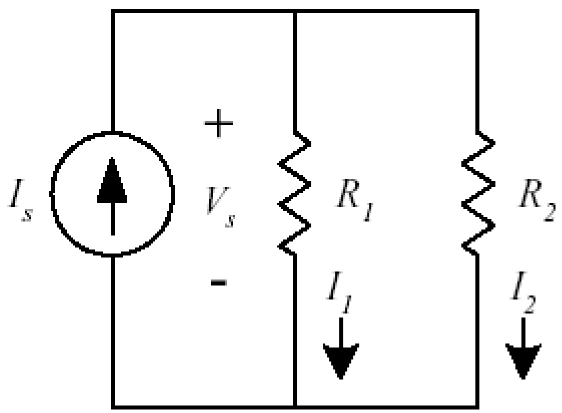
- Resistors in parallel have a more complicated relationship
- Easier to express in terms of conductance
- For two resistors: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Voltage Divider



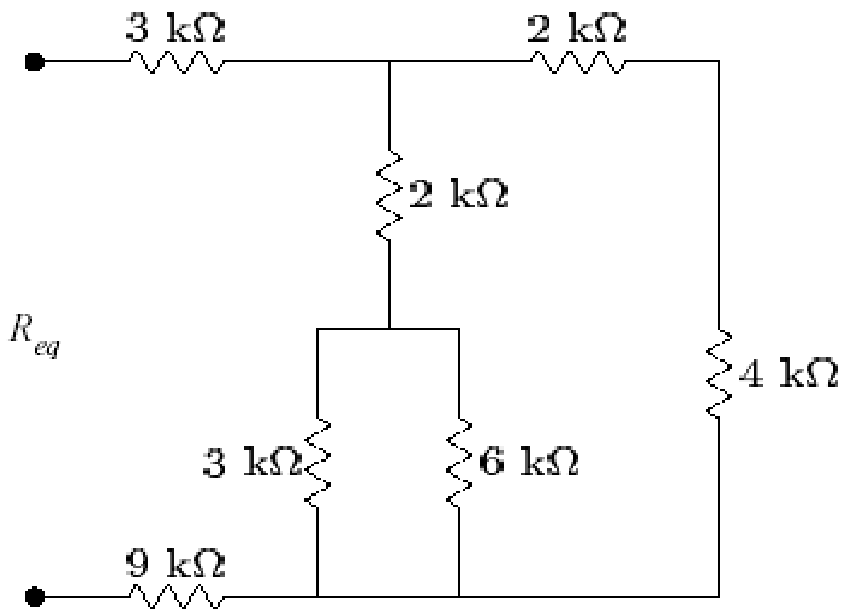
$$
\begin{aligned}
R_{eq} &= R_1 + R_2 \\
I_s &= \frac{V_s}{R_{eq}} \\
&= \frac{V_s}{R_1 + R_2} \\
V_2 &= I_s R_2 \\
&= V_s \frac{R_2}{R_1 + R_2} \\
V_1 &= I_s R_1 \\
&= V_s \frac{R_1}{R_1 + R_2}
\end{aligned}
$$

Current Divider



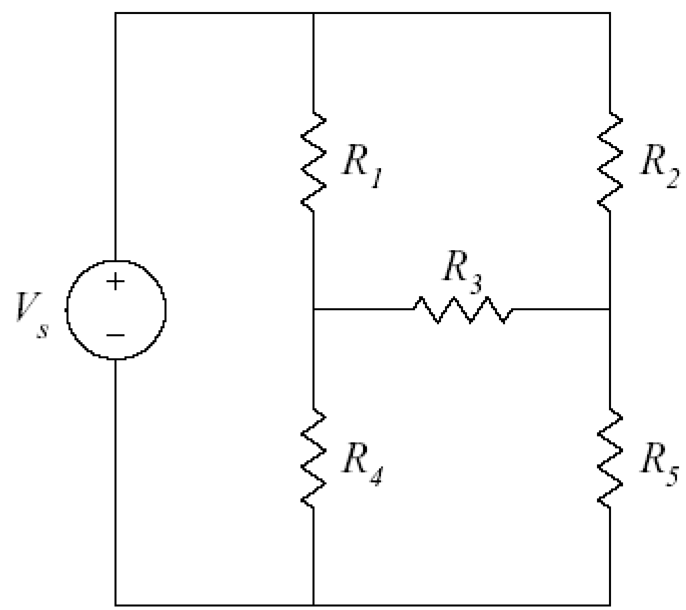
$$
\begin{aligned}
V_s &= I_s \frac{R_1 R_2}{R_1 + R_2} \\
I_2 &= \frac{V_s}{R_2} \\
&= I_s \frac{R_1}{R_1 + R_2} \\
I_1 &= \frac{V_s}{R_1} \\
&= I_s \frac{R_2}{R_1 + R_2}
\end{aligned}
$$

Resistor Network



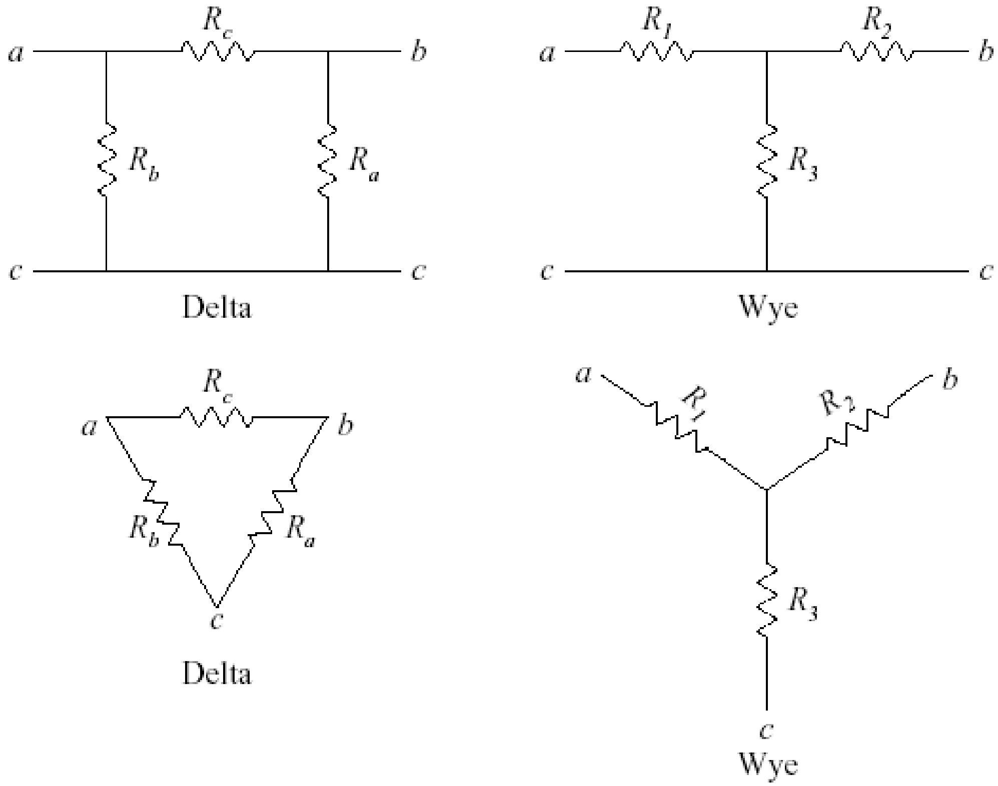
Find R_{eq}.

Resistor Network - Comments



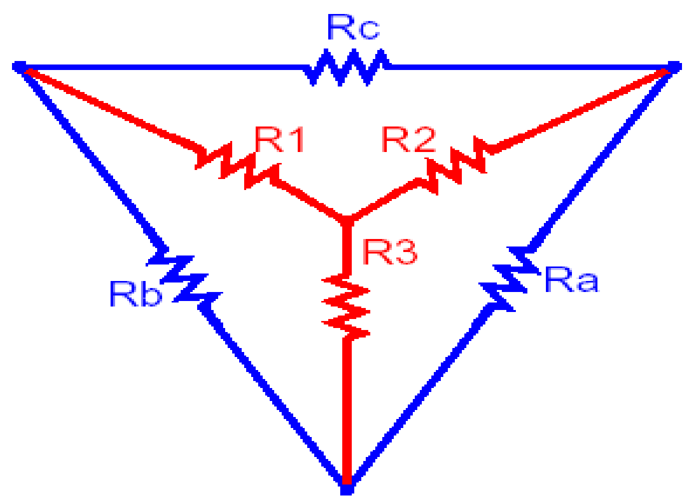
- Knowing the equivalent and parallel equivalents of resistors is not quite adequate
- There are some configurations that require one more tool

Delta ←→ Wye Transformations



- Every Delta network is functionally equivalent to a Wye network (and vice versa).

Delta ←→ Wye Transformations



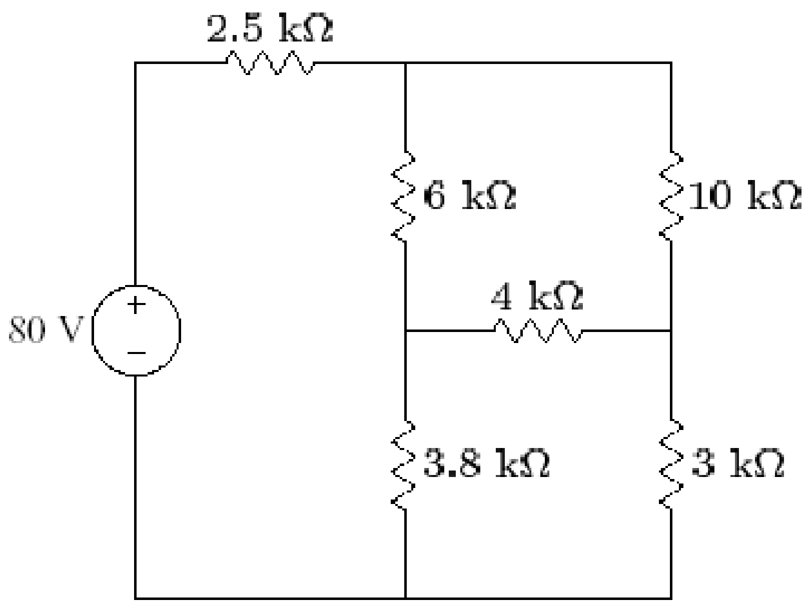
The following must be satisfied for equivalence

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_2 = \frac{R_cR_a}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Example – Delta ←→ Wye Transformations



Find R_{eq} and Power delivered by source

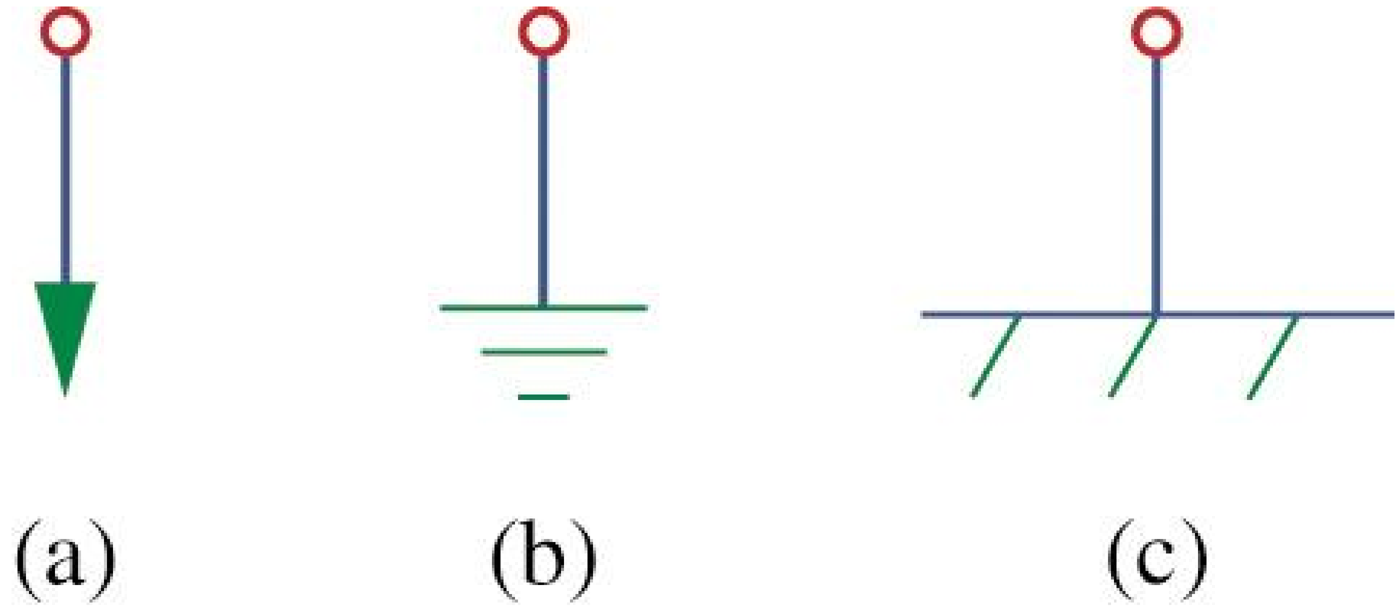
Methods of Analysis

- Introduction
- Nodal analysis
- Nodal analysis with voltage source
- Mesh analysis
- Mesh analysis with current source
- Nodal and mesh analyses by inspection
- Nodal versus mesh analysis

3.2 Nodal Analysis

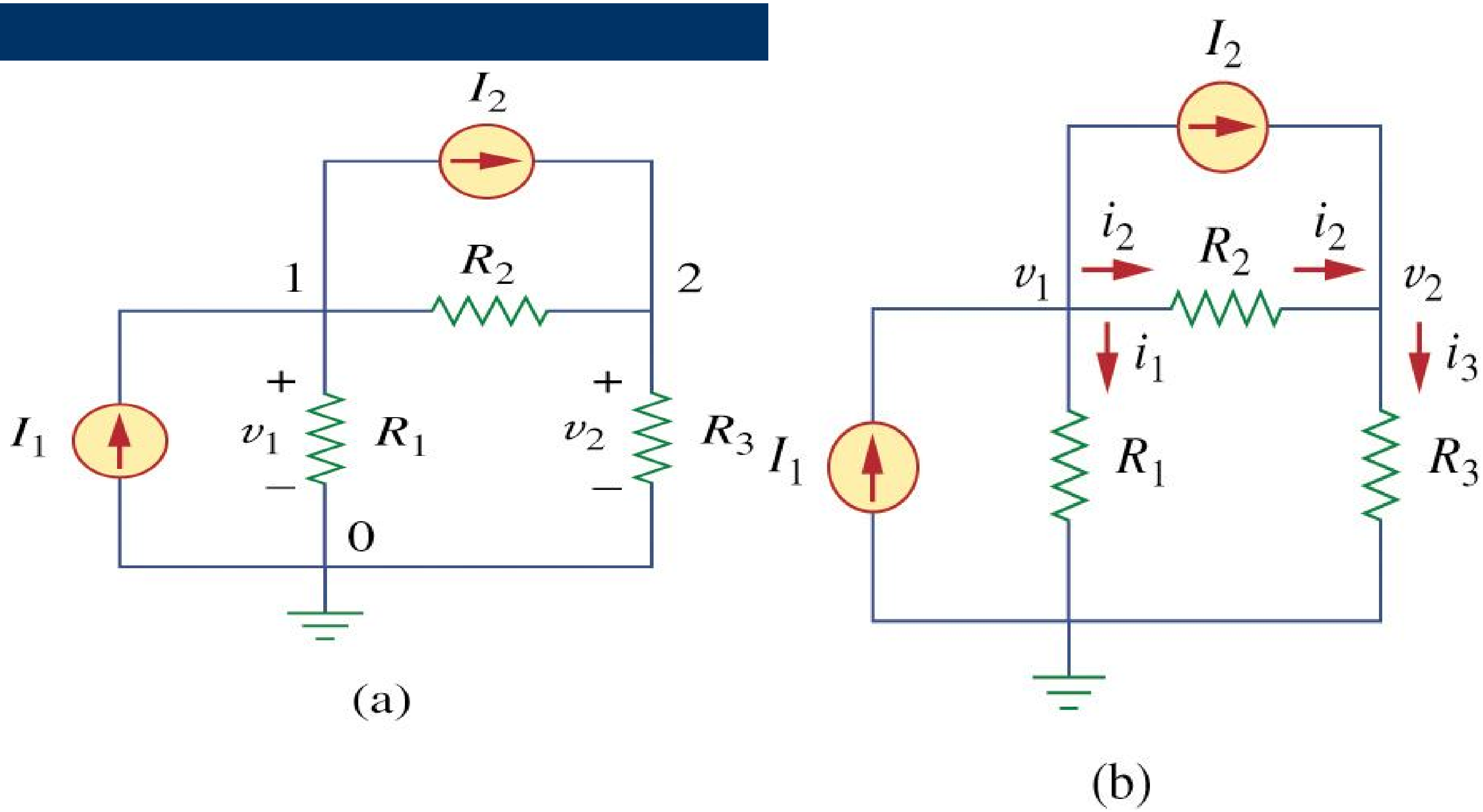
- Steps to Determine Node Voltages:
  1. Select a node as the reference node. Assign voltage v_1, v_2, …v_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
  2. Apply KCL to each of the n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
  3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Figure 3.1



Common symbols for indicating a reference node,
(a) common ground, (b) ground, (c) chassis.

Figure 3.2



Typical circuit for nodal analysis

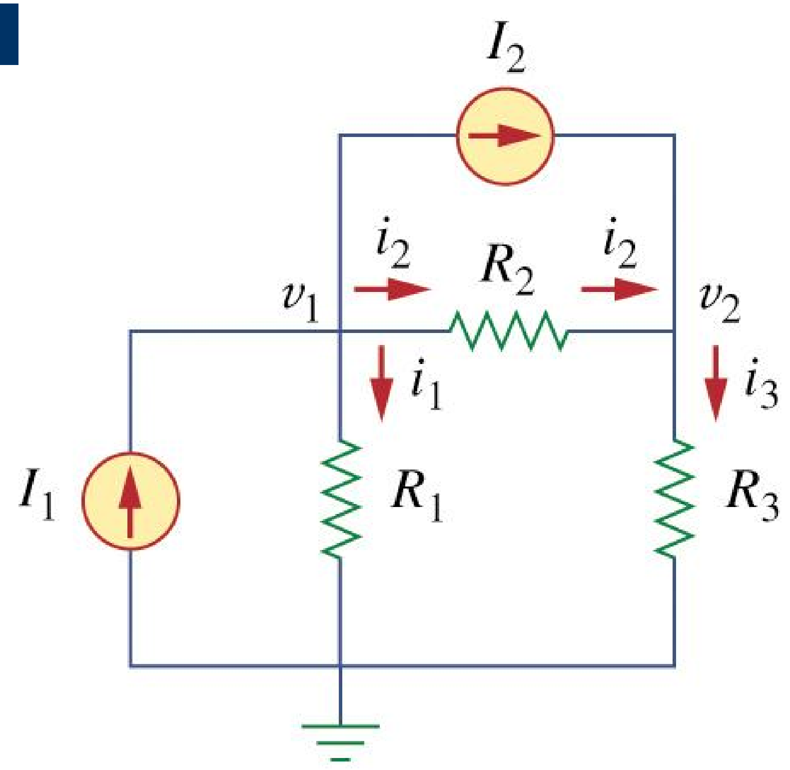
$$I_1 = I_2 + i_1 + i_2$$
$$I_2 + i_2 = i_3$$

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$



(b)

$$\Rightarrow I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

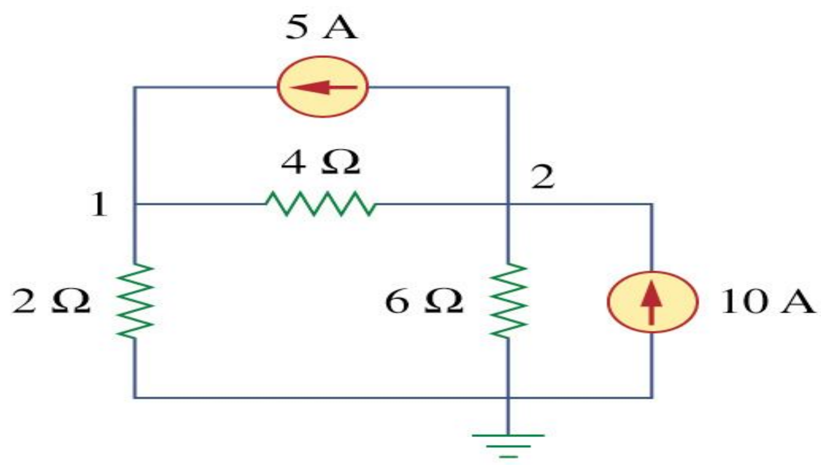
$$\Rightarrow I_1 - I_2 = G_1 v_1 + G_2 (v_1 - v_2)$$

$$I_2 = -G_2 (v_1 - v_2) + G_3 v_2$$

$$\Rightarrow \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Example 3.1

- Calculus the node voltage in the circuit shown in Fig. 3.3(a)



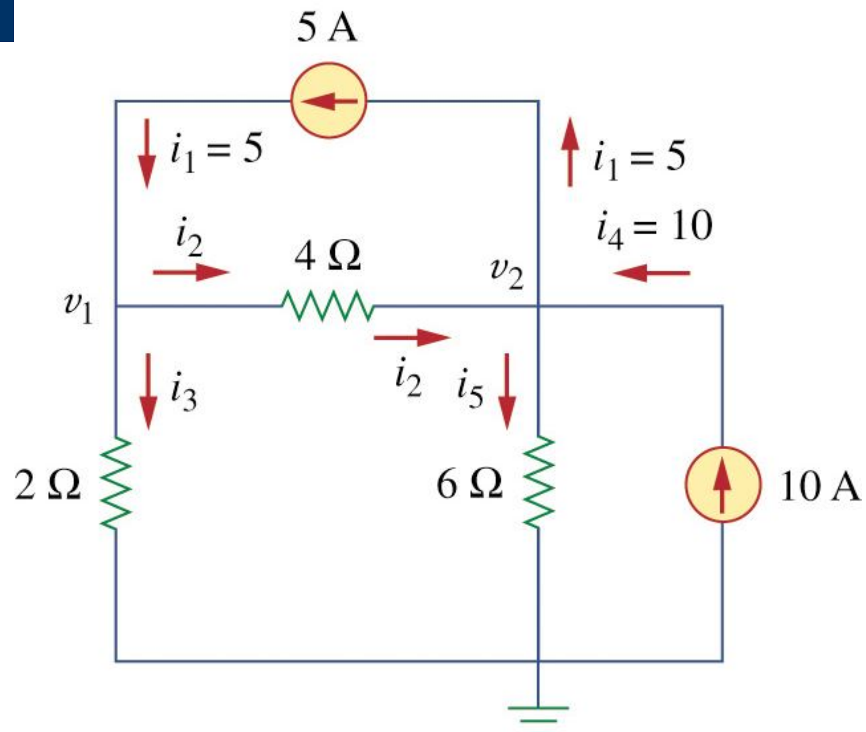
(a)

Example 3.1

- At node 1

$$i_1 = i_2 + i_3$$

$$\Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$



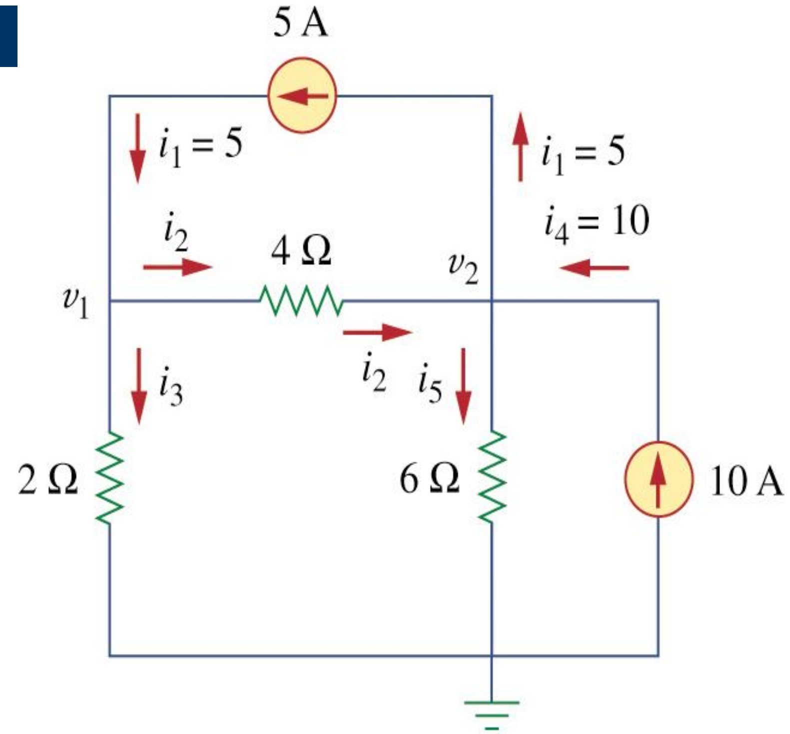
(b)

Example 3.1

- At node 2

$$i_2 + i_4 = i_1 + i_5$$

$$\Rightarrow 5 = \frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6}$$



(b)

Example 3.1

- In matrix form:

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{6}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

Practice Problem 3.1

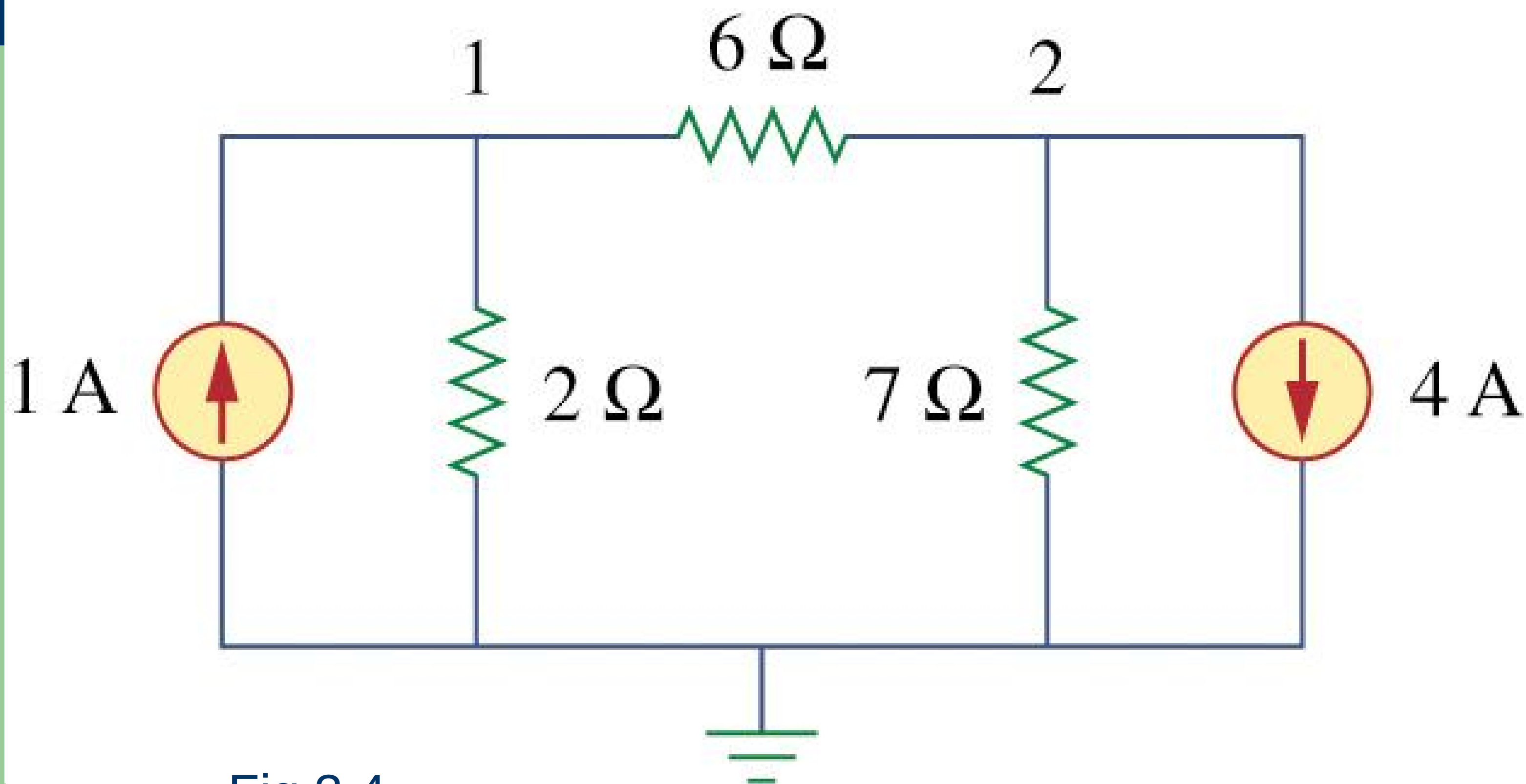
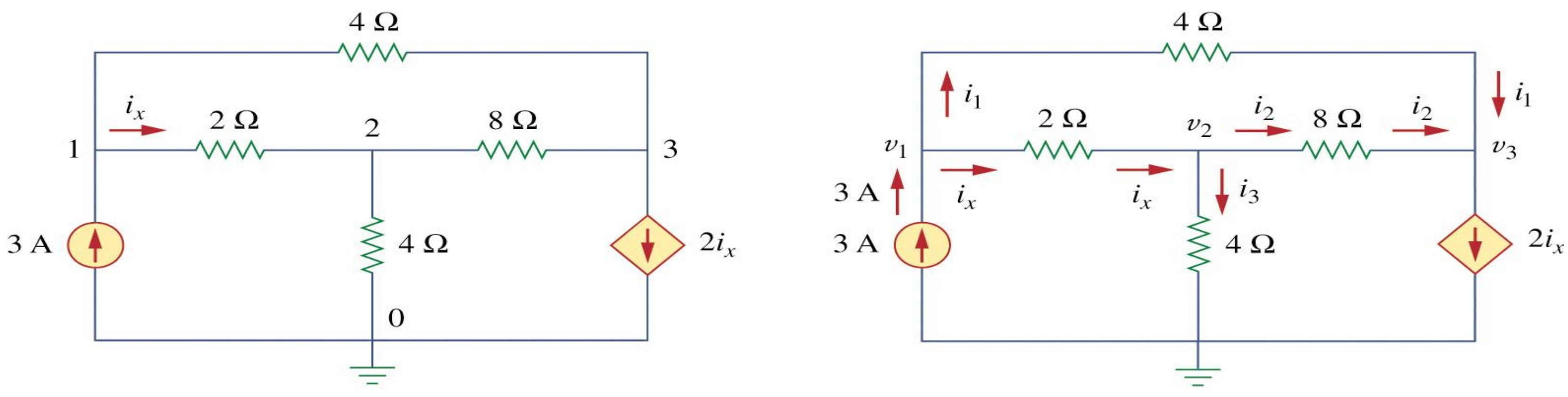


Fig 3.4

Example 3.2

- Determine the voltage at the nodes in Fig. 3.5(a)



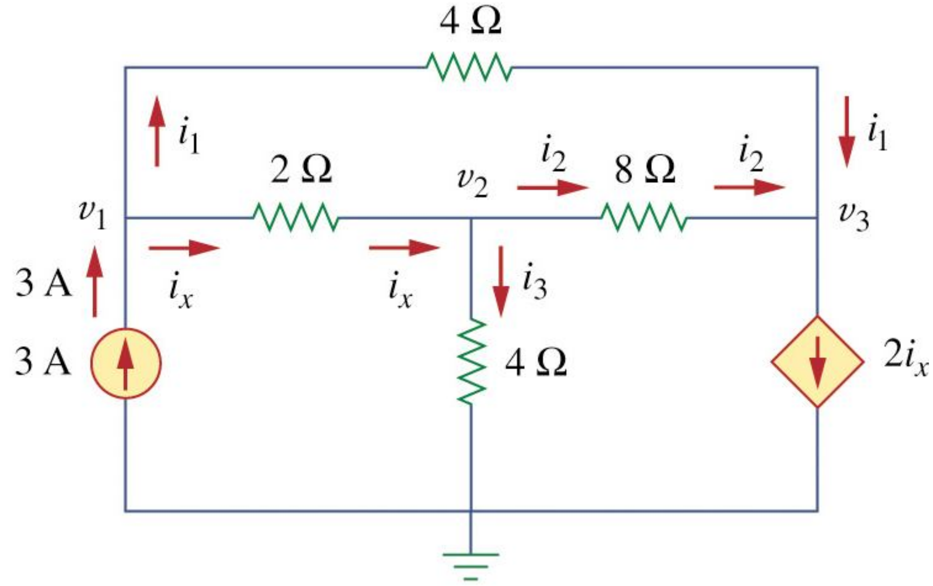
(a) (b)

Example 3.2

- At node 1,

$$3 = i_1 + i_x$$

$$\Rightarrow 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

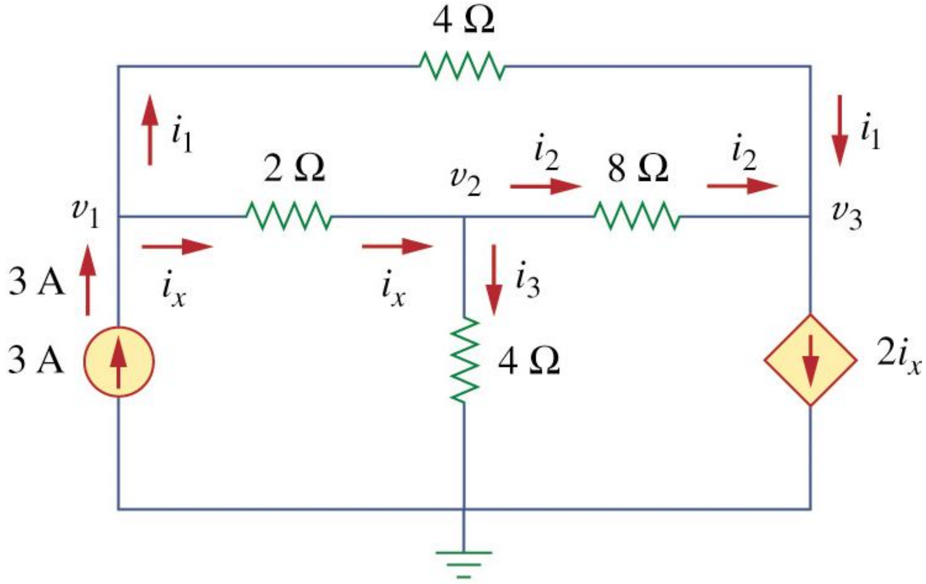


(b)

Example 3.2

- At node 2 $i_x = i_2 + i_3$

$$\Rightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$



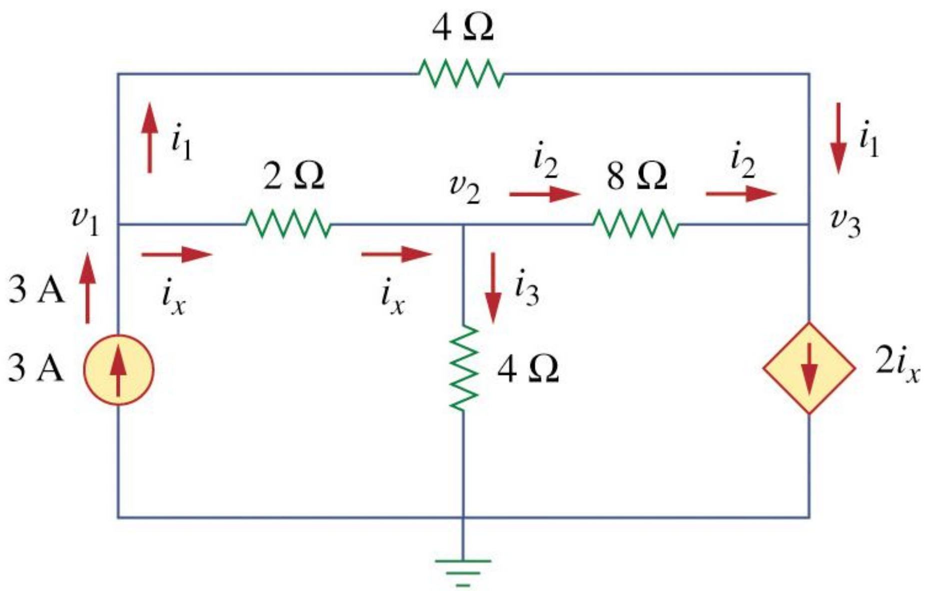
(b)

Example 3.2

- At node 3

$$i_1 + i_2 = 2i_x$$

$$\Rightarrow \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$



(b)

Example 3.2

- In matrix form:

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{7}{8} & -\frac{1}{8} \\ \frac{3}{4} & -\frac{9}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

3.3 Nodal Analysis with Voltage Sources

- Case 1: The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.
- Case 2: The voltage source is connected between two nonreferenced nodes: a generalized node (supernode) is formed.

3.3 Nodal Analysis with Voltage Sources

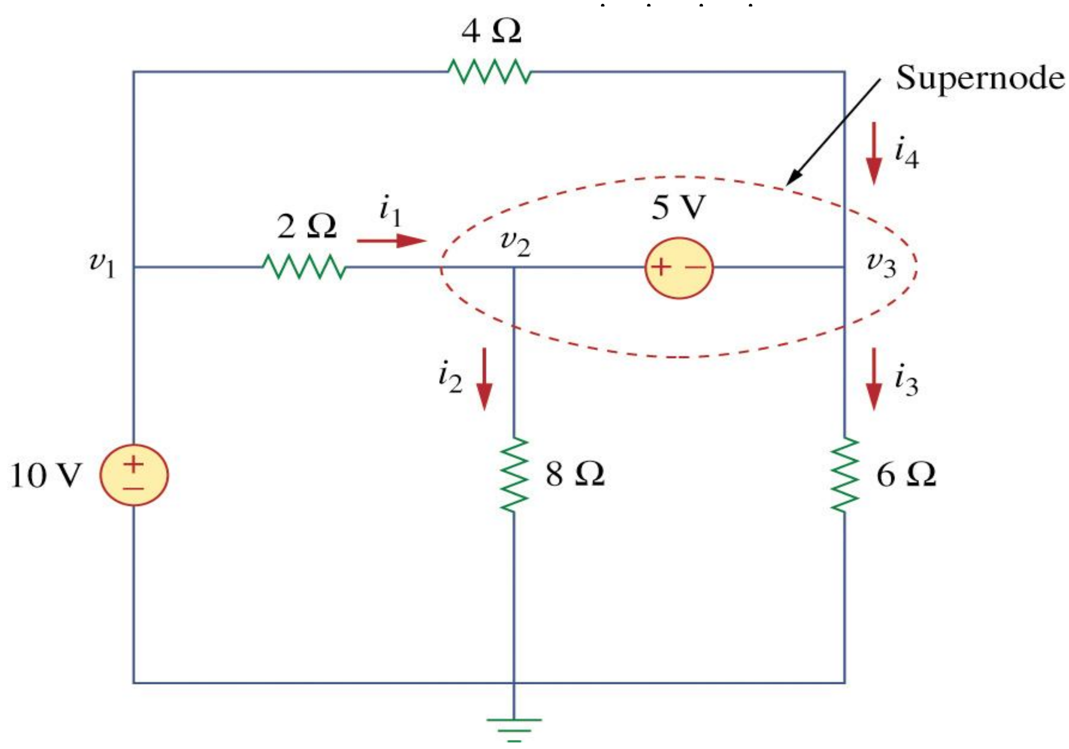
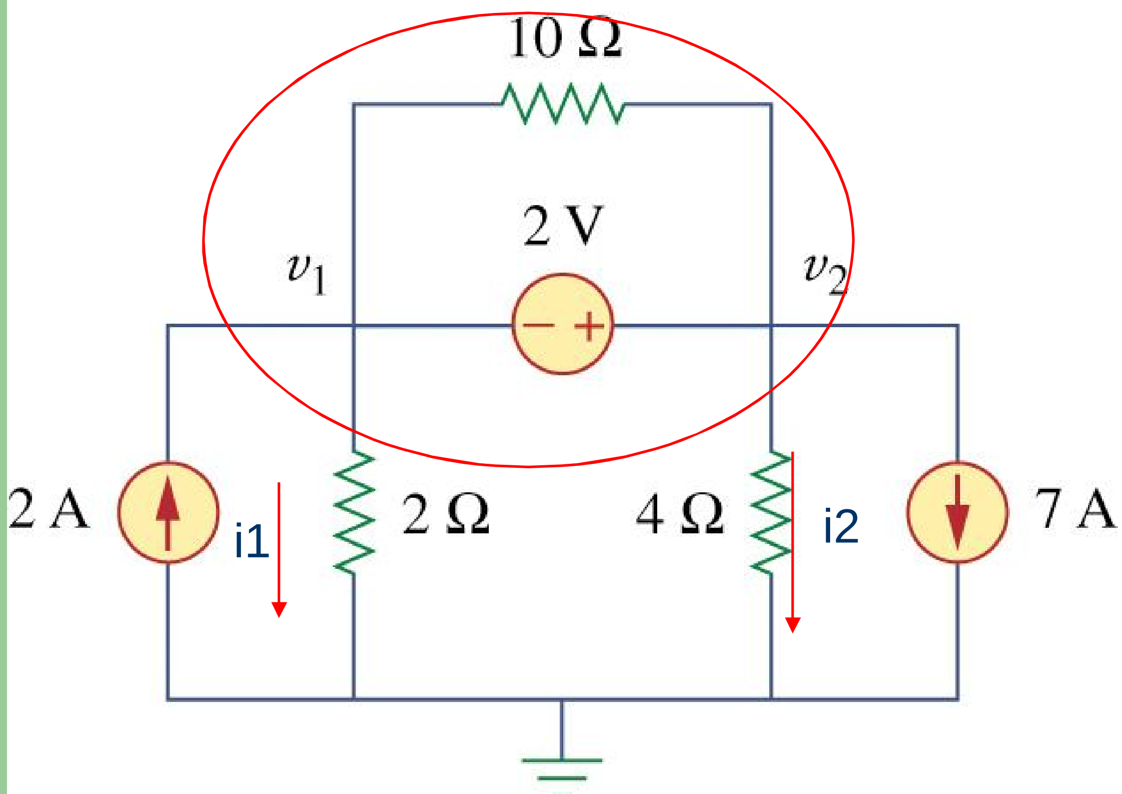


Fig. 3.7 A circuit with a supernode.

- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.
- The required two equations for regulating the two nonreference node voltages are obtained by the KCL of the supernode and the relationship of node voltages due to the voltage source.

Example 3.3

- For the circuit shown in Fig. 3.9, find the node [redacted]



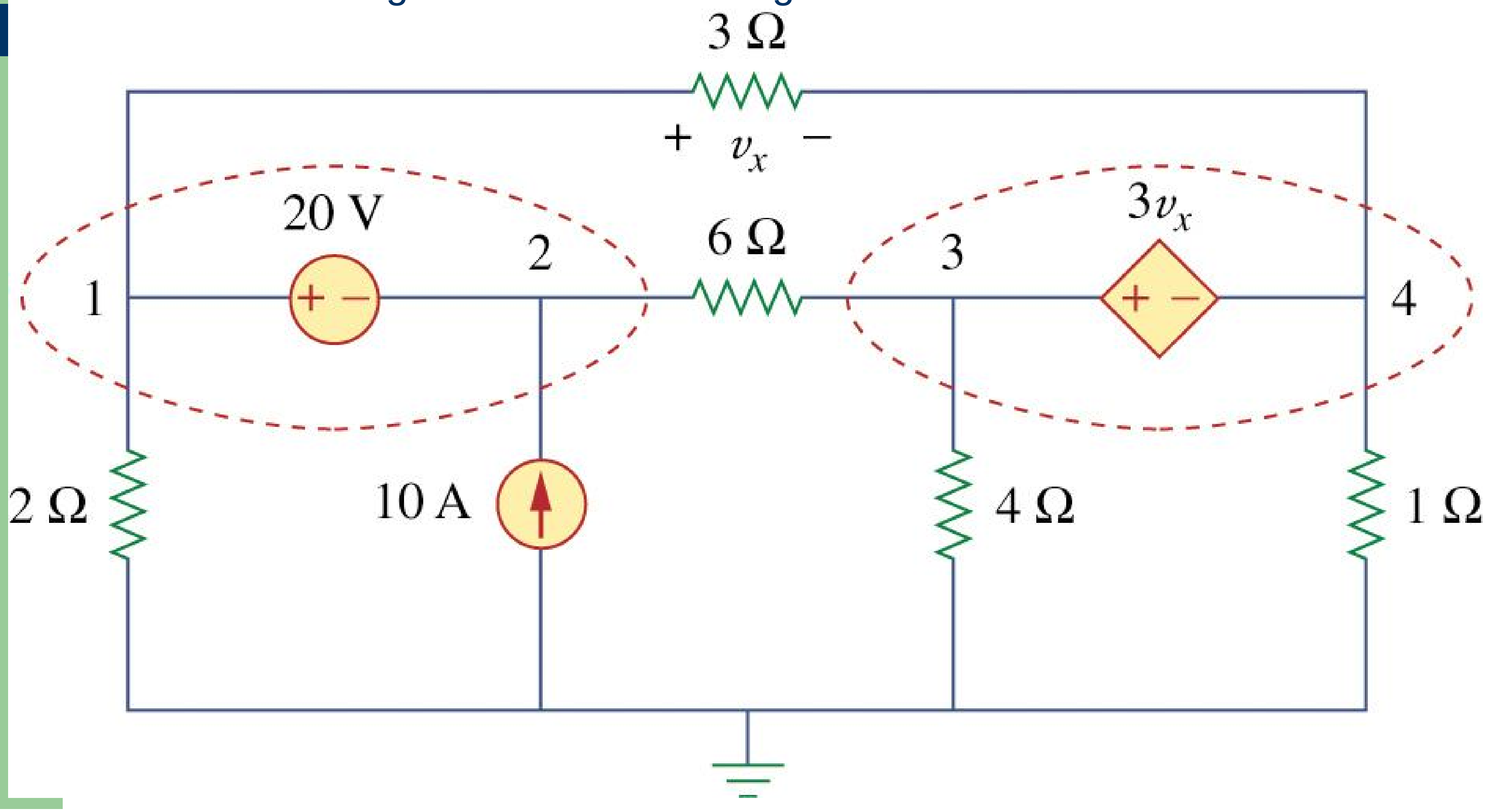
$$2-7-i1-i2=0$$

$$2-7-\frac{v_1}{2}-\frac{v_2}{4}=0$$

$$v_1-v_2=-2$$

Example 3.4

Find the node voltages in the circuit of Fig. 3.12.

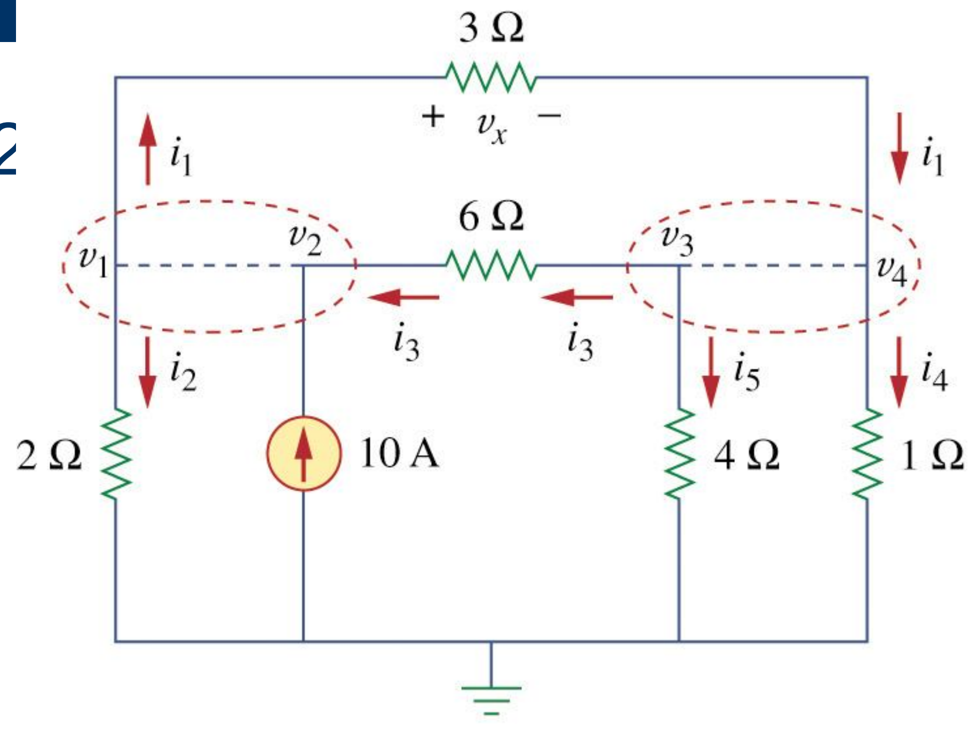


Example 3.4

- At suopernode 1-2

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

$$v_1 - v_2 = 20$$



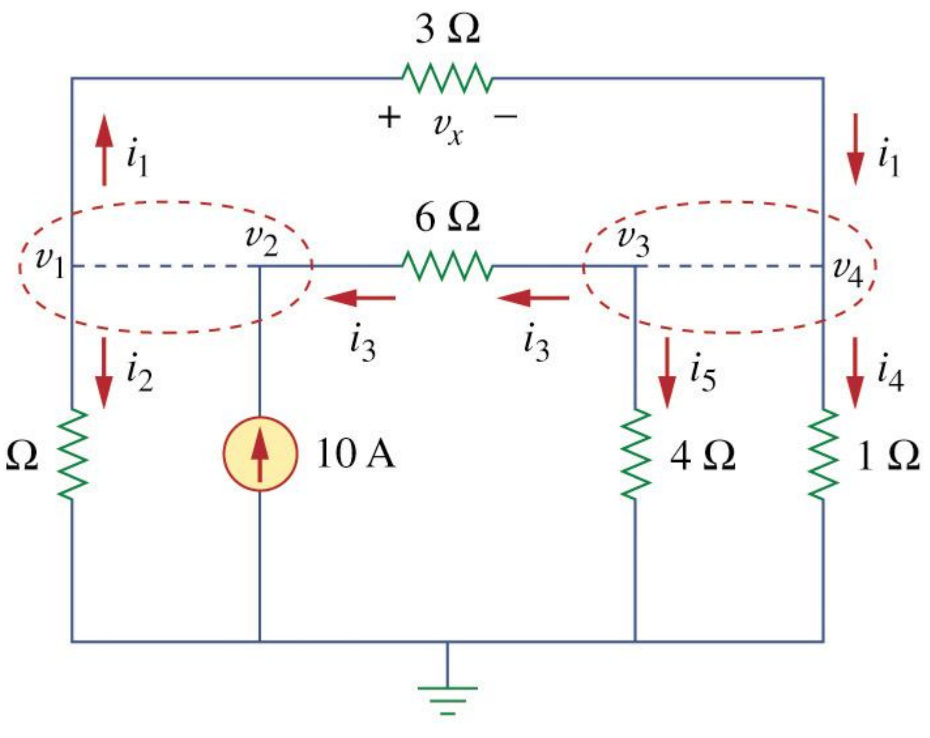
(a)

Example 3.4

- At supernode 3-4,

$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$v_3 - v_4 = 3(v_1 - v_4)$$

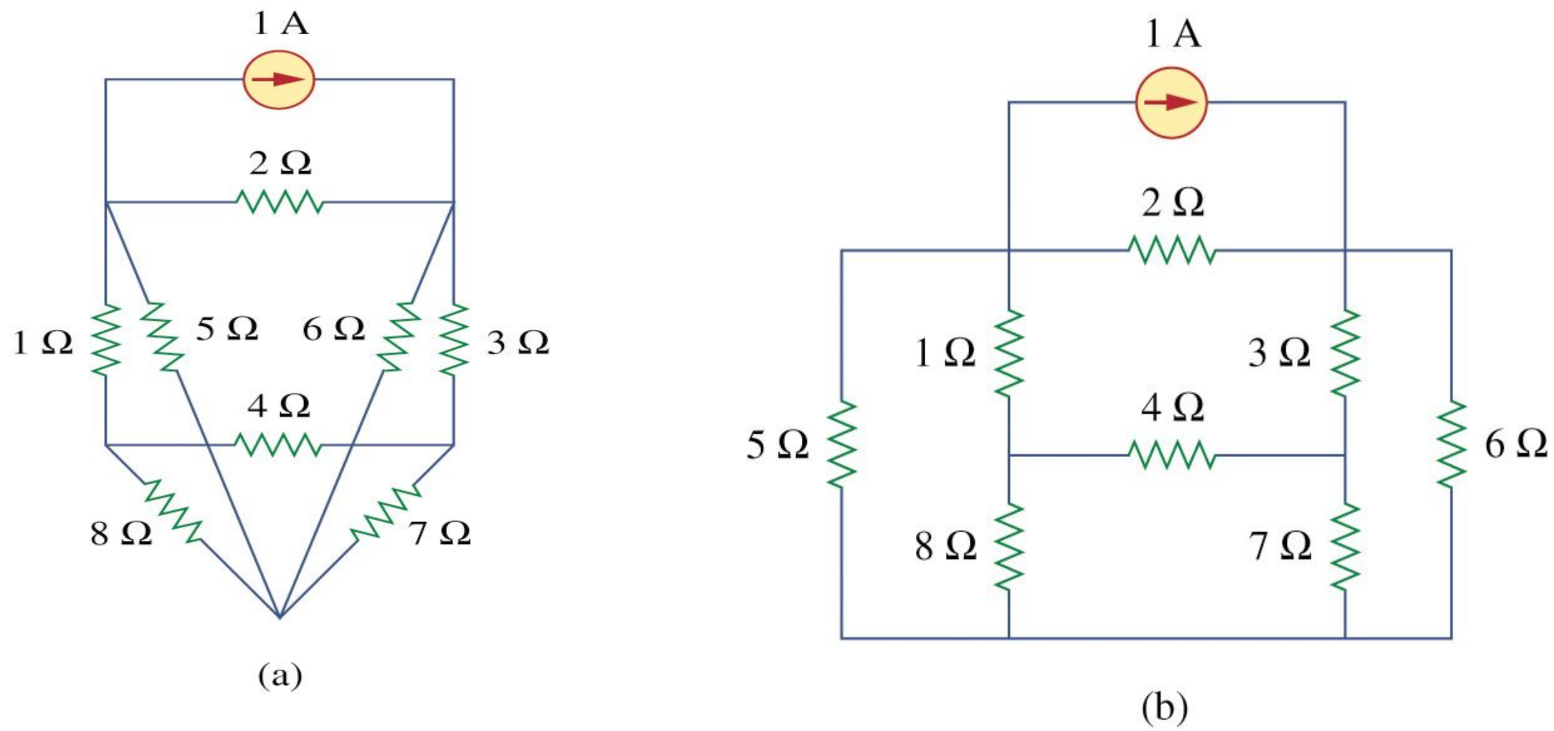


(a)

3.4 Mesh Analysis

- Mesh analysis: another procedure for analyzing circuits, applicable to planar circuit.
- A Mesh is a loop which does not contain any other loops within it

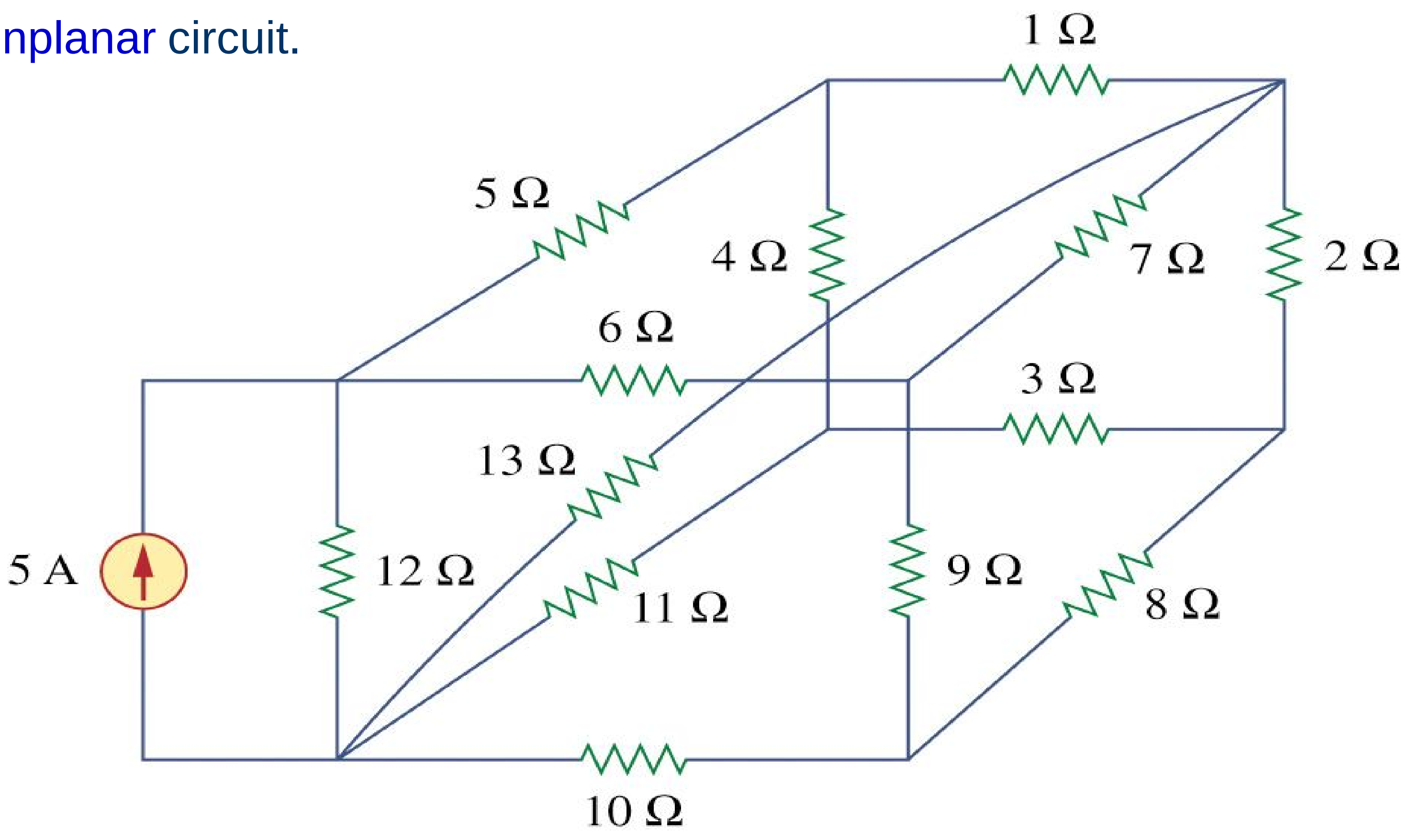
Fig. 3.15



(a) A Planar circuit with crossing branches,
(b) The same circuit redrawn with no crossing branches.

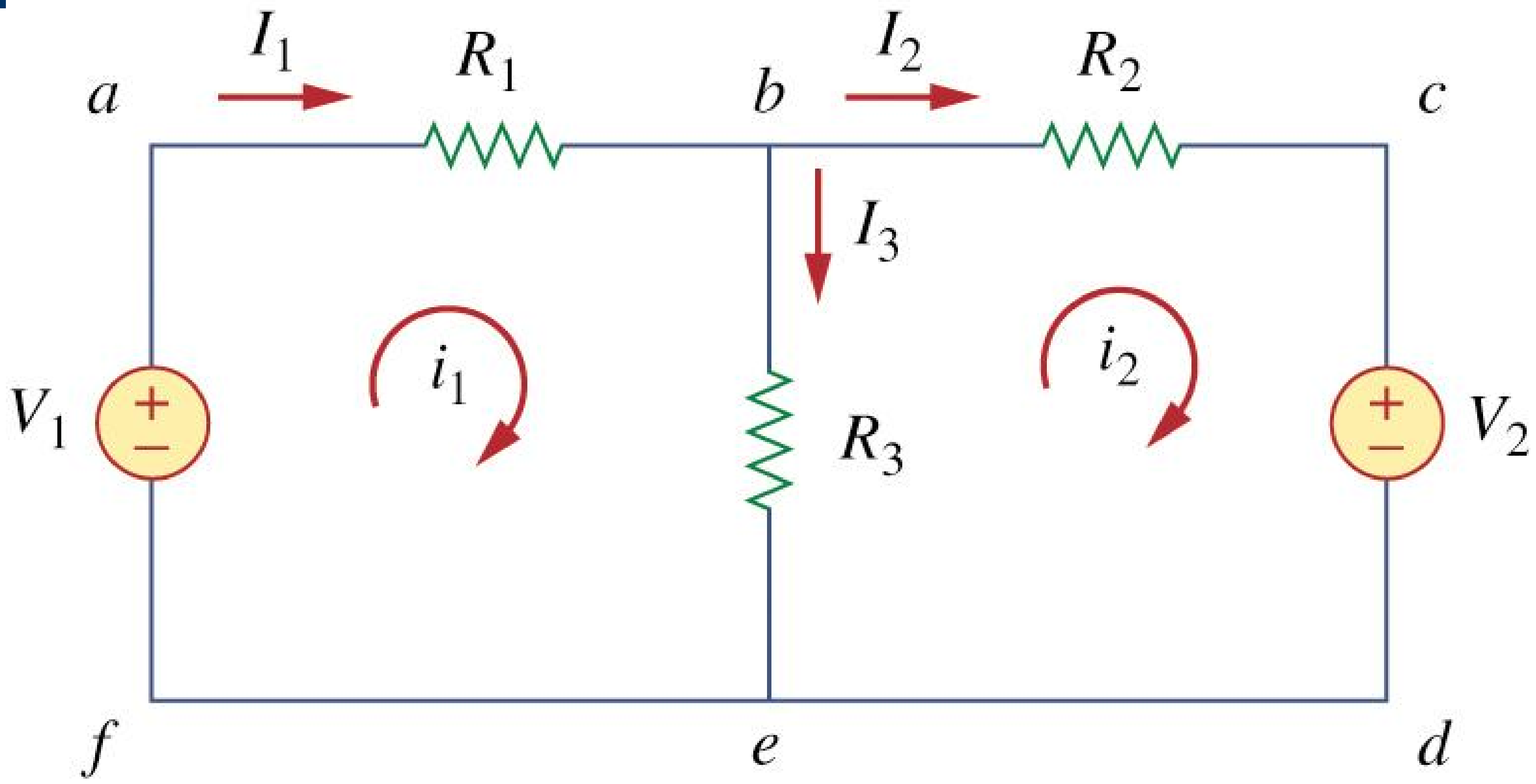
Fig. 3.16

A nonplanar circuit.



- Steps to Determine Mesh Currents:
  1. Assign mesh currents i_1, i_2, .., i_n to the n meshes.
  2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
  3. Solve the resulting *n* simultaneous equations to get the mesh currents.

Fig. 3.17



A circuit with two meshes.

- Apply KVL to each mesh. For mesh 1,

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1$$

- For mesh 2,

$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

- Solve for the mesh currents.

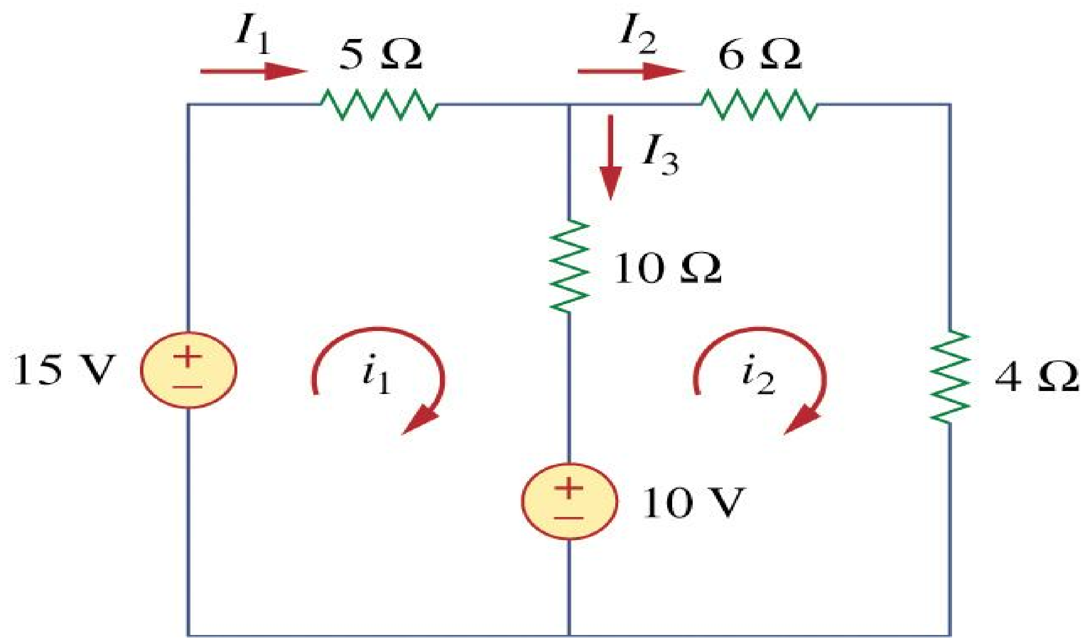
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

- Use *i* for a mesh current and *I* for a branch current. It's evident from Fig. 3.17 that

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$$

Example 3.5

- Find the branch current I_1, I_2, and I_3 using mesh analysis.



Example 3.5

- For mesh 1,

$$-15+5i_1+10(i_1-i_2)+10=0$$

$$\boxed{3i_1-2i_2=1}$$

- For mesh 2,

$$6i_2+4i_2+10(i_2-i_1)-10=0$$

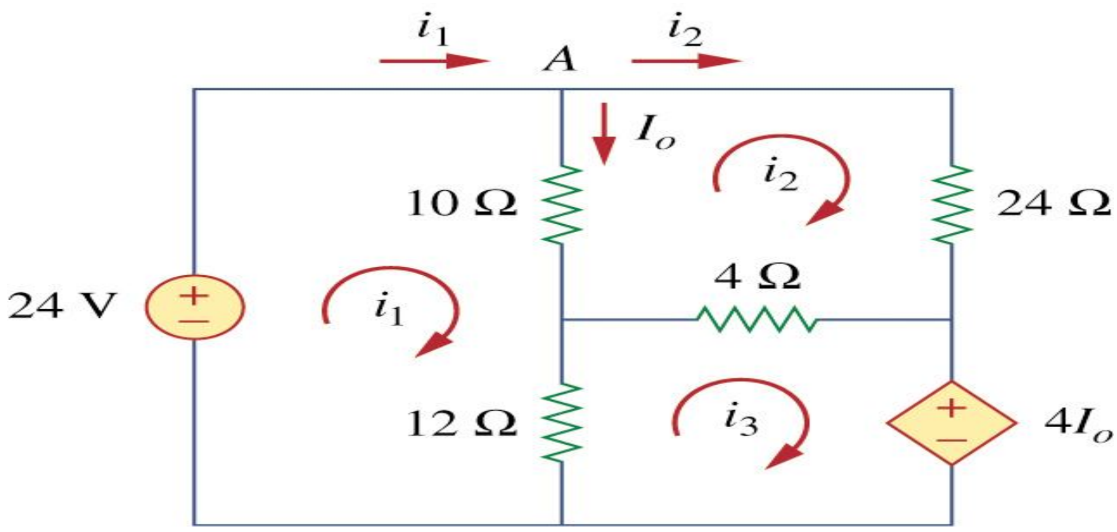
$$\boxed{i_1=2i_2-1}$$

- We can find i_1 and i_2 by substitution method or Cramer's rule. Then,

$$I_1=i_1, \quad I_2=i_2, \quad I_3=i_1-i_2$$

Example 3.6

- Use mesh analysis to find the current I_0 in the circuit of Fig. 3.20.



Example 3.6

- Apply KVL to each mesh. For mesh 1,

$$-24+10(i_1-i_2)+12(i_1-i_3)=0$$
$$11i_1-5i_2-6i_3=12$$

- For mesh 2,

$$24i_2+4(i_2-i_3)+10(i_2-i_1)=0$$
$$-5i_1+19i_2-2i_3=0$$

Example 3.6

- For mesh 3,

$$4I_0 + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$\text{At node A, } I_0 = I_1 - i_2,$$

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$-i_1 - i_2 + 2i_3 = 0$$

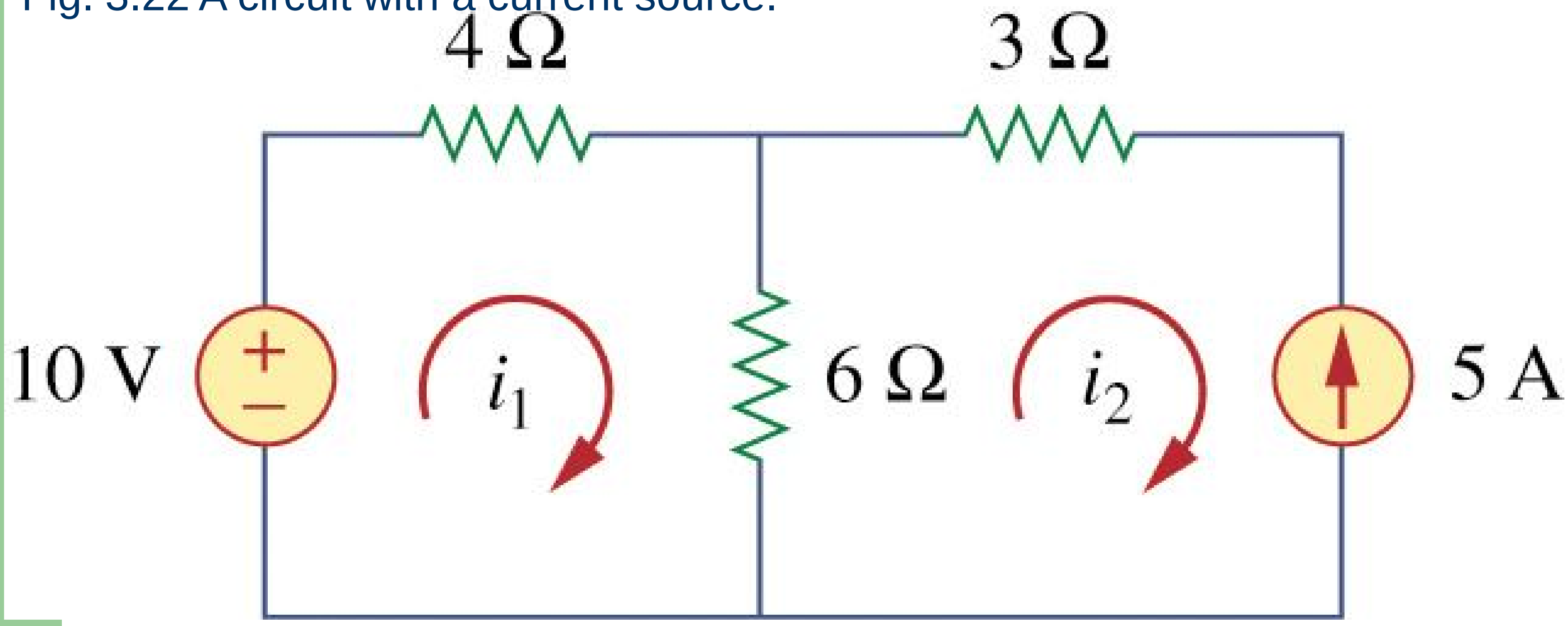
- In matrix from Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

we can calculus i_1, i_2 and i_3 by Cramer's rule, and find I_0.

3.5 Mesh Analysis with Current Sources

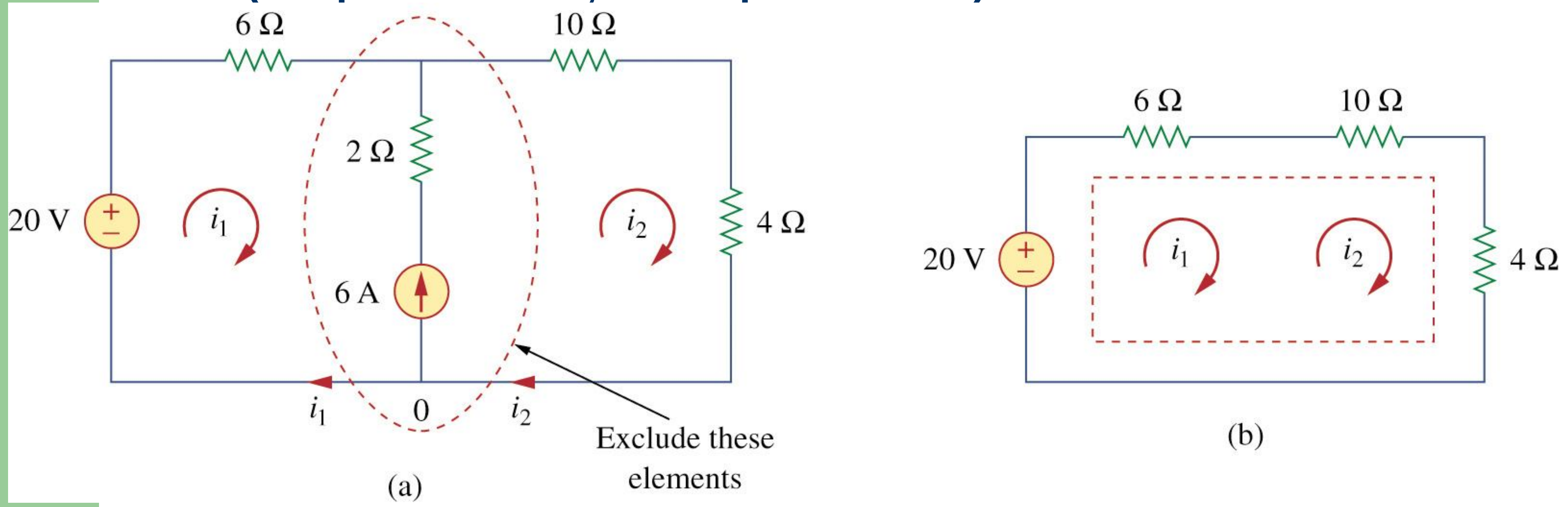
Fig. 3.22 A circuit with a current source.



- Case 1 $i_1 = -2\,\mathrm{A}$
  - Current source exist only in one mesh
  - One mesh variable is reduced
- Case 2
  - Current source exists between two meshes, a super-mesh is obtained.

Fig. 3.23

- a supermesh results when two meshes have a (dependent , independent) current source

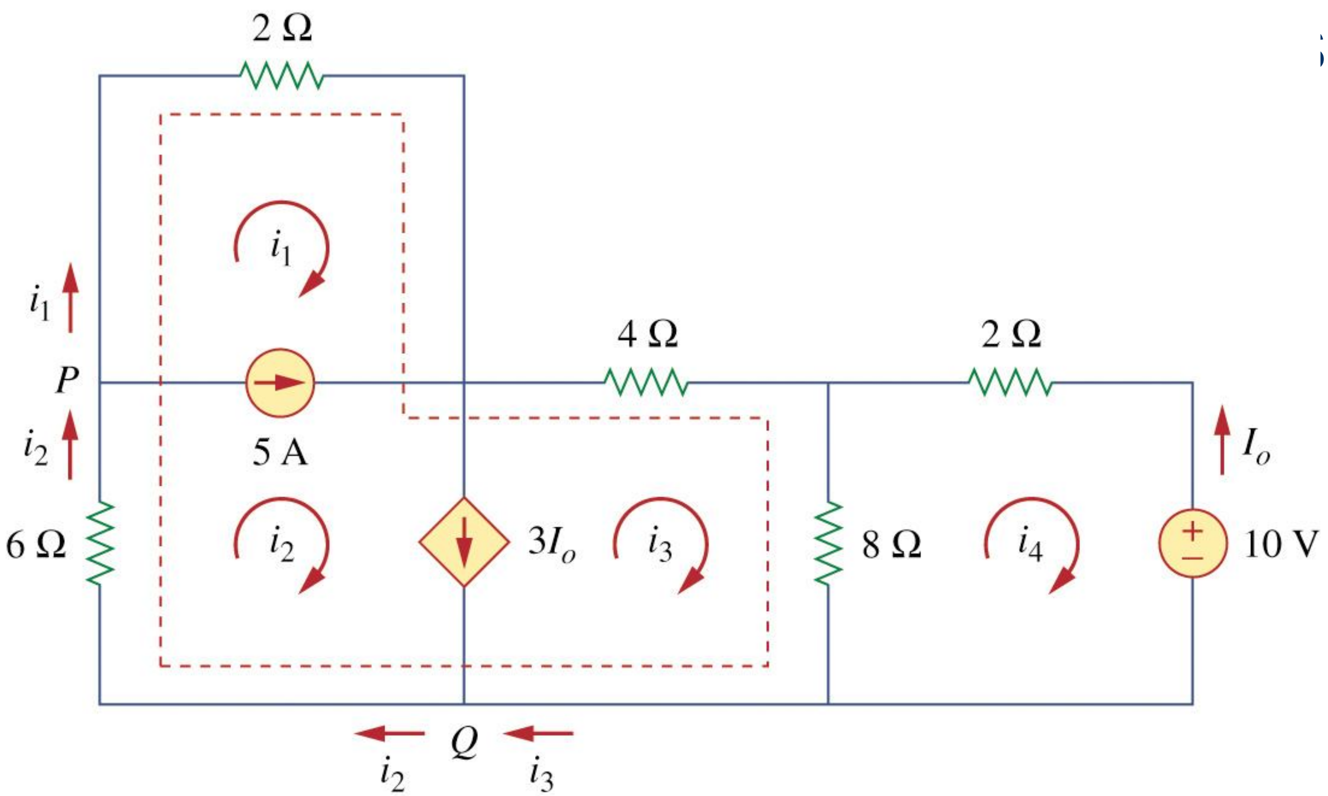


Properties of a Supermesh

1. The current is not completely ignored
   – provides the constraint equation necessary to solve for the mesh current.
2. A supermesh has no current of its own.
3. Several current sources in adjacency form a bigger supermesh.

Example 3.7

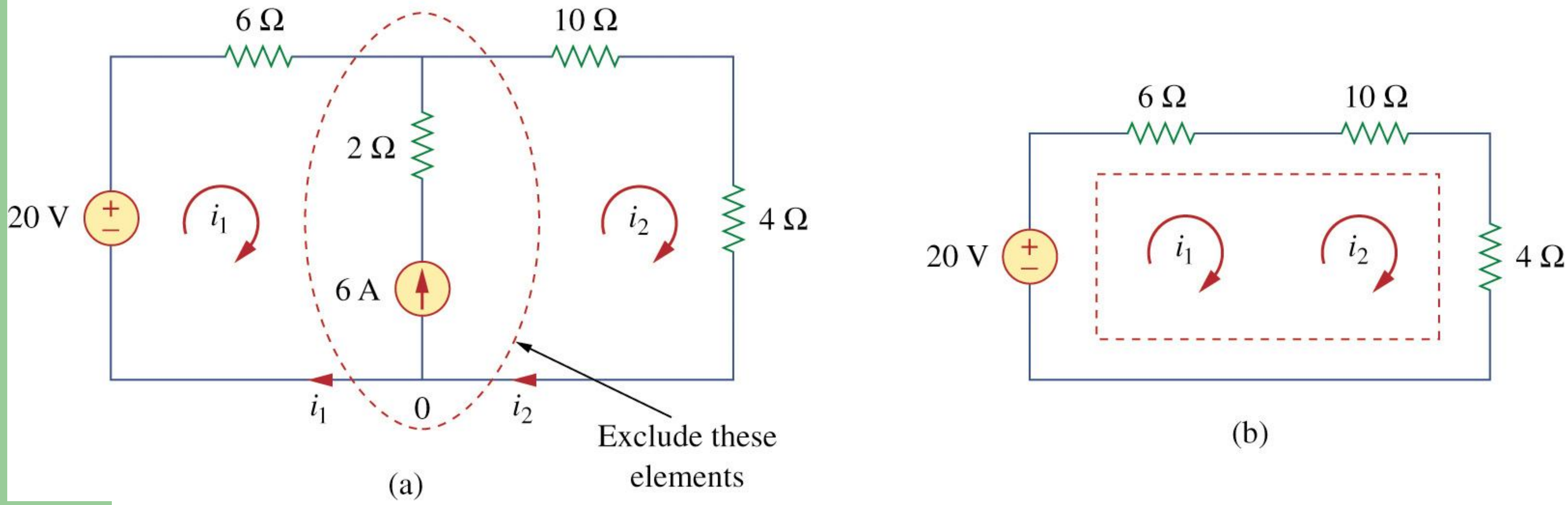
- ing



- If a supermesh consists of two meshes, two equations are needed; one is obtained using KVL and Ohm's law to the supermesh and the

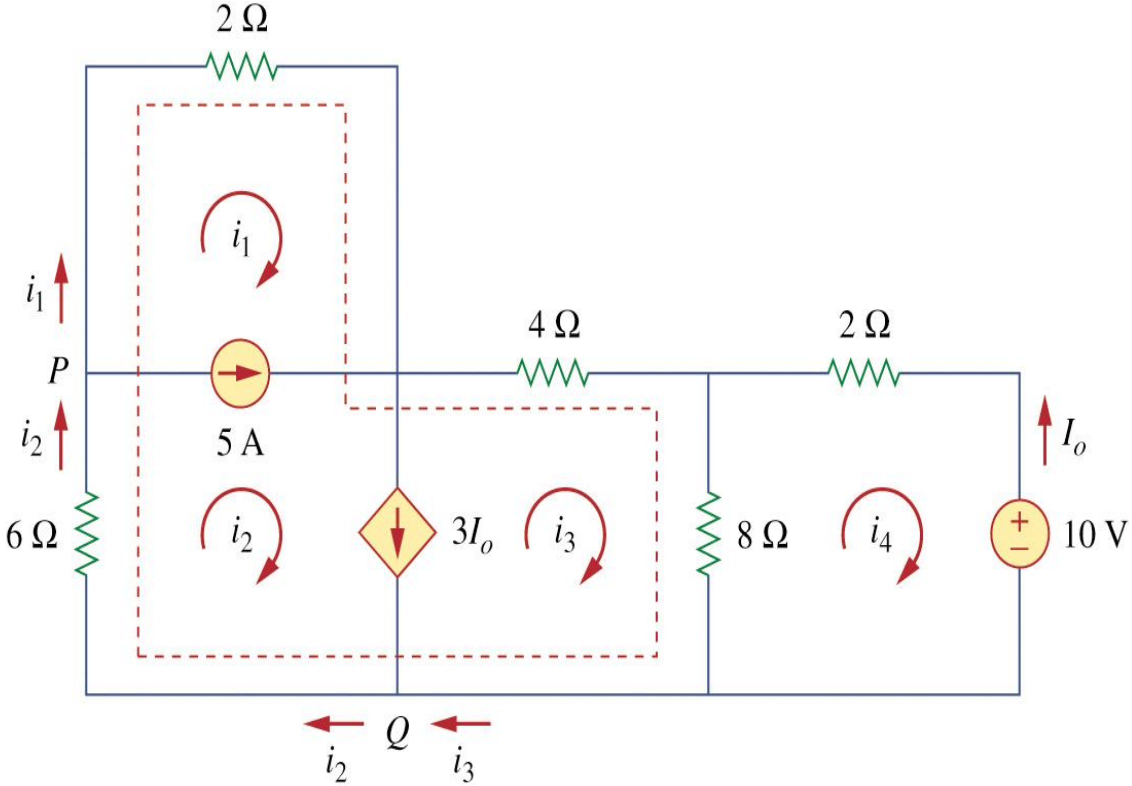
$$6i_1 + 14i_2 = 20$$

$$i_1 - i_2 = -6$$



- Similarly, a supermesh formed from three meshes needs three equations: one is from the supermesh and the other two equations are obtained from the two current sources.

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

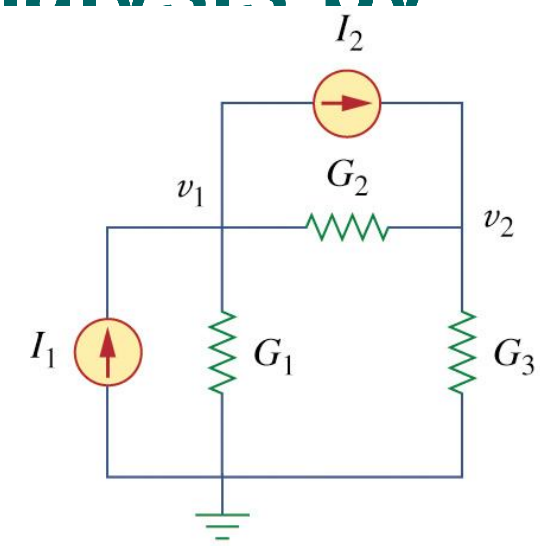


$$10 = 0$$

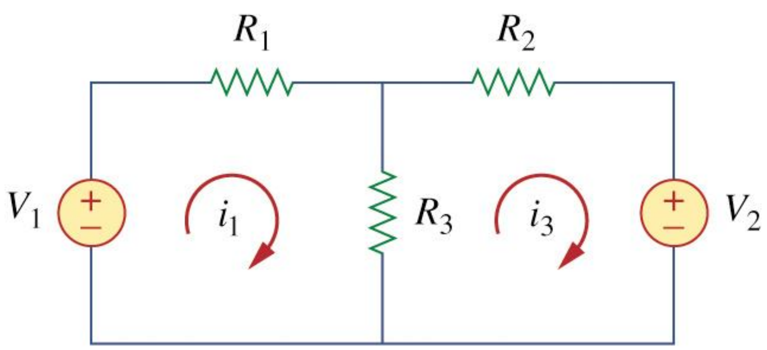
3.6 Nodal and Mesh Analysis by Inspection

The analysis equations can be obtained by direct inspection

(a) For circuits with only resistors and independent current sources
(b) For planar circuits with only resistors and independent voltage sources



(a)



(b)

- In the Fig. 3.26 (a), the circuit has two nonreference nodes and the node equations

$$I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2) \quad (3.7)$$

$$I_2 + G_2 (v_1 - v_2) = G_3 v_2 \quad (3.8)$$

$$\rightarrow MATRIX$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

- In general, the node voltage equations in terms of the conductance is

or simply

Gv = i

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

where *G* : the conductance matrix,
v : the output vector, *i* : the input vector

- The circuit has two nonreference nodes and the node equations were derived as

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

- In general, if the circuit has N meshes, the mesh-current equations as the resistances term is

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

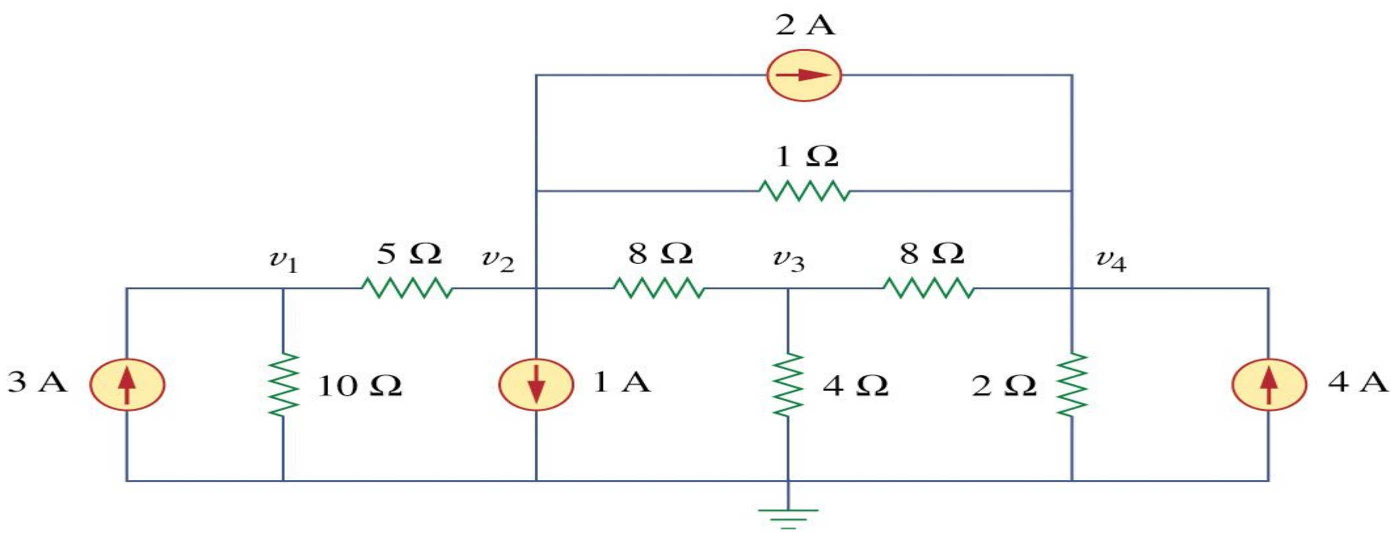
or simply

Rv = i

where *R* : the resistance matrix,
i : the output vector, *v* : the input vector

Example 3.8

- Write the node voltage matrix equations in Fig.3.27.



Example 3.8

- The circuit has 4 nonreference nodes, so

$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \quad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \quad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

- The off-diagonal terms are

$$G_{12} = -\frac{1}{5} = -0.2, \quad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \; G_{23} = -\frac{1}{8} = -0.125, \quad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \; G_{32} = -0.125, \; G_{34} = -0.125$$

$$G_{41} = 0, \; G_{42} = -1, \; G_{43} = -0.125$$

Example 3.8

$$i_1 = 3, \quad i_2 = -1 - 2 = -3, \quad i_3 = 0, \quad i_4 = 2 + 4 = 6$$

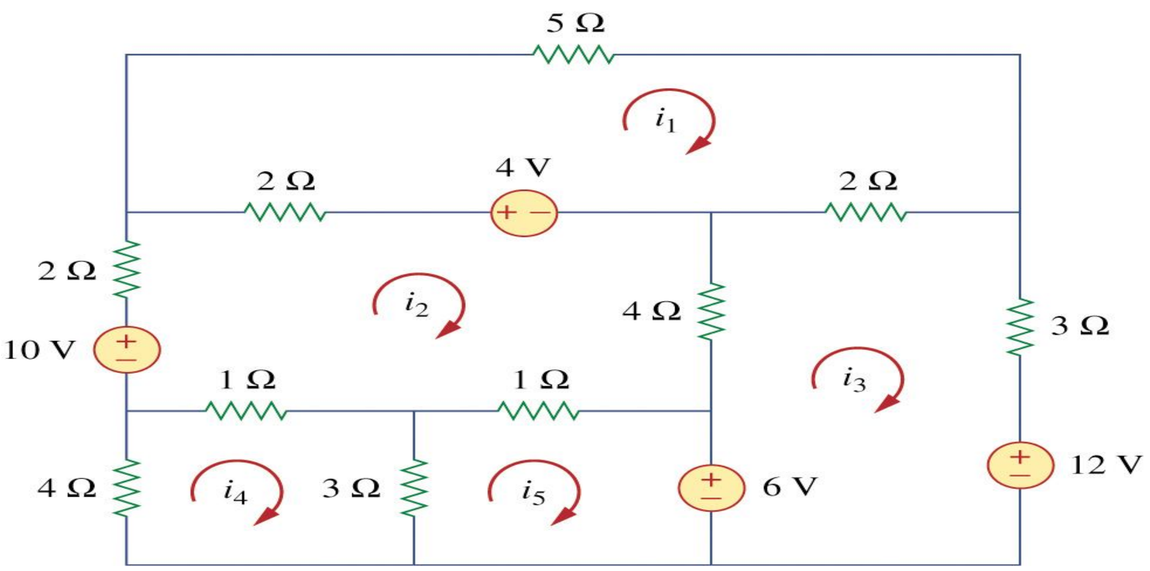
- The input current vector *i* in amperes

- The node-voltage equations are

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

Example 3.9

- Write the mesh current equations in Fig.3.27.



Example 3.9

- The input voltage vector *v* in volts

$$v_1 = 4, \quad v_2 = 10 - 4 = 6,$$
$$v_3 = -12 + 6 = -6, \quad v_4 = 0, \quad v_5 = -6$$

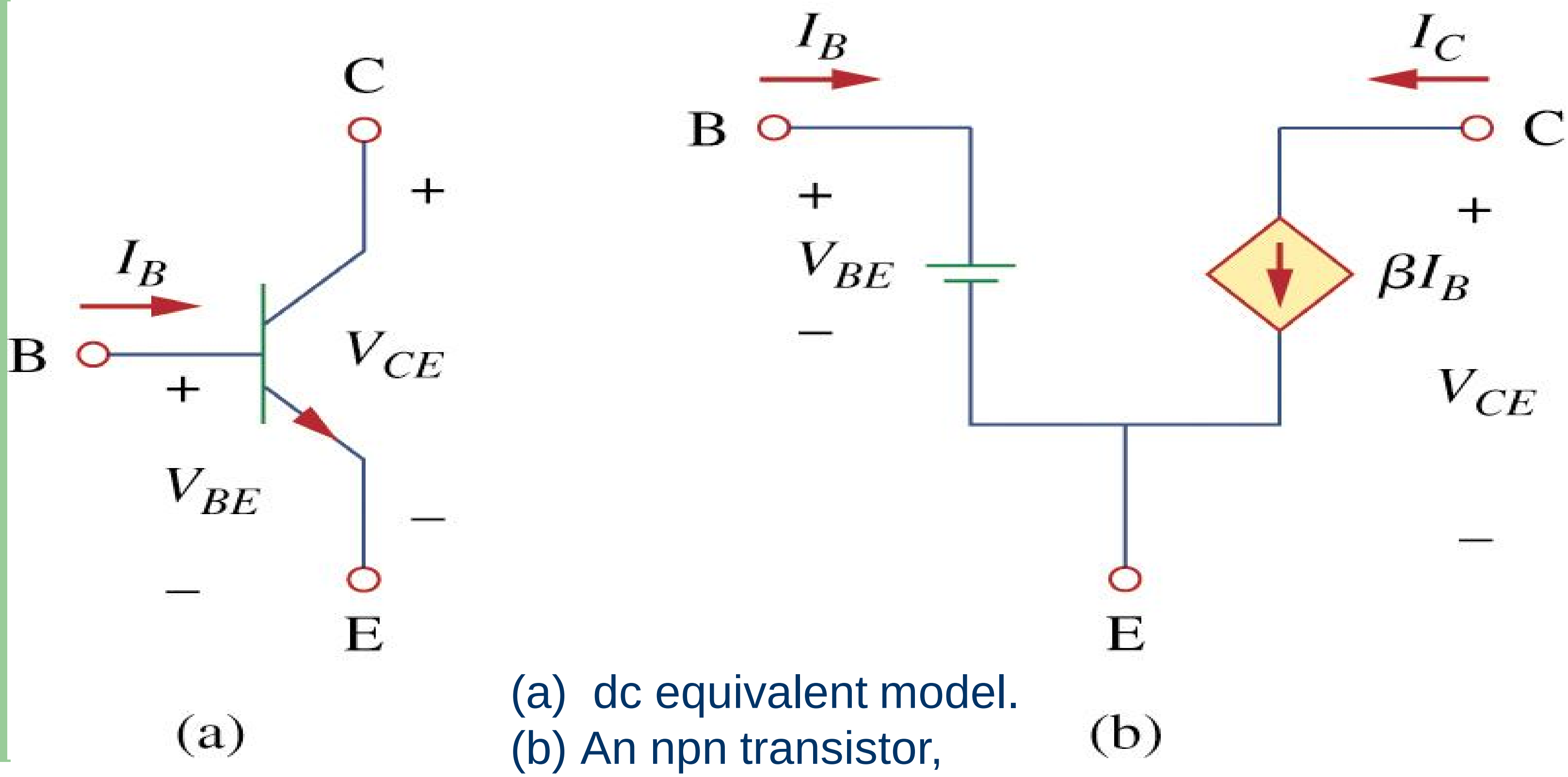
- The mesh-current equations are

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

3.7 Nodal Versus Mesh Analysis

- Both nodal and mesh analyses provide a systematic way of analyzing a complex network.
- The choice of the better method dictated by two factors.
  - First factor : nature of the particular network. The key is to select the method that results in the smaller number of equations.
  - Second factor : information required.

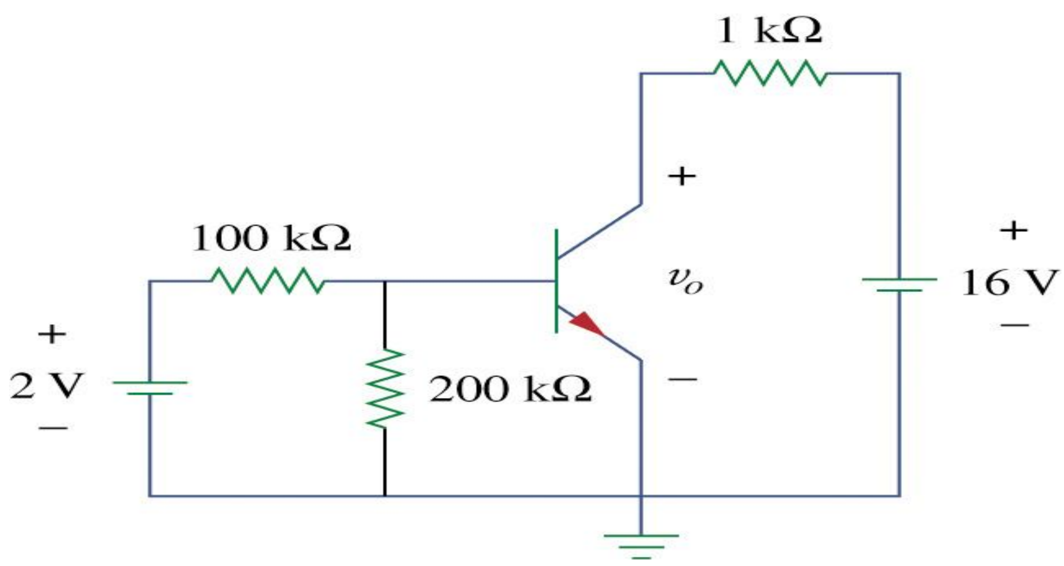
BJT Circuit Models



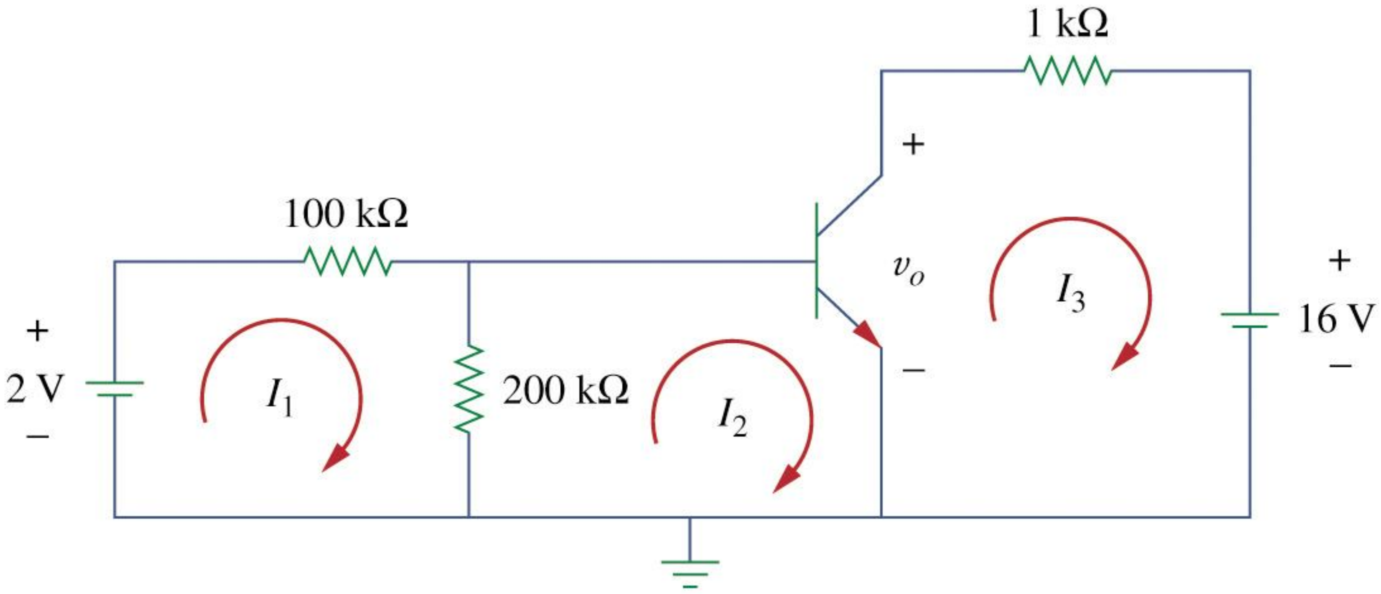
(a) dc equivalent model.
(b) An npn transistor,

Example 3.13

- For the BJT circuit in Fig.3.43, β=150 and V_{BE} = 0.7 V. Find v_0.

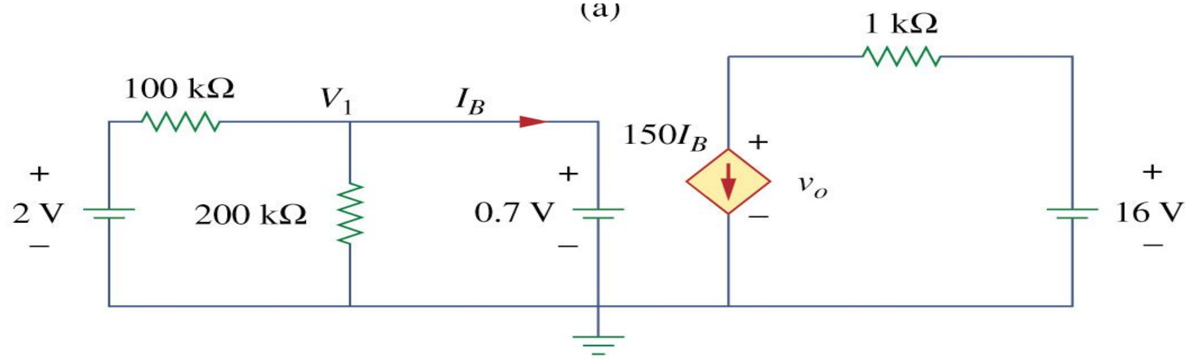


Example 3.13

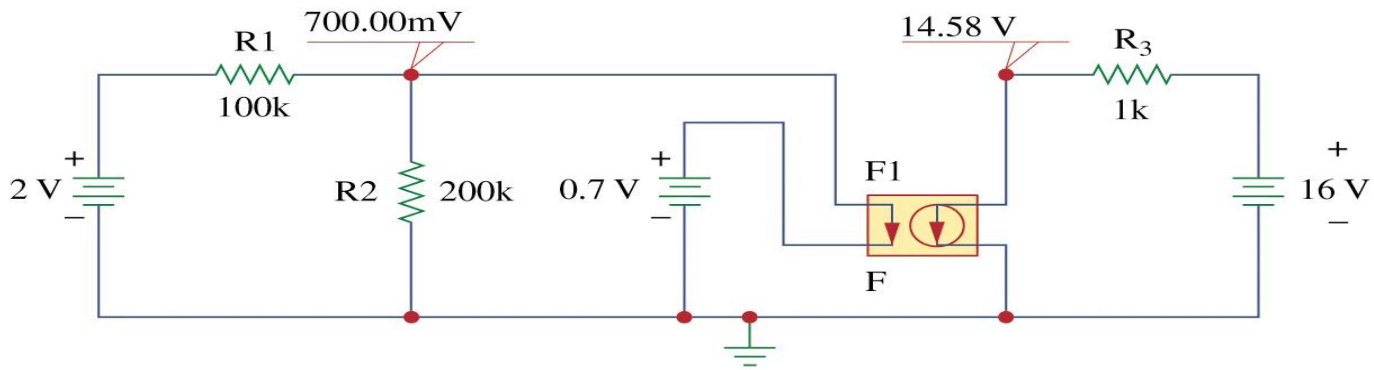


Example 3.13

(a)



(b)



(c)

3.10 Summary

1. Nodal analysis: the application of KCL at the nonreference nodes
   - A circuit has fewer node equations
2. A supernode: two nonreference nodes
3. Mesh analysis: the application of KVL
   - A circuit has fewer mesh equations
4. A supermesh: two meshes

UNIT II – SINUSOIDAL STEADY STATE ANALYSIS **9**

–-Phasor

– Sinusoidal steady state response concepts of impedance and admittance

– Analysis of simple circuits

– Power and power factors

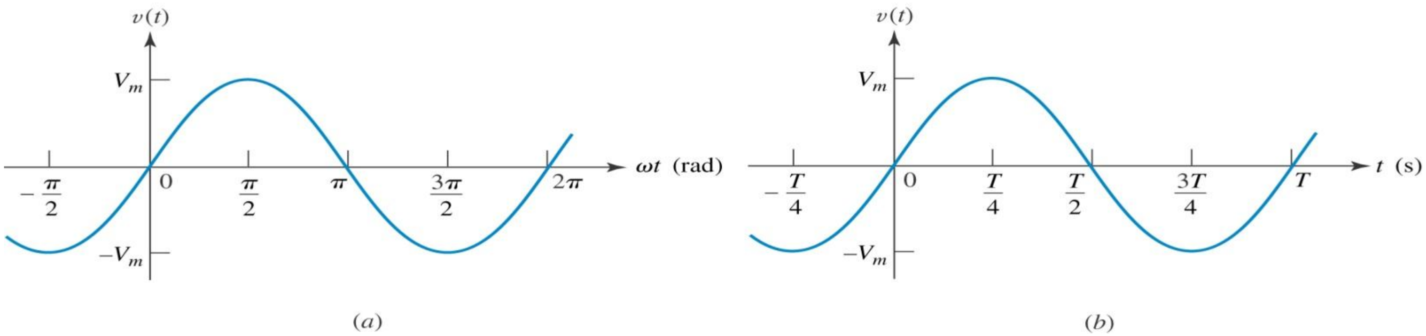
–– Solution of three phase balanced circuits and three phase unbalanced circuits

–-Power measurement in three phase circuits.

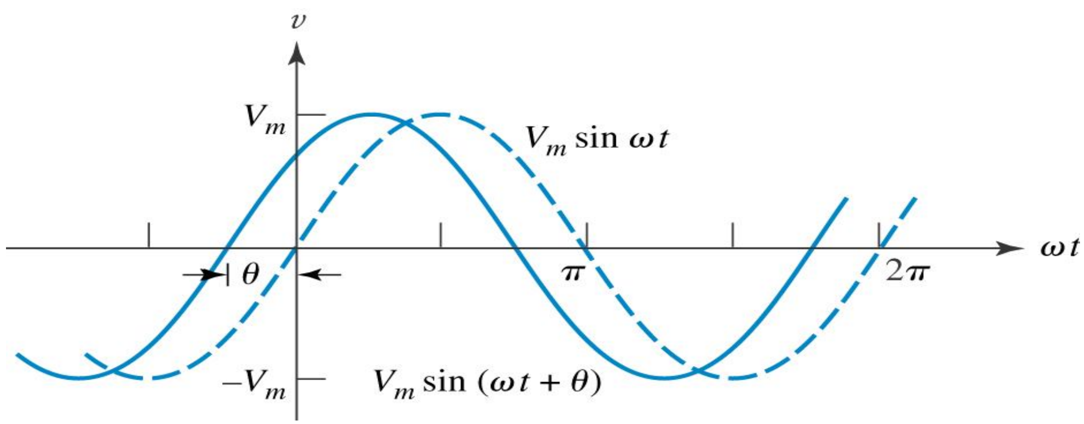
Sinusoidal Steady State Response

Sinusoidal Steady State Response

1. Identify the frequency, angular frequency, peak value, RMS value, and phase of a sinusoidal signal.
2. Solve steady-state ac circuits using phasors and complex impedances.
3. Compute power for steady-state ac circuits.
4. Find Thévenin and Norton equivalent circuits.
5. Determine load impedances for maximum power transfer.



The sinusoidal function $v(t)$ = $V_M \sin \omega t$ is plotted (a) versus ωt and (b) versus t.



The sine wave $V_M \sin(\omega t + \theta)$ leads $V_M \sin \omega t$ by θ radian

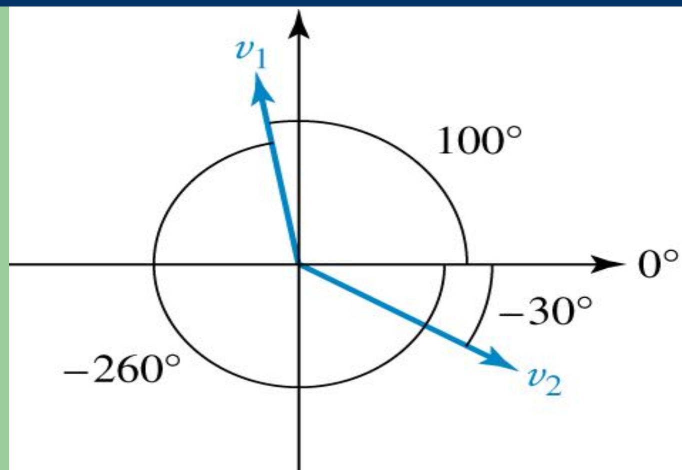
Frequency $f = \frac{1}{T}$ **Angular frequency** $\omega = \frac{2\pi}{T}$

$$\omega = 2\pi f$$

$$\sin z = \cos(z - 90^o)$$

$$\sin \omega t = \cos(\omega t - 90^o)$$

$$\sin(\omega t + 90^o) = \cos \omega t$$



Representation of the two vectors v_1 **and** v_2**.**

In this diagram, v1 leads v2 by 100º + 30º = 130º, although it could also be argued that v_2 **leads** v_1 **by 230º.**

Generally we express the phase difference by an angle less than or equal to 180º in magnitude.

Euler's identity

$$\theta = \omega t$$

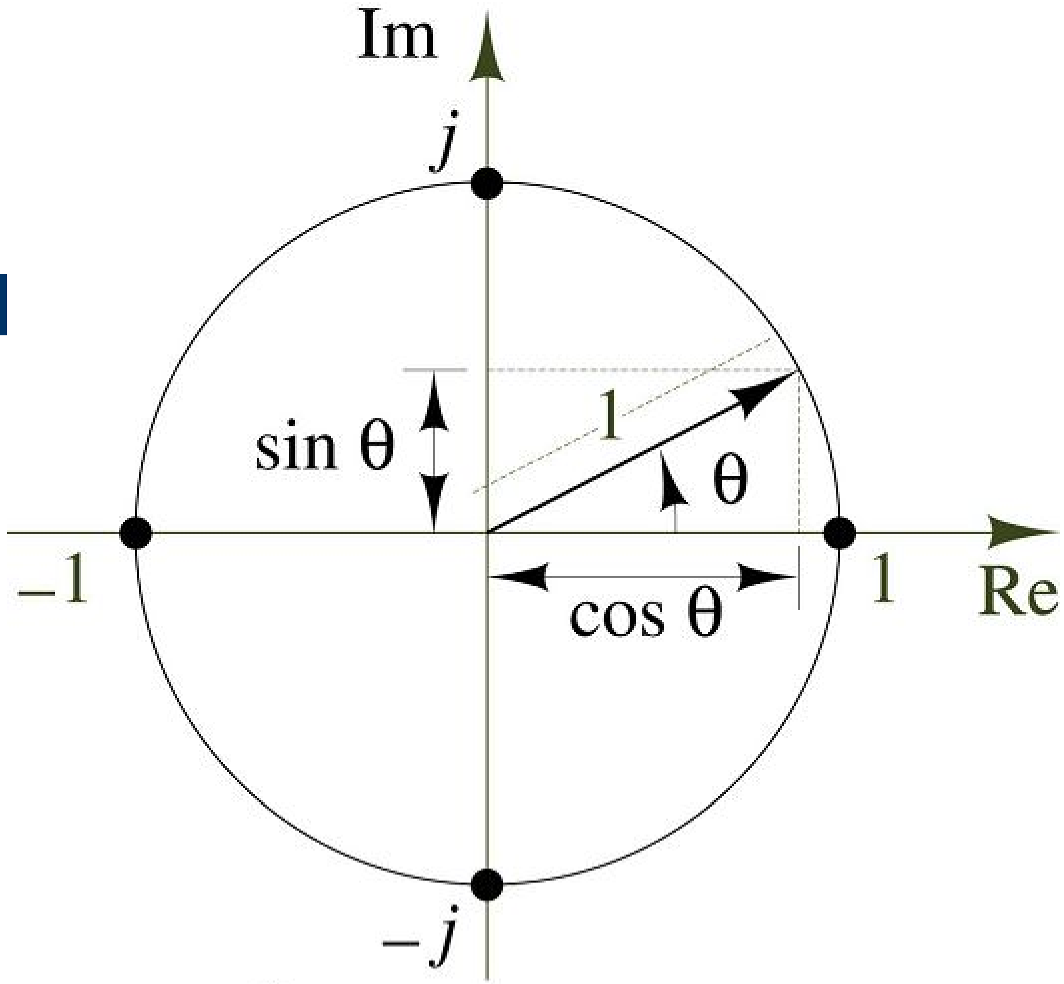
$$\omega = 2\pi f$$

$$A\cos\omega t = A\cos 2\pi f t$$

In Euler expression,

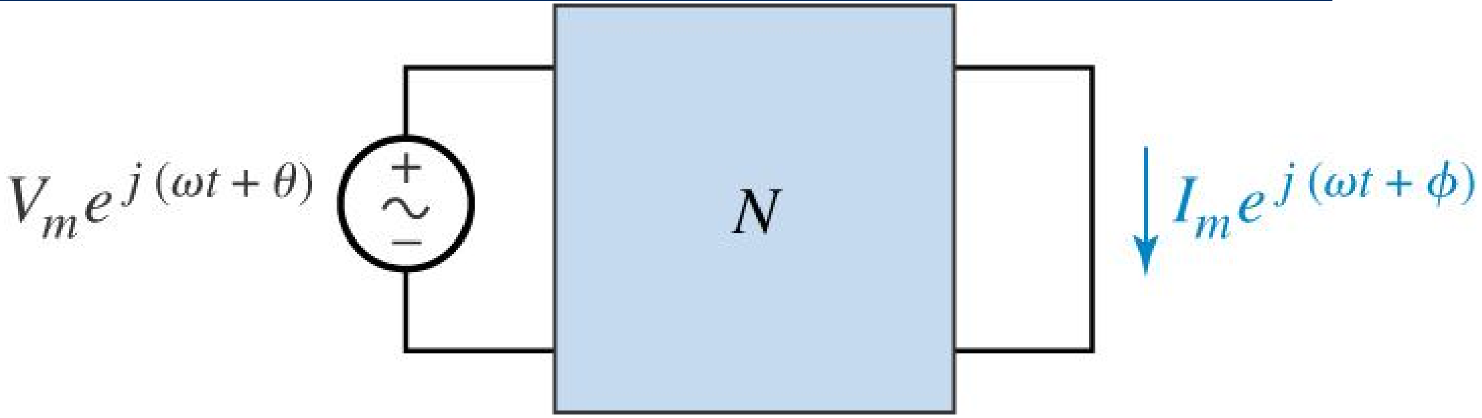
A *cos* ωt = *Real* (A $e^{j\omega t}$)
A *sin* ωt = *Im*(A $e^{j\omega t}$)

Any sinusoidal function can be expressed as in Euler form.

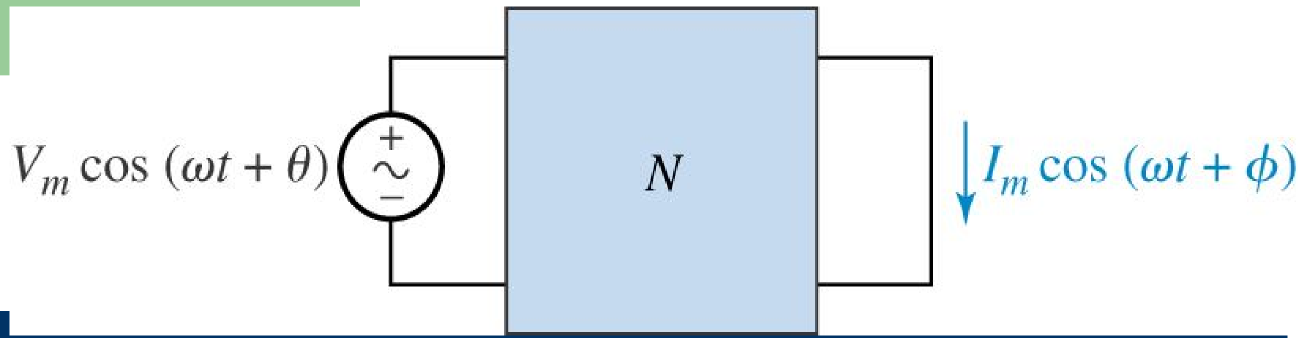


$$e^{j\theta} = \cos\theta + j\sin\theta$$

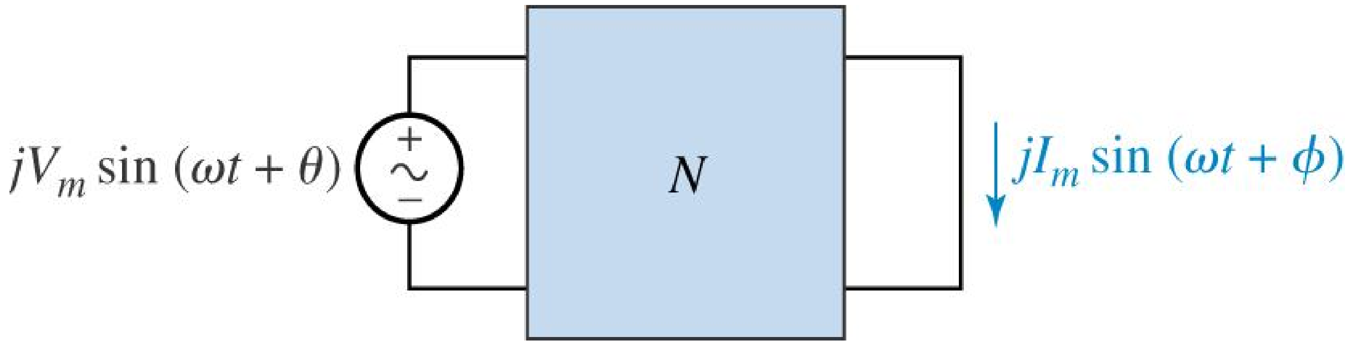
Applying Euler's Identity



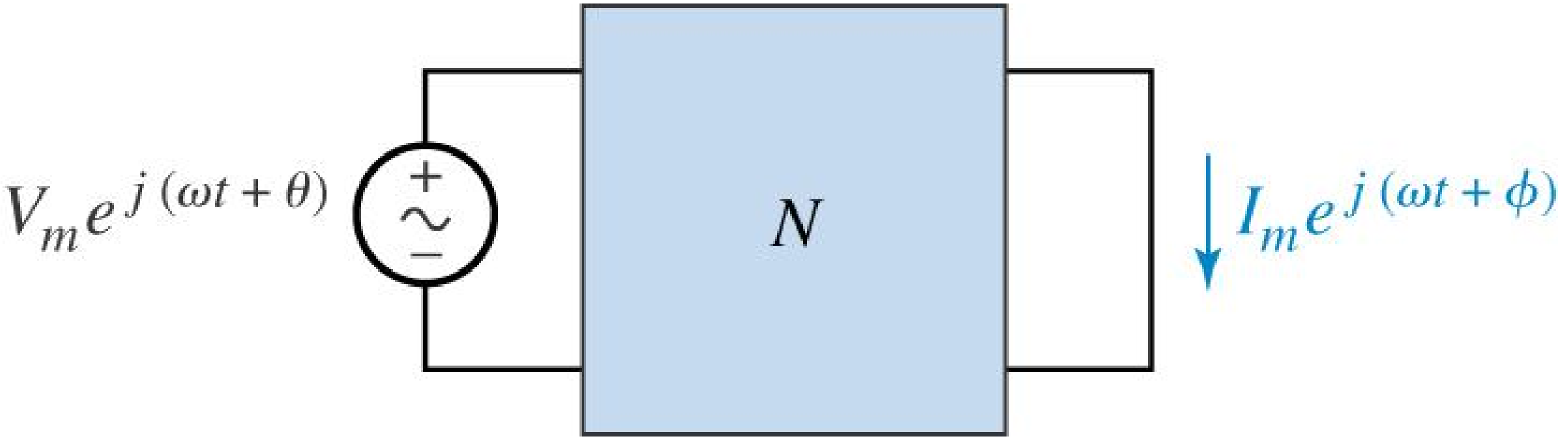
The complex forcing function $V_m e^{j(\omega t + \theta)}$ produces the complex response $I_m e^{j(\omega t + \phi)}$.



The sinusoidal forcing function V_m cos (ω *t* + θ) produces the steady-state response I_m cos (ω *t* + φ).



The imaginary sinusoidal input *j* V_m sin (ω *t* + θ) produces the imaginary sinusoidal output response *j* I_m sin (ω *t* + φ).



$$\mathrm{Re}(V_m\, e^{j(\omega t + \theta)}) \rightarrow \mathrm{Re}(I_m\, e^{j(\omega t + \phi)})$$

$$\mathrm{Im}(V_m\, e^{j(\omega t + \theta)}) \rightarrow \mathrm{Im}(I_m\, e^{j(\omega t + \phi)})$$

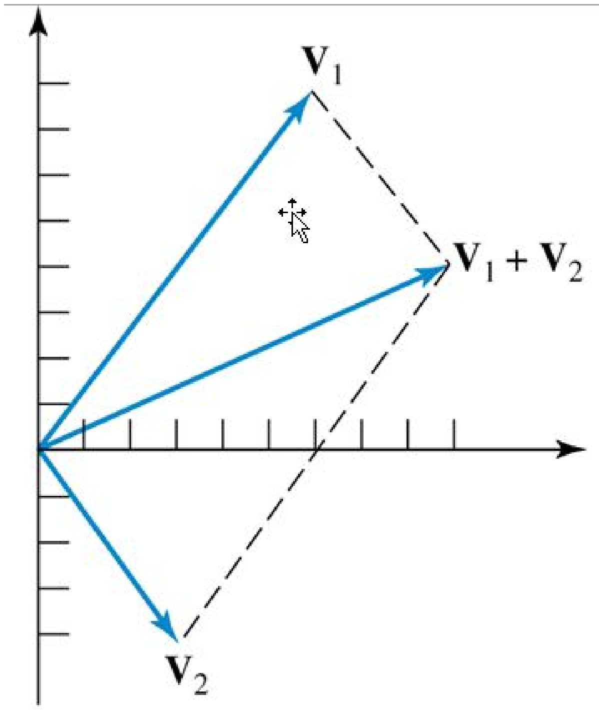
Phasor Definition

$$\text{Time function: } v_1(t) = V_1 \cos(\omega t + \theta_1)$$

$$\text{Phasor: } \mathbf{V}_1 = V_1 \angle \theta_1$$

$$\mathbf{V}_1 = \text{Re}(e^{j(\omega t + \theta_1)})$$

$$\mathbf{V}_1 = \text{Re}(e^{j(\theta_1)}) \textit{ by dropping } \omega t$$



A phasor diagram showing the sum of

$\mathbf{V}_1 = 6 + j8$ V and $\mathbf{V}_2 = 3 - j4$ V,

$\mathbf{V}_1 + \mathbf{V}_2 = 9 + j4$ V = **Vs**

$\mathbf{Vs} = \mathbf{A}e^{j\theta}$

$\mathbf{A} = [9^2 + 4^2]^{1/2}$

$\boldsymbol{\theta} = \tan^{-1}(4/9)$

$\mathbf{Vs} = 9.85\angle 24.0^{o}$ V.

Phasors Addition

Step 1: Determine the phase for each term.

Step 2: Add the phase's using complex arithmetic.

Step 3: Convert the Rectangular form to polar form.

Step 4: Write the result as a time function.

Conversion of rectangular to polar form

$$v_1(t) = 20\cos(\omega t - 45^\circ)$$

$$v_2(t) = 10\cos(\omega t - 30^\circ)$$

$$\mathbf{V}_1 = 20\angle -45^\circ$$

$$\mathbf{V}_2 = 10\angle -30^\circ$$

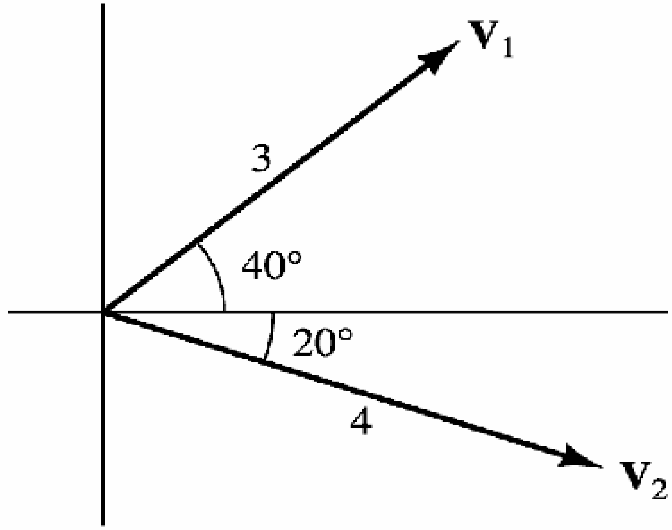
$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2$$
$$= 20\angle -45^\circ + 10\angle -30^\circ$$
$$= 14.14 - j14.14 + 8.660 - j5$$
$$= 23.06 - j19.14$$
$$= 29.97\angle -39.7^\circ$$

$$\mathrm{V}s = Ae^{j\theta}$$

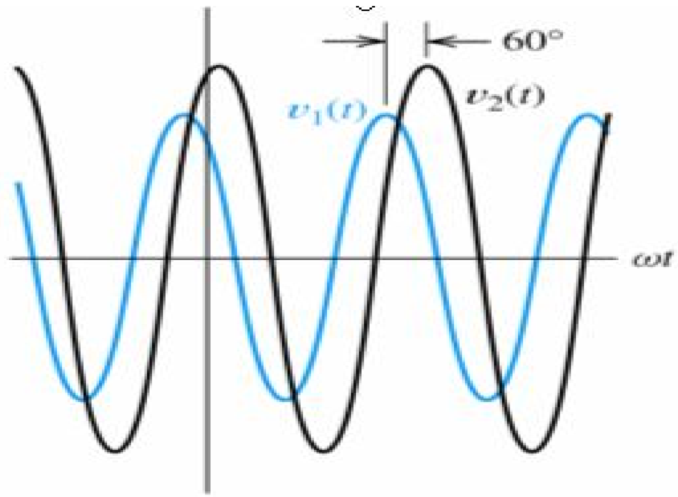
$$A = \sqrt{23.06^2 + (-19.14)^2} = 29.96,\ \theta = \tan^{-1}\frac{-19.14}{23.06} = -39.7^\circ$$

$$v_s(t) = 29.97\cos\left(\omega t - 39.7^\circ\right)$$

Phase relation ship



Because the vectors rotate counterclockwise, $\mathbf{v}_1$ leads $\mathbf{v}_2$ by 60° (or, equivalently, $\mathbf{v}_2$ lags $\mathbf{v}_1$ by 60°.)



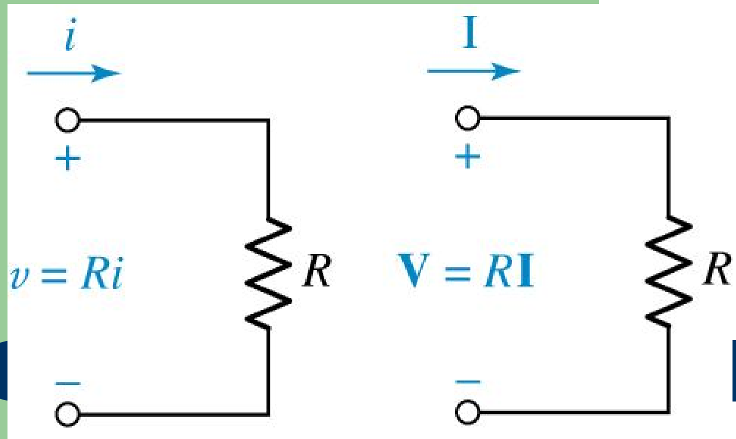
The peaks of $v_1(t)$ occur 60° before the peaks of $v_2(t)$. In other words, $v_1(t)$ leads $v_2(t)$ by 60°.

COMPLEX IMPEDANCE

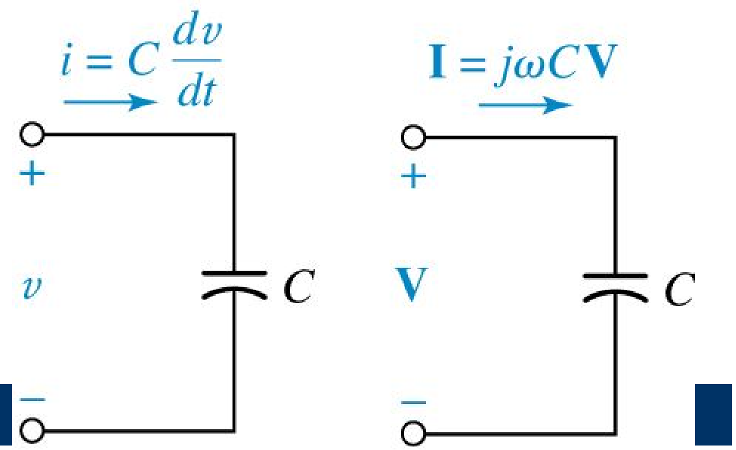
$$\mathbf{V}_L = j\omega L \times \mathbf{I}_L$$

$$Z_L = j\omega L = \omega L \angle 90^\circ$$

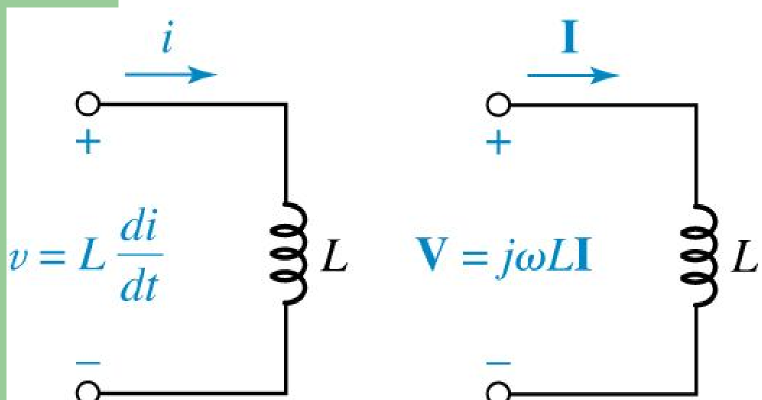
$$\mathbf{V}_L = Z_L \mathbf{I}_L$$



(a)



(b)



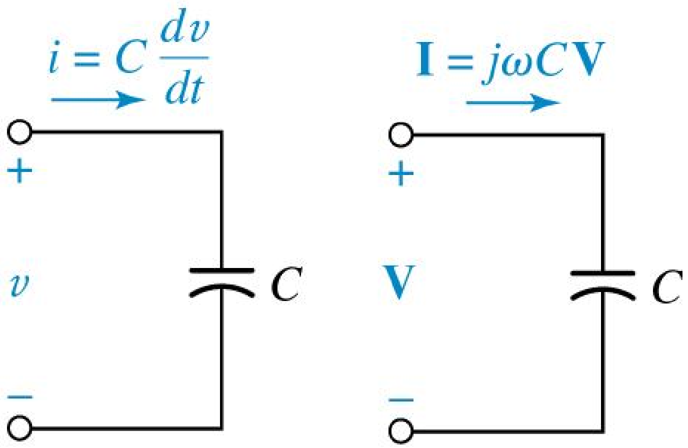
(c)

In the phasor domain,

(a) a resistor R is represented by an impedance of the same value;

(b) a capacitor C is represented by an impedance 1/*jωC*;

(c) an inductor L is represented by an impedance *jωL*.



$$\mathrm{V} = Ve^{j\omega t}$$

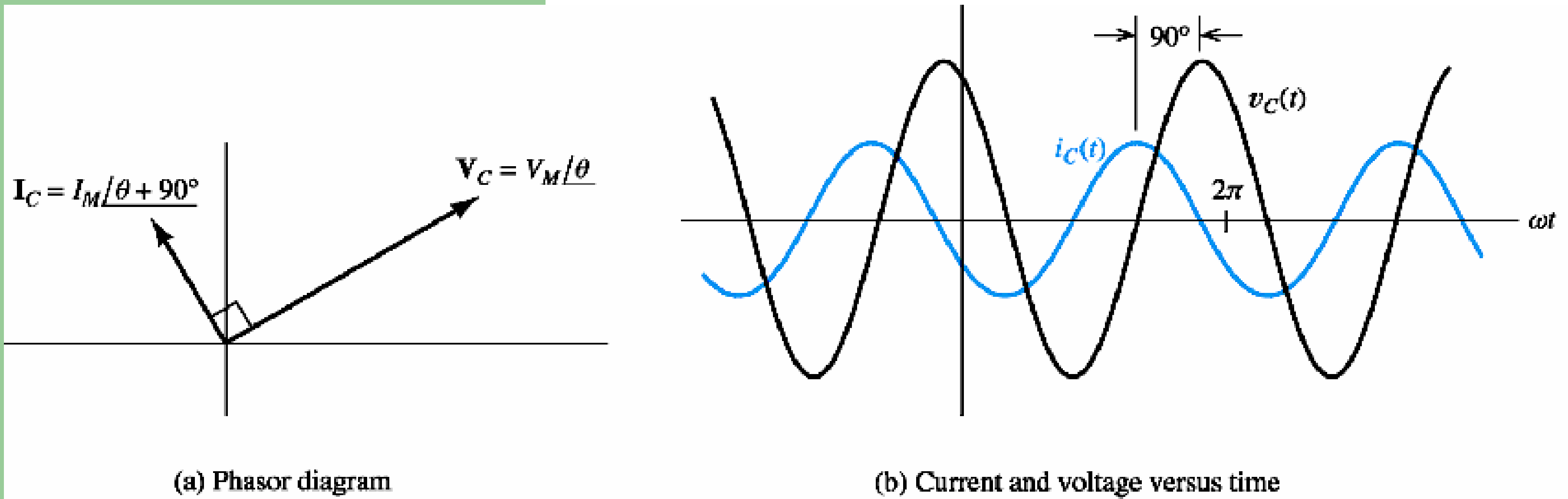
$$\mathrm{I} = C\frac{d\,\mathrm{V}}{dt} = C\frac{d\,Ve^{j\omega t}}{dt} = j\omega CVe^{j\omega t}$$

$$\mathrm{I} = j\omega C\mathrm{V} \Rightarrow \frac{\mathrm{V}}{\mathrm{I}} = \frac{1}{j\omega C} = Zc$$

Zc is defined as the impedance of a capacitor

The impedance of a capacitor is 1/jωC.

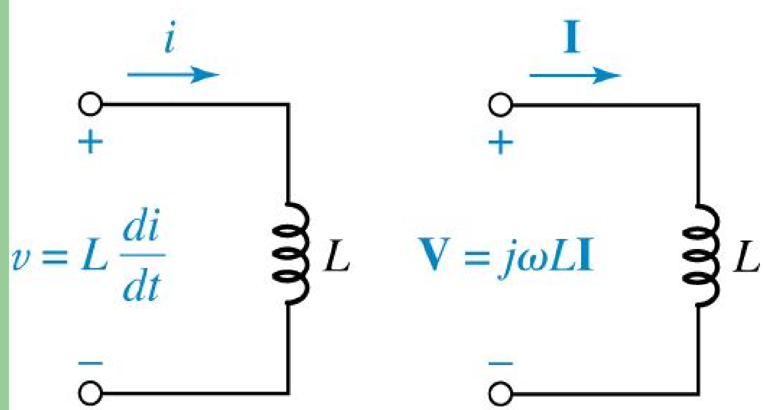
$$As\ \mathrm{I} = j\omega C\mathrm{V},\quad if\ v = V\cos\omega t,\ then\ i = \omega CV\cos(\omega t + 90^\circ)$$



Current leads voltage by 90° in a pure capacitance.

$$As\ \mathrm{I} = j\omega C\,\mathrm{V}, \quad if\ \ v = V_M \cos\omega t, then\ i = \omega C V_M \cos(\omega t + 90^\circ)$$

$$I_M = \omega C V_M$$



$$\mathrm{I} = Ie^{j\omega t}$$

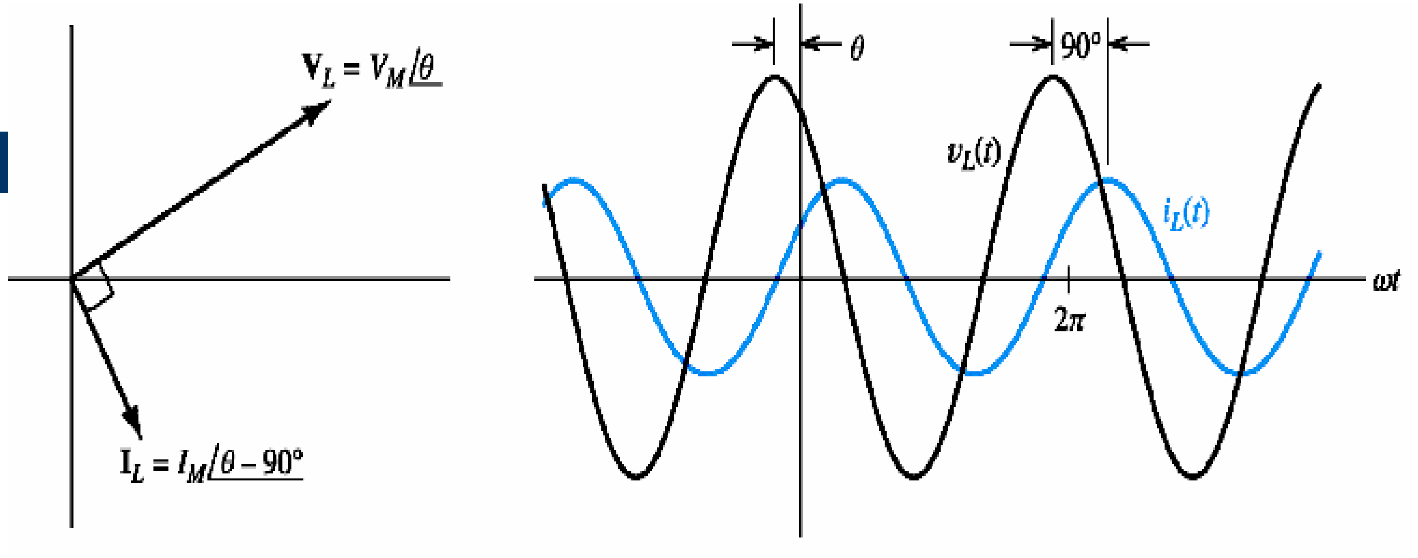
$$\mathrm{V} = L\frac{d\,\mathrm{I}}{dt} = L\frac{d\,Ie^{j\omega t}}{dt} = j\omega LIe^{j\omega t}$$

$$\mathrm{V} = j\omega L\mathrm{I} \Rightarrow \frac{\mathrm{V}}{\mathrm{I}} = j\omega L = Z_L$$

Z_L is the impedance of an inductor. The impedance of a inductor is $j\omega L$

$$As\ \mathrm{V} = j\omega C\mathrm{I},\quad if\ i = I\cos\omega t,\ then\ v = \omega LI\cos(\omega t + 90^\circ)$$

$$or\ i = I\cos(\omega t - 90^\circ),\ and\ v = \omega LI\cos\omega t.$$



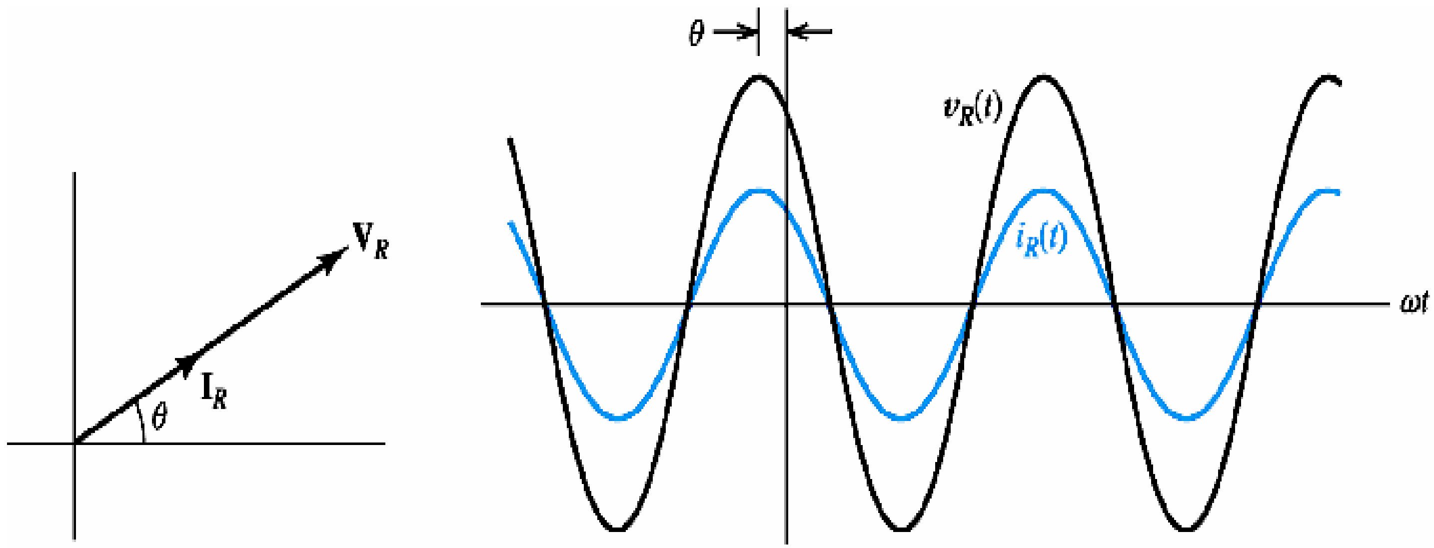
(a) Phasor diagram

(b) Current and voltage versus time

Current lags voltage by 90° in a pure inductance.

$$As\ \mathrm{V} = j\omega C\mathrm{I},\quad if\ i = I_M \cos\omega t,\ then\ v = \omega L I_M \cos(\omega t + 90°)$$

$$or\ i = I_M \cos(\omega t - 90°),\ and\ v = \omega L I_M \cos\omega t,\ V_M = \omega L I_M$$



(a) Phasor diagram

(b) Current and voltage versus time

For a pure resistance, current and voltage are in phase.

Complex Impedance in Phasor Notation

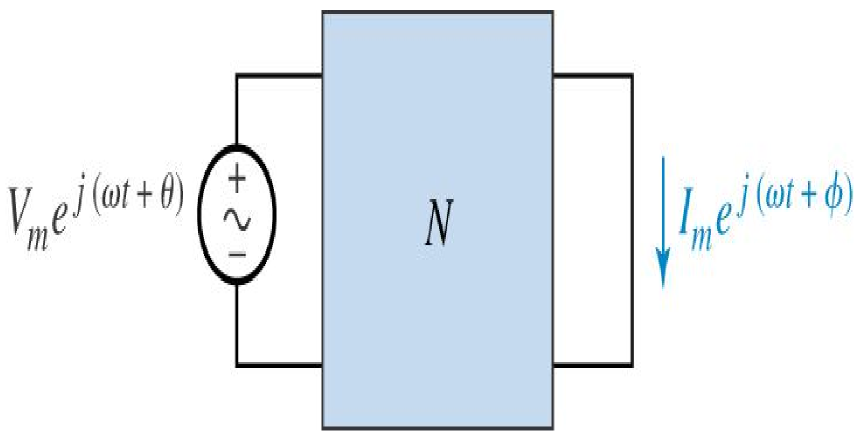
$$\mathbf{V}_C = Z_C \mathbf{I}_C$$

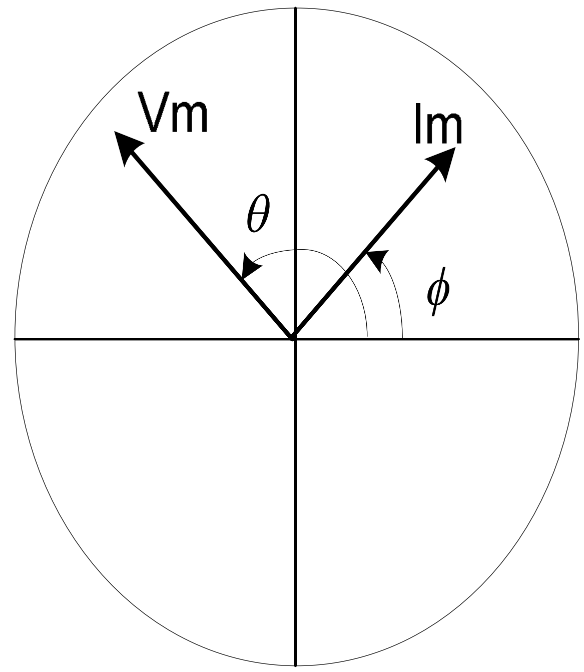
$$Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

$$\mathbf{V}_L = Z_L \mathbf{I}_L$$

$$Z_L = j\omega L = \omega L \angle 90^\circ$$

$$\mathbf{V}_R = R\mathbf{I}_R$$

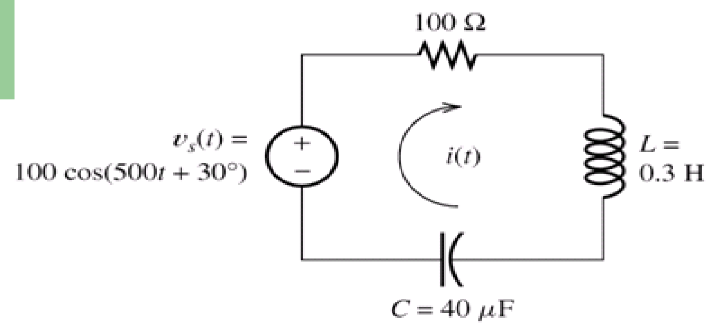

$V_m e^{j(\omega t+\theta)}$
N
$I_m e^{j(\omega t+\phi)}$

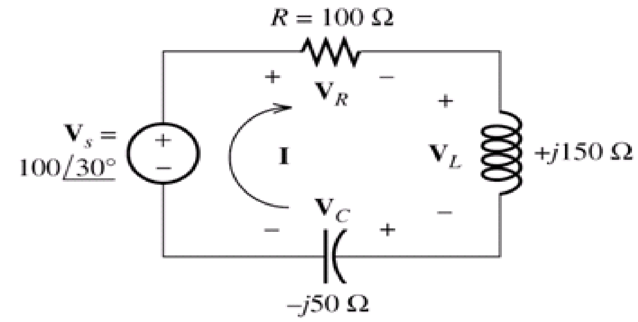

Vm
Im
θ
ϕ

Kirchhoff's Laws in Phasor Form

We can apply KVL directly to phasors. The sum of the phasor voltages equals zero for any closed path.

The sum of the phasor currents entering a node must equal the sum of the phasor currents leaving.





$$Z_{total} = 100 + j(150 - 50)$$

$$= 100 + j100 = 141.4\angle 45^\circ$$

$$\mathrm{I} = \frac{V}{Z_{total}} = \frac{100\angle 30^\circ}{141.4\angle 45^\circ} = 0.707\angle 30^\circ - 45^\circ$$

$$= 0.707\angle -15^\circ$$

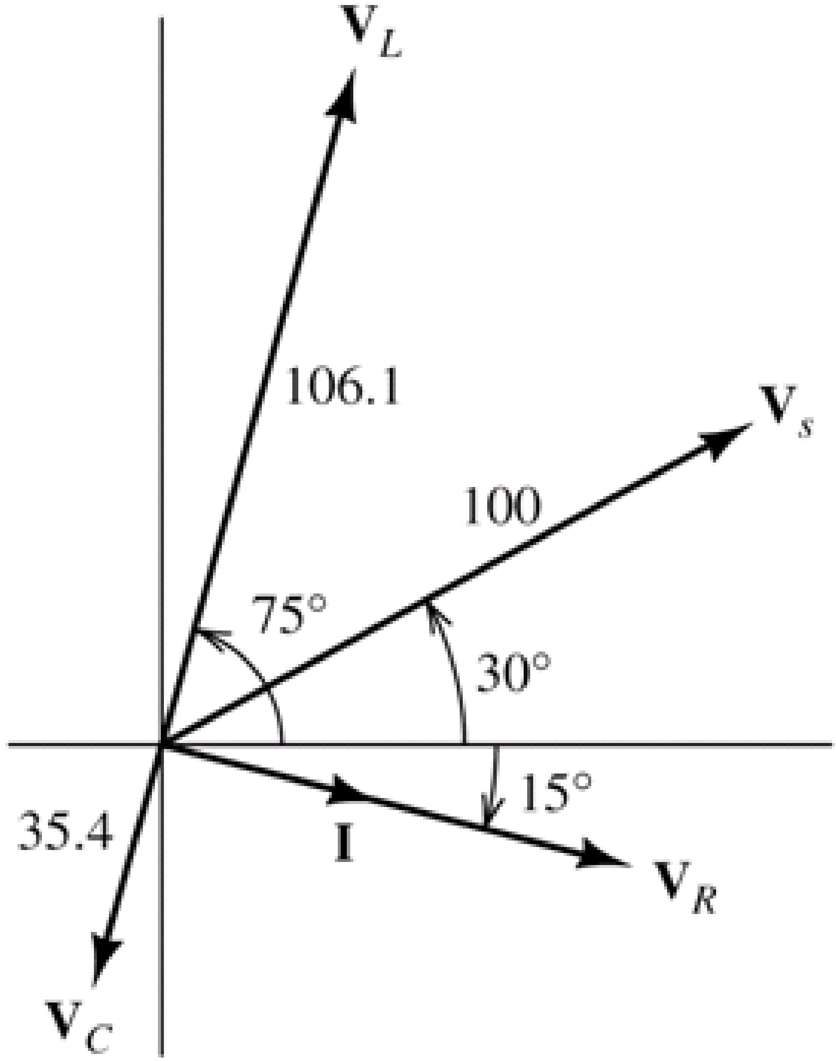
$$V_R = 100 \times \mathrm{I} = 70.7\angle -15^\circ,\ v_R(t) = 70.7\cos(500t - 15^\circ)$$

$$V_L = j150 \times \mathrm{I} = 150\angle 90^\circ \times 0.707\angle -15^\circ = 106.05\angle 90^\circ - 15^\circ$$

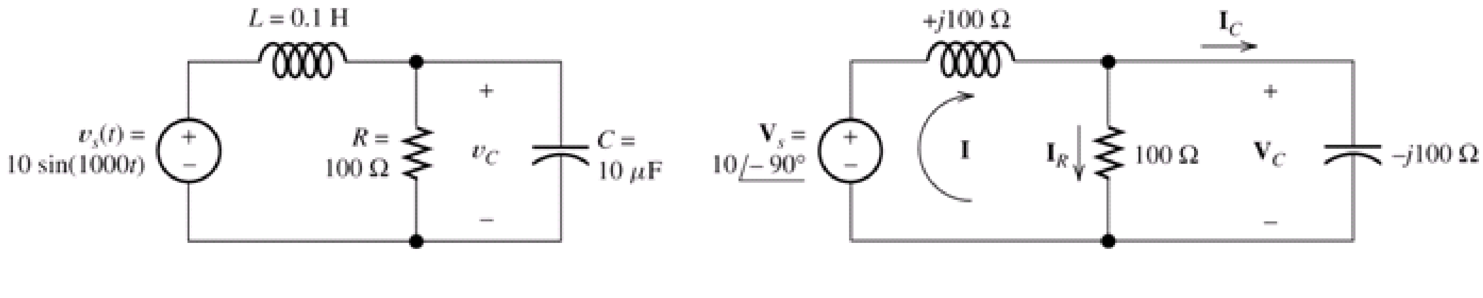
$$= 106.05\angle 75^\circ,\ v_L(t) = 106.05\cos(500t + 75^\circ)$$

$$V_C = -j50 \times \mathrm{I} = 50\angle -90^\circ \times 0.707\angle -15^\circ = 35.35\angle -90^\circ - 15^\circ$$

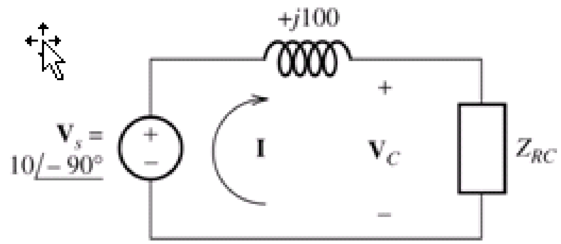
$$= 35.35\angle -105^\circ,\ v_L(t) = 35.35\cos(500t - 105^\circ)$$



$\mathbf{V}_L$
106.1
$\mathbf{V}_s$
100
75°
30°
15°
35.4
$\mathbf{I}$
$\mathbf{V}_R$
$\mathbf{V}_C$



(a) (b)

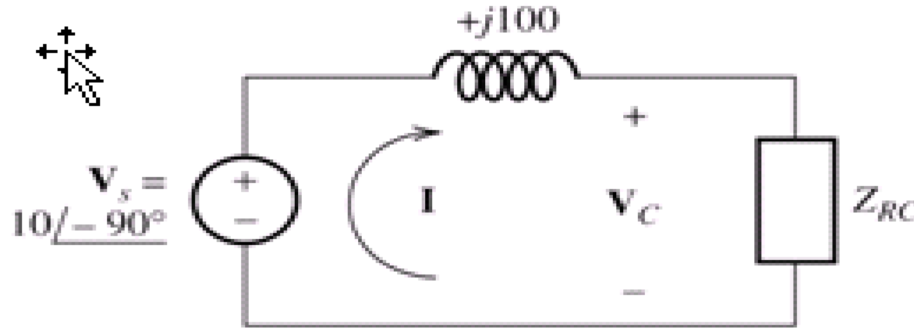


$$Z_{RC} = \frac{1}{1/R + 1/Zc} = \frac{1}{1/100 + 1/(-j100)}$$

$$As \frac{1}{-j100} = \frac{1}{-j100} \times \frac{j}{j} = \frac{j0.01}{-j^2} = j0.01$$

$$Z_{RC} = \frac{1}{0.01 + j0.01} = \frac{1\angle 0^\circ}{0.01414\angle 45^\circ} = 70.71\angle -45^\circ$$

$$= 50 - j50$$

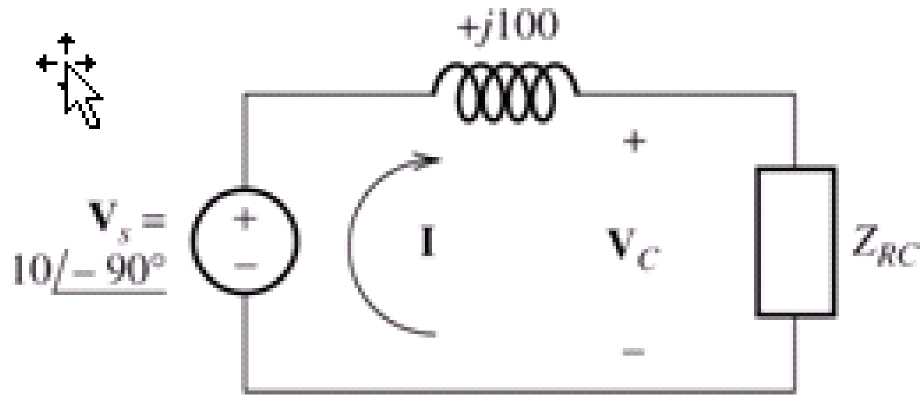


$$Vc = Vs\frac{Z_{RC}}{Z_L + Z_{RC}}\ (voltage\ division)$$

$$= 10\angle -90^\circ \frac{70.71\angle -45^\circ}{j100 + 50 - j50} = 10\angle -90^\circ \frac{70.71\angle -45^\circ}{50 + j50}$$

$$= 10\angle -45^\circ \frac{70.71\angle -45^\circ}{70.71\angle 45^\circ} = 10\ \angle -135^\circ$$

$$v_c(t) = 10\cos(1000t - 135^\circ) = -10\cos 1000t\ \text{V}$$

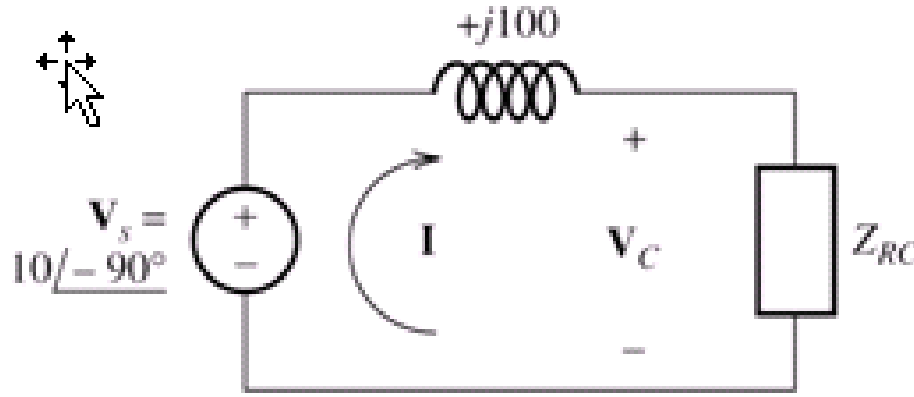


$$I = \frac{Vs}{Z_L + Z_{RC}}$$

$$= \frac{10\angle -90^\circ}{j100 + 50 - j50} = \frac{10\angle -90^\circ}{50 + j50}$$

$$= \frac{10\angle -90^\circ}{70.71\angle 45^\circ} = 0.414\angle -135^\circ$$

$$i(t) = 0.414\cos(1000t - 135^\circ)$$



$$I_R = \frac{V_C}{R}$$

$$= \frac{10\angle -135^\circ}{100} = 0.1\angle -135^\circ$$

$$i_R(t) = 0.1\cos(1000t - 135^\circ)$$

$$I_C = \frac{V_C}{Zc} = \frac{10\angle -135^\circ}{-j100} = \frac{10\angle -135^\circ}{100\angle -90^\circ} = 0.1\angle -45^\circ$$

$$i_R(t) = 0.1\cos(1000t - 45^\circ)\ \text{A}$$

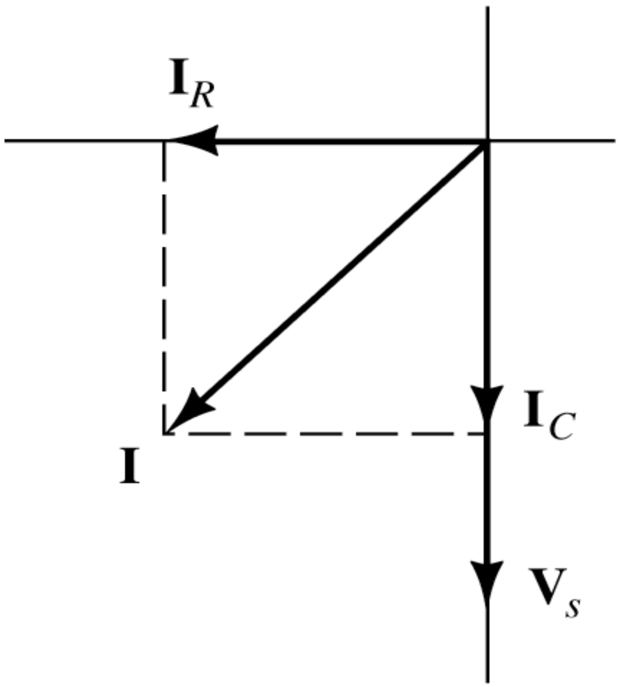
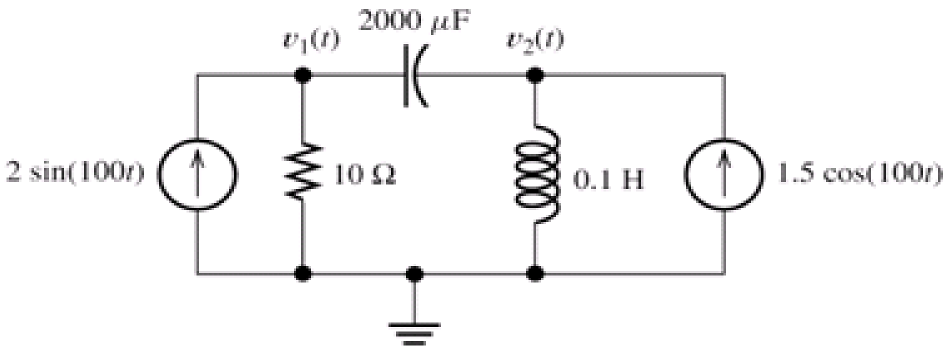
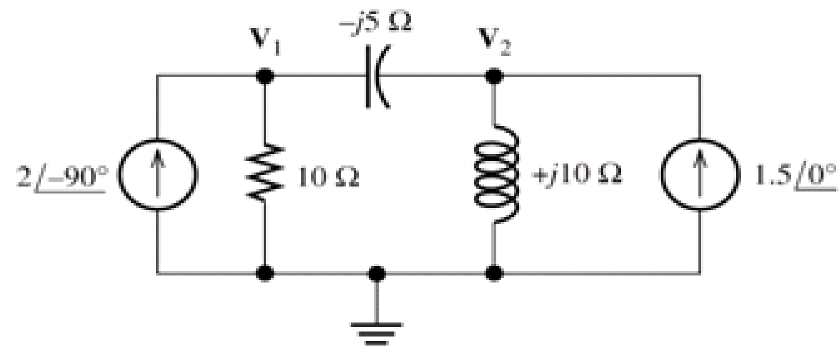


Figure 5.14 Phasor diagram for Example 5.4.





Solve by nodal analysis

$$\frac{V_1}{10} + \frac{V_1 - V_2}{-j5} = 2\angle -90^\circ = -j2 \quad eq(1)$$

$$\frac{V_2}{j10} + \frac{V_2 - V_1}{-j5} = 1.5\angle 0^\circ = 1.5 \quad eq(2)$$

$$\frac{V_1}{10}+\frac{V_1-V_2}{-j5}=2\angle -90^\circ=-j2 \quad eq(1) \qquad \frac{V_2}{j10}+\frac{V_2-V_1}{-j5}=1.5\angle 0^\circ=1.5 \qquad eq(2)$$

$From\, eq\,(1)$

$$0.1V_1+j0.2V_1-j0.2V_2=-j2 \qquad As\ \frac{1}{j}=\frac{1}{j}\times\frac{j}{j}=\frac{j}{-1}=-j,\ \frac{1}{-j}=j$$

$$(0.1+j0.2)V_1-j0.2V_2=-j2$$

$From\, eq\,(2)$

$$-j0.2V_1+j0.1V_2=1.5$$

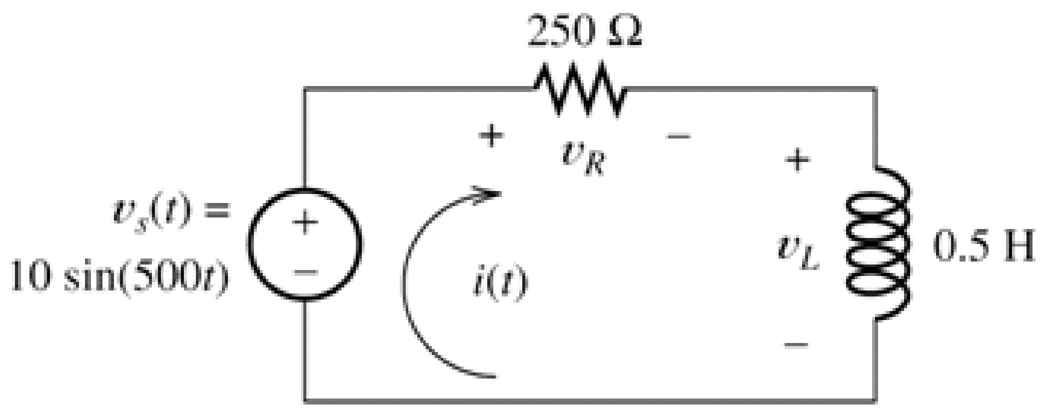
$Solving\, V_1\ by\ \ eq(1)+2\times eq(2)$

$$(0.1-j0.2)V_1=3-j2$$

$$V_1=\frac{3-j2}{0.1-j0.2}=\frac{3.6\angle -33.69^\circ}{0.2236\angle -63.43^\circ}=16.1\angle -33.69^\circ+63.43^\circ$$

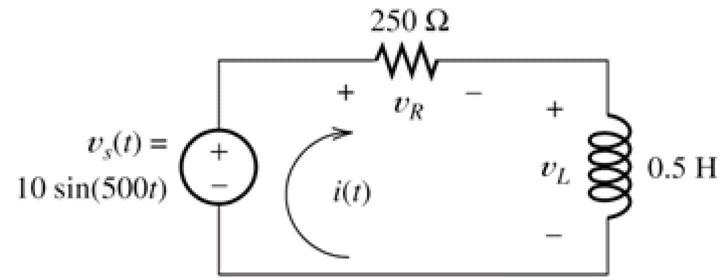
$$=16.1\angle 29.74^\circ$$

$$v_1=16.1\cos(100t+29.74^\circ)\ \text{V}$$



Vs= - j10, Z_L=jωL=j(0.5×500)=j250

Use mesh analysis,



$$-Vs + V_R + V_Z = 0$$

$$-(-j10) + I \times 250 + I \times (j250) = 0$$

$$I = \frac{-j10}{250 + j250} = \frac{10\angle -90^\circ}{353.33\angle 45^\circ} = 0.028\angle -90^\circ - 45^\circ$$

$$I = 0.028\angle -135^\circ$$

$$i = 0.028\cos(500t - 135^\circ)\text{A}$$

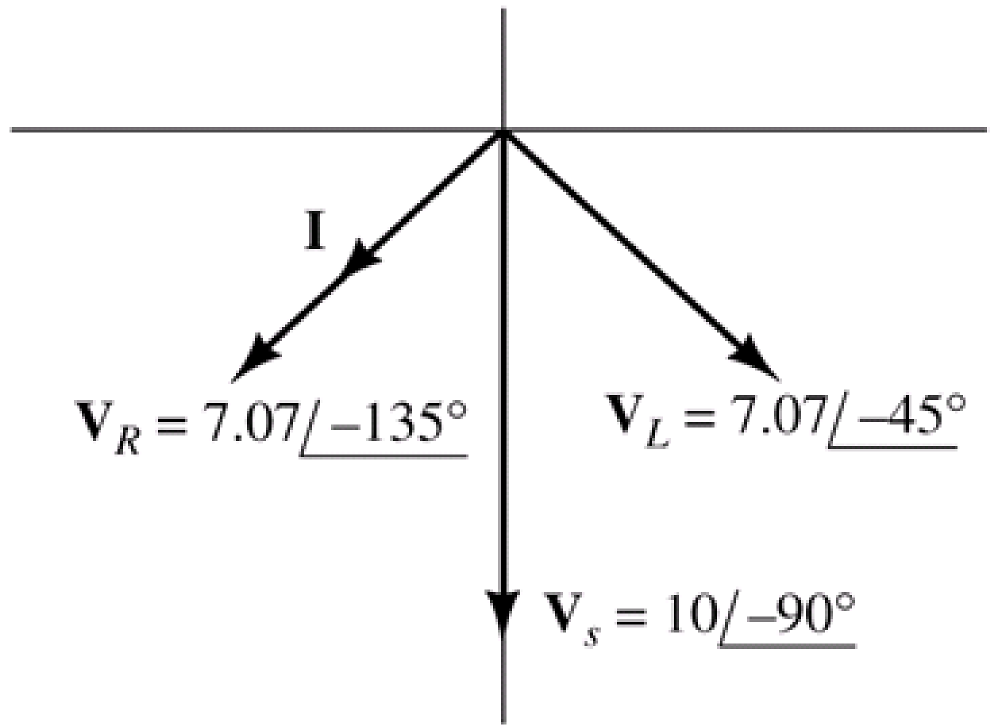
$$V_L = I \times Z_L = (0.028\angle -135^\circ) \times 250\angle 90^\circ$$

$$= 7\angle -45^\circ$$

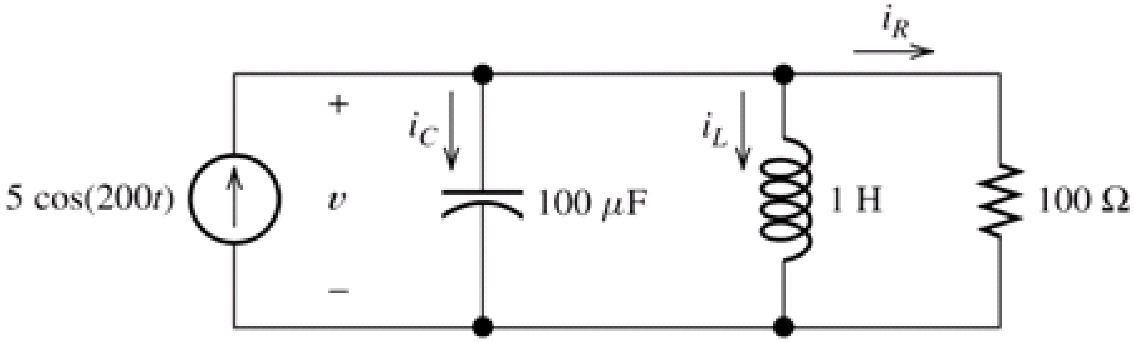
$$v_L(t) = 7\cos(500t - 45^\circ)\ \text{V}$$

$$V_R = I \times R = (0.028\angle -135^\circ) \times 250 = 7\angle -135^\circ$$

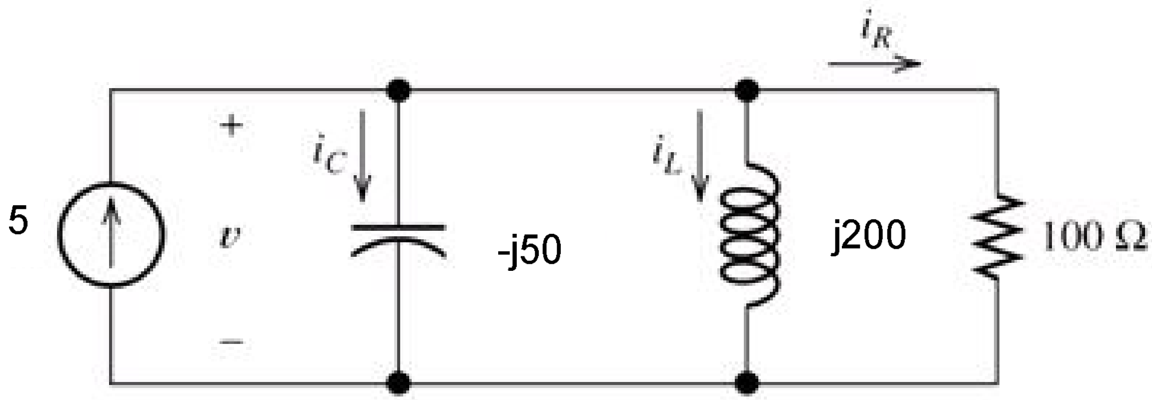
$$v_R(t) = 7\cos(500t - 135^\circ)$$



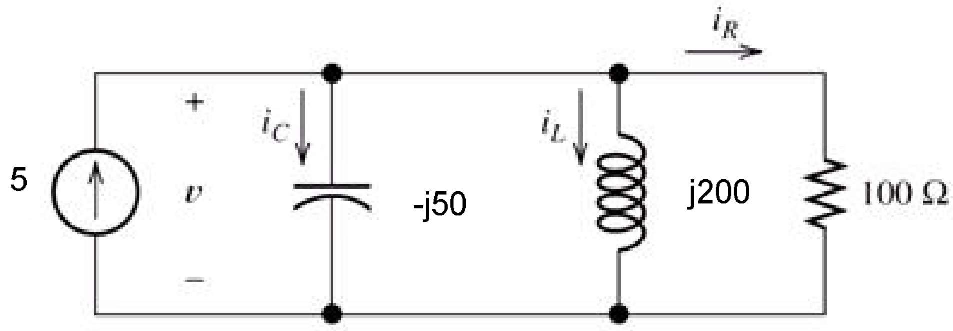
I
$V_R = 7.07\angle -135°$
$V_L = 7.07\angle -45°$
$V_s = 10\angle -90°$



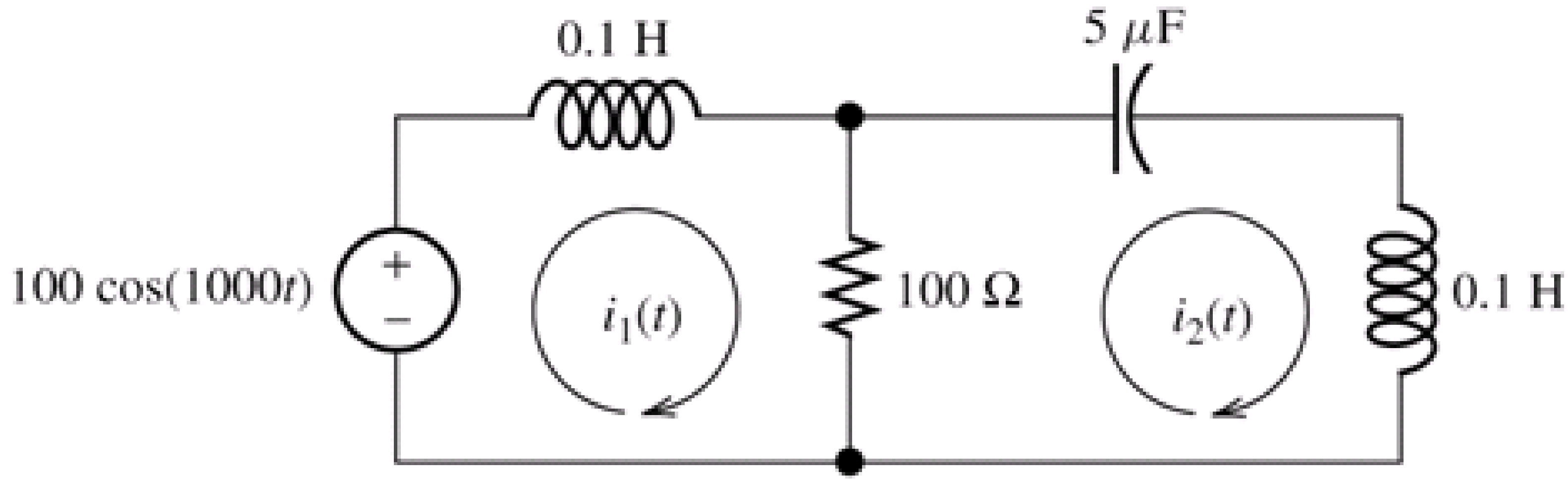
i_R
+
i_C
i_L
5 cos(200t)
v
100 μF
1 H
100 Ω
−

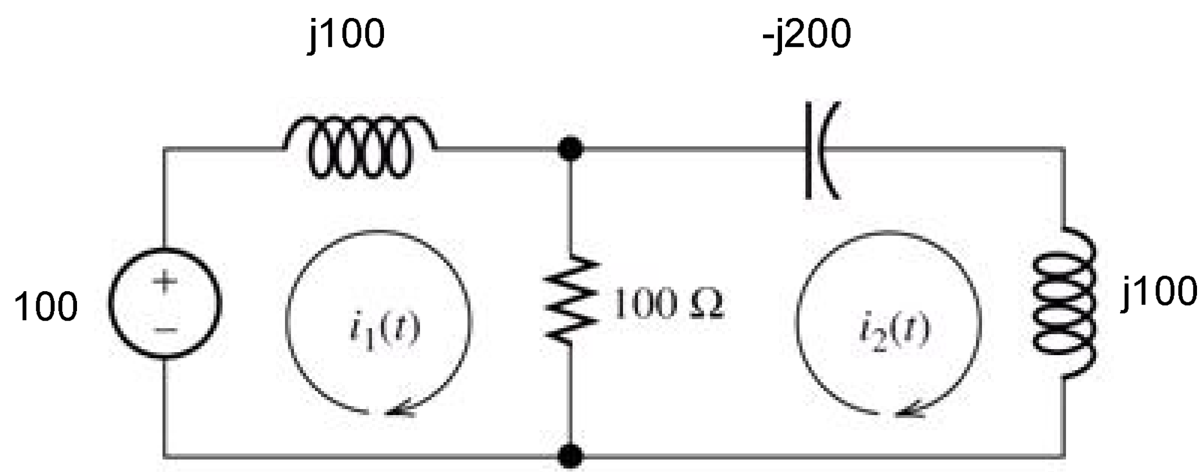


i_R
+
i_C
i_L
5
v
-j50
j200
100 Ω
−



i_R
+
i_C
i_L
5
v
-j50
j200
100 Ω
−


0.1 H
5 μF
100 cos(1000t)
i1(t)
100 Ω
i2(t)
0.1 H


j100
-j200
100
i1(t)
100 Ω
i2(t)
j100

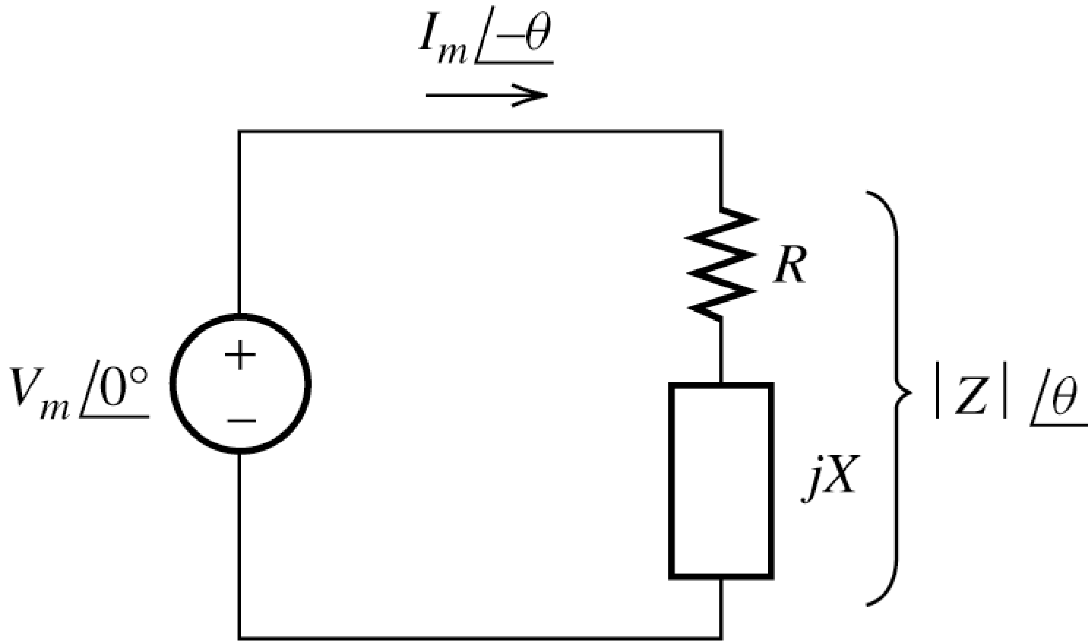


Figure 5.19 A voltage source delivering power to a load impedance $Z = R + jX$.

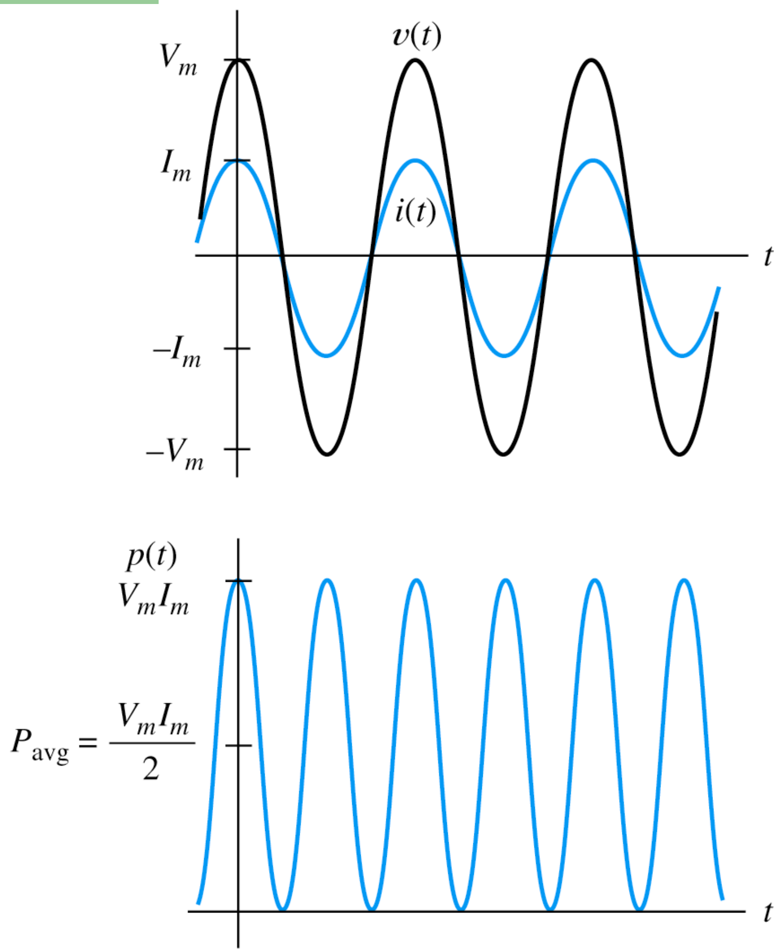
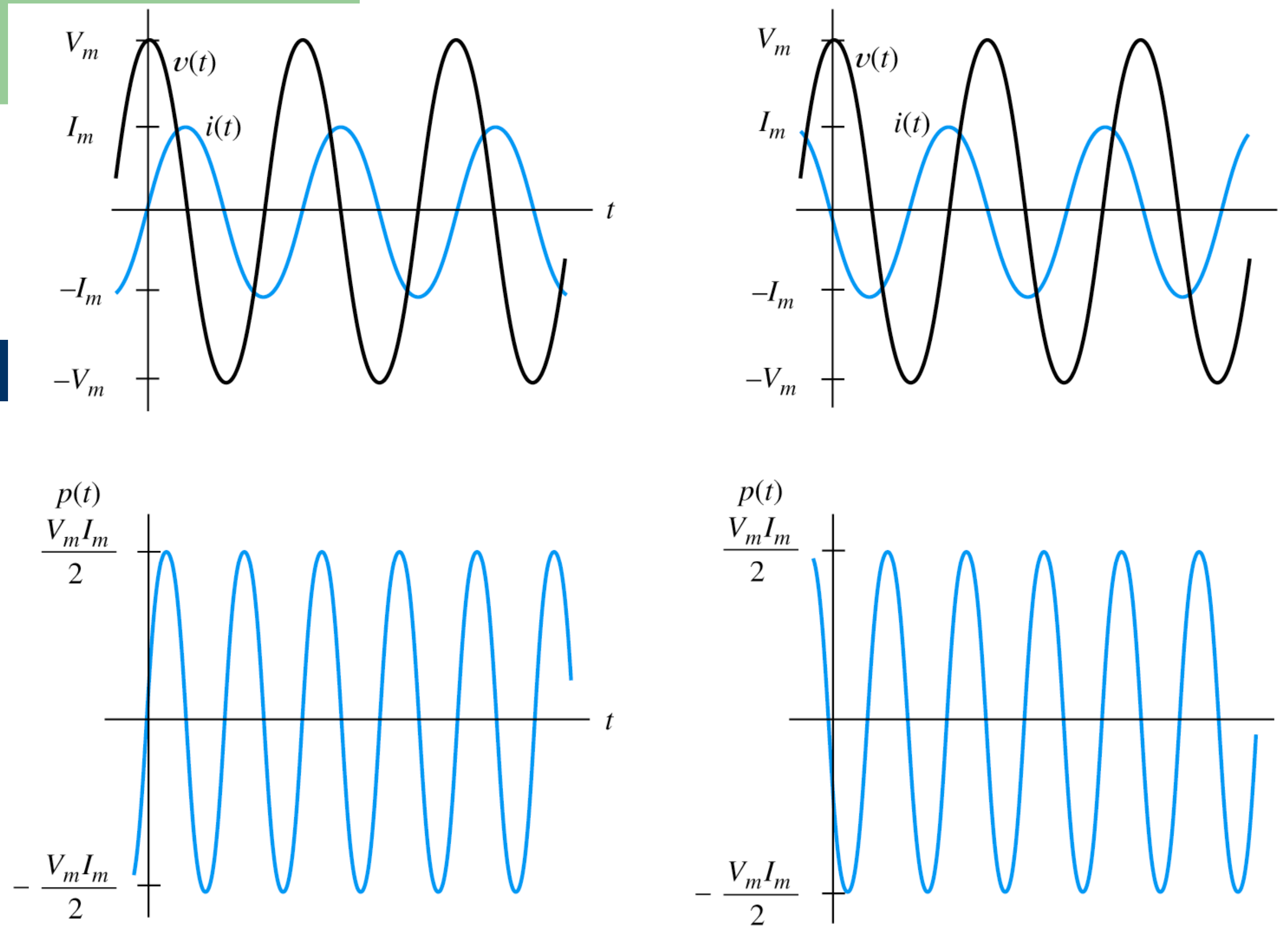


Figure 5.20 Current, voltage, and power versus time for a purely resistive load.



(a) Pure inductive load

(b) Pure capacitive load

Figure 5.21 Current, voltage, and power versus time for pure energy-storage elements.

AC Power Calculations

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$$

$$\text{PF} = \cos(\theta)$$

$$\theta = \theta_v - \theta_i$$

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta)$$

$$\text{apparent power} = V_{\text{rms}} I_{\text{rms}}$$

$$P^2 + Q^2 = \left(V_{\text{rms}} I_{\text{rms}}\right)^2$$

$$P = I_{\text{rms}}^2 R \qquad P = \frac{V_{R\text{rms}}^2}{R}$$

$$Q = I_{\text{rms}}^2 X \qquad Q = \frac{V_{X\text{rms}}^2}{X}$$

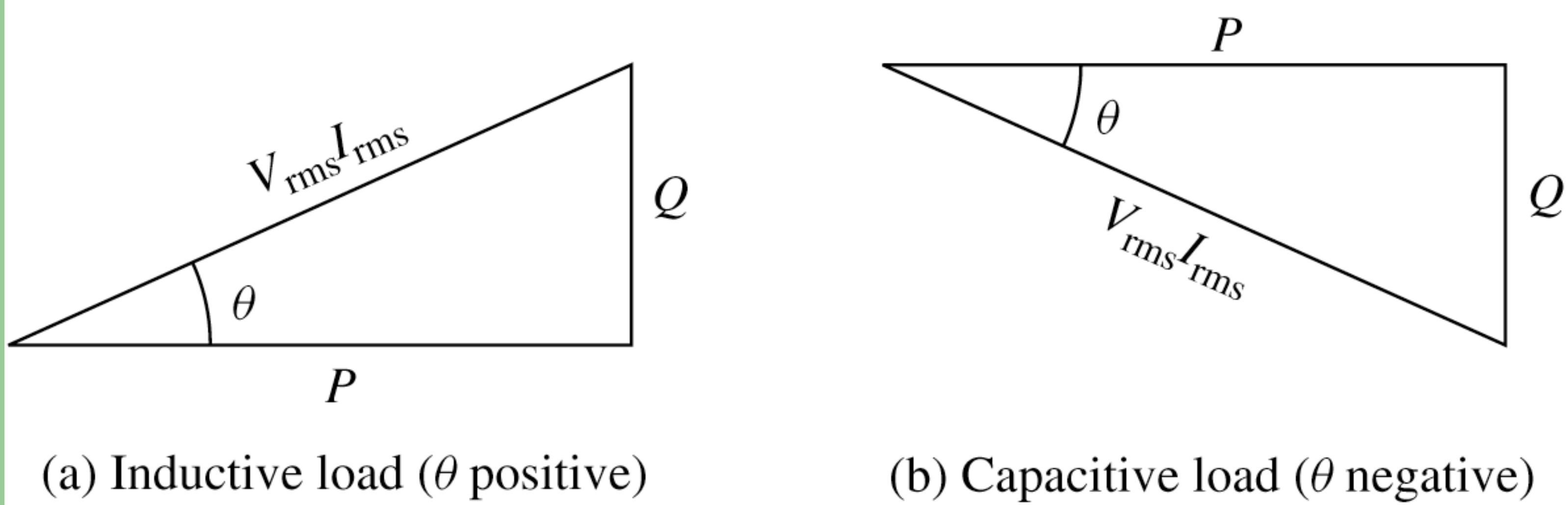


Figure 5.22 Power triangles for inductive and capacitive loads.

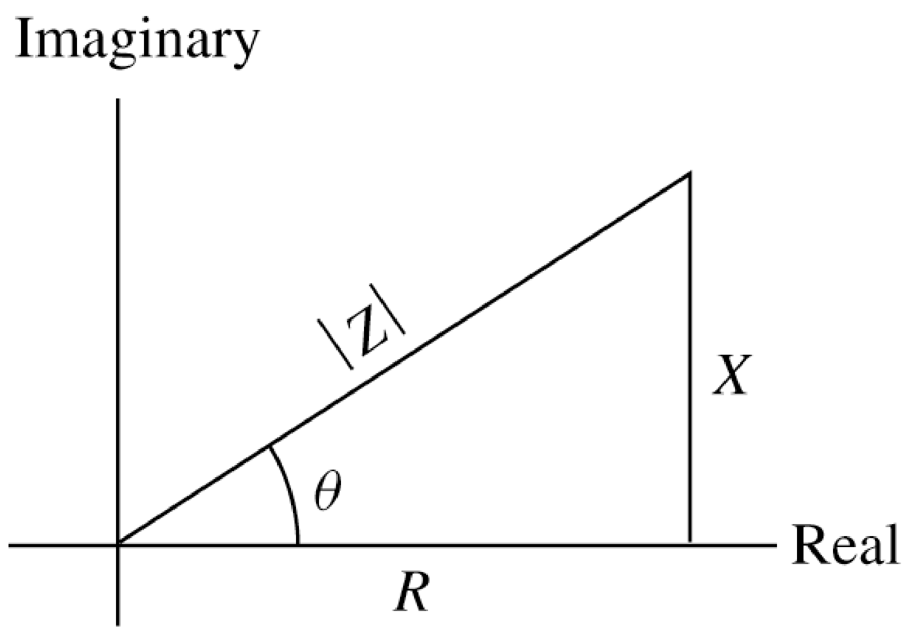


Figure 5.23 The load impedance in the complex plane.

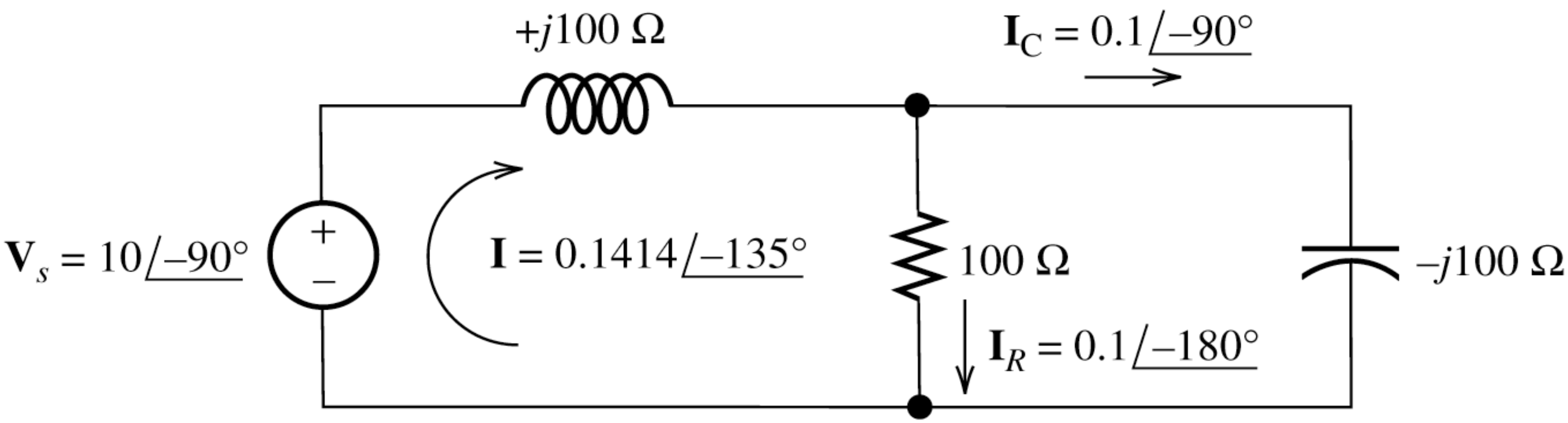


Figure 5.24 Circuit and currents for Example 5.6.

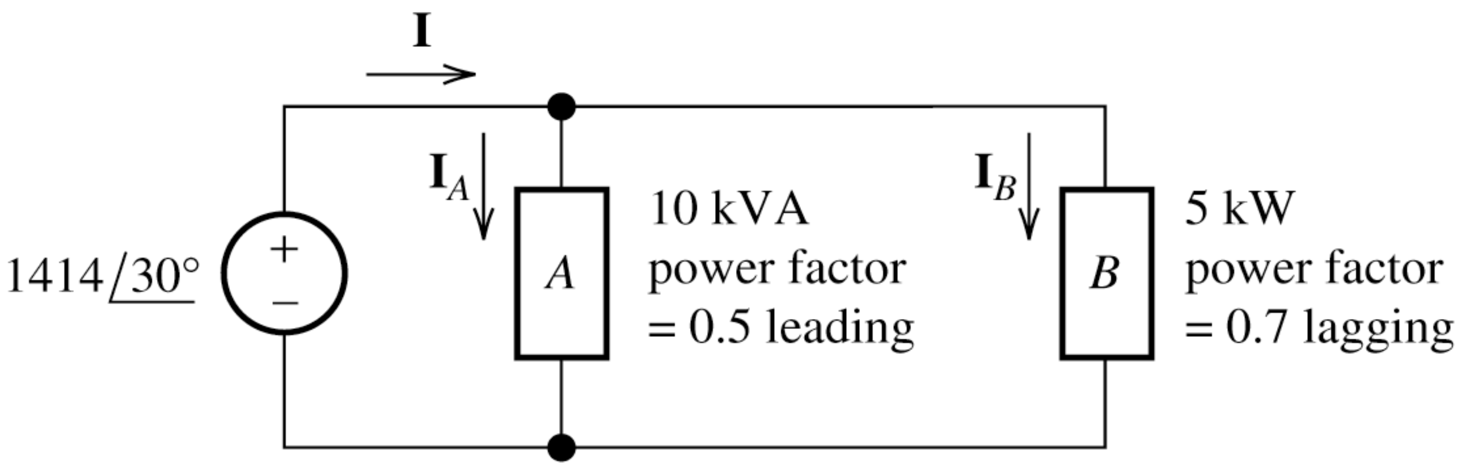


Figure 5.25 Circuit for Example 5.7.

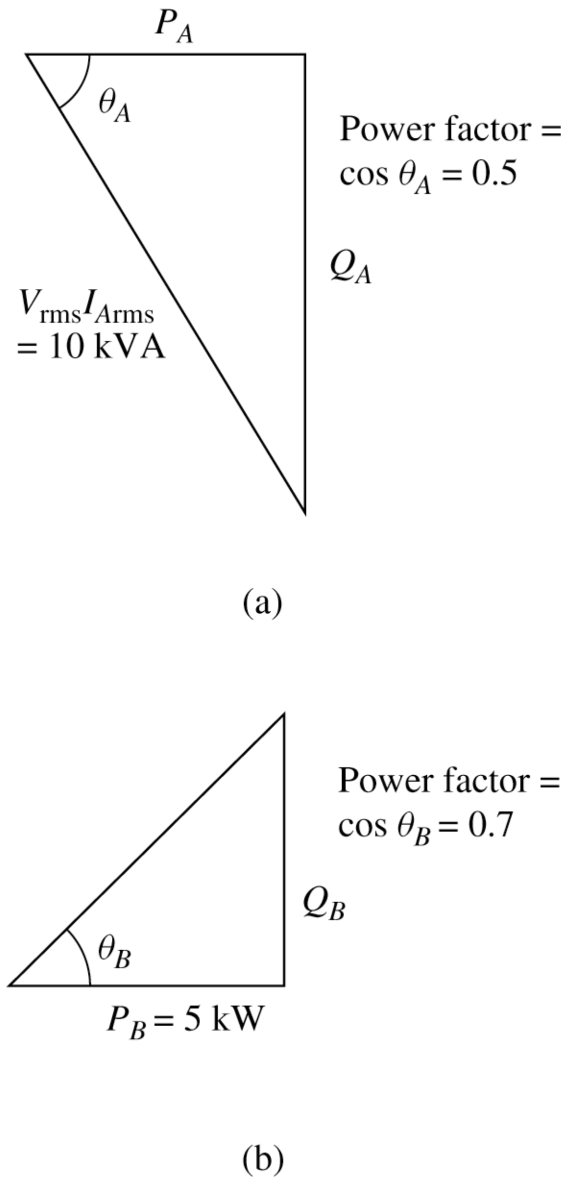


Figure 5.26 Power triangles for loads A and B of Example 5.7.

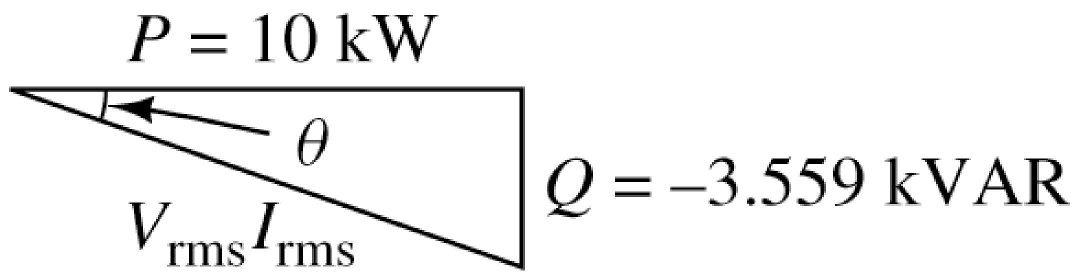


Figure 5.27 Power triangle for the source of Example 5.7.

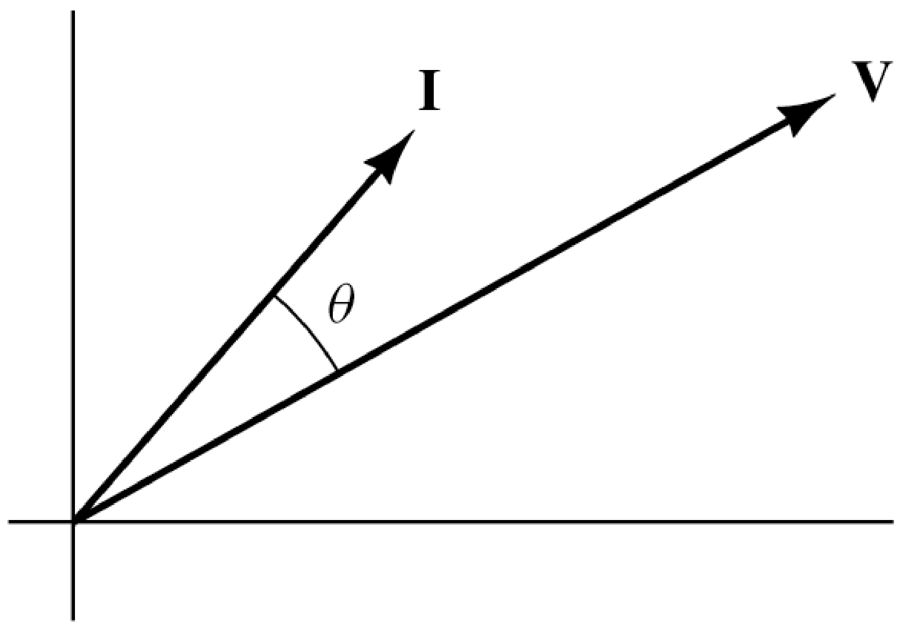


Figure 5.28 Phasor diagram for Example 5.7.

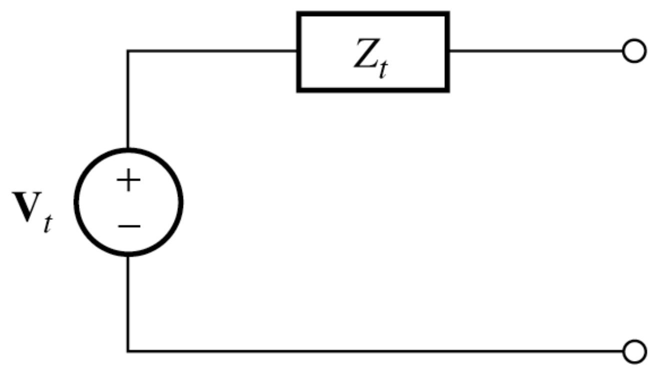


Figure 5.29 The Thévenin equivalent for an ac circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.

THÉVENIN EQUIVALENT CIRCUITS

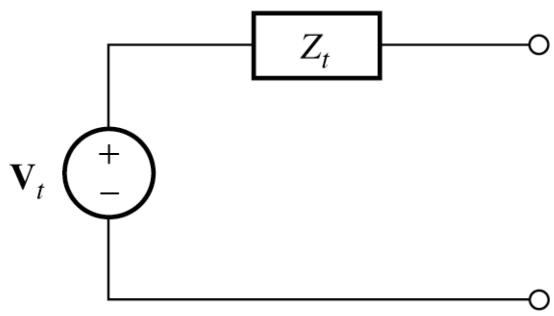


Figure 5.29 The Thévenin equivalent for an ac circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.

The Thévenin voltage is equal to the open-circuit phasor voltage of the original circuit.

$$\mathbf{V}_t = \mathbf{V}_{\mathrm{oc}}$$

We can find the Thévenin impedance by zeroing the independent sources and determining the impedance looking into the circuit terminals.

The Thévenin impedance equals the open-circuit voltage divided by the short-circuit current.

$$Z_t = \frac{\mathbf{V}_{\mathrm{oc}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{\mathbf{V}_t}{\mathbf{I}_{\mathrm{sc}}}$$

$$\mathbf{I}_n = \mathbf{I}_{\mathrm{sc}}$$

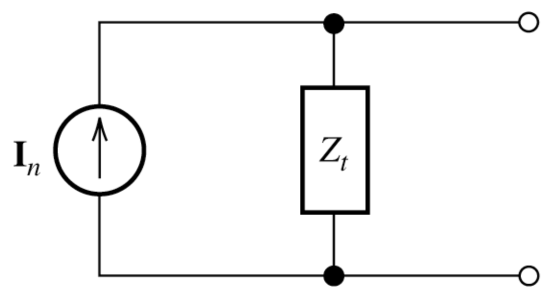


Figure 5.30 The Norton equivalent circuit consists of a phasor current source $\mathbf{I}_n$ in parallel with the complex impedance Z_t.

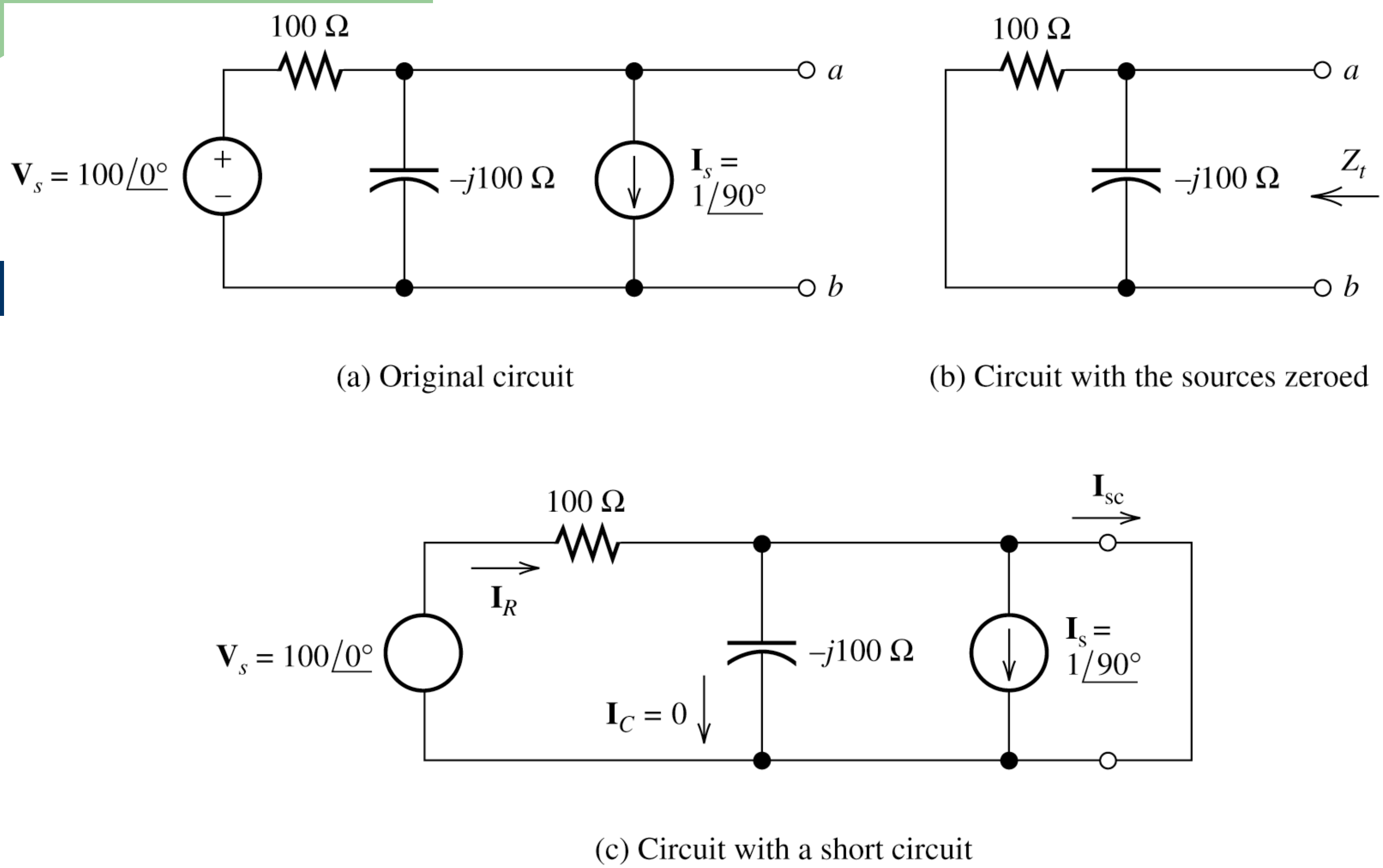


Figure 5.31 Circuit of Example 5.9.

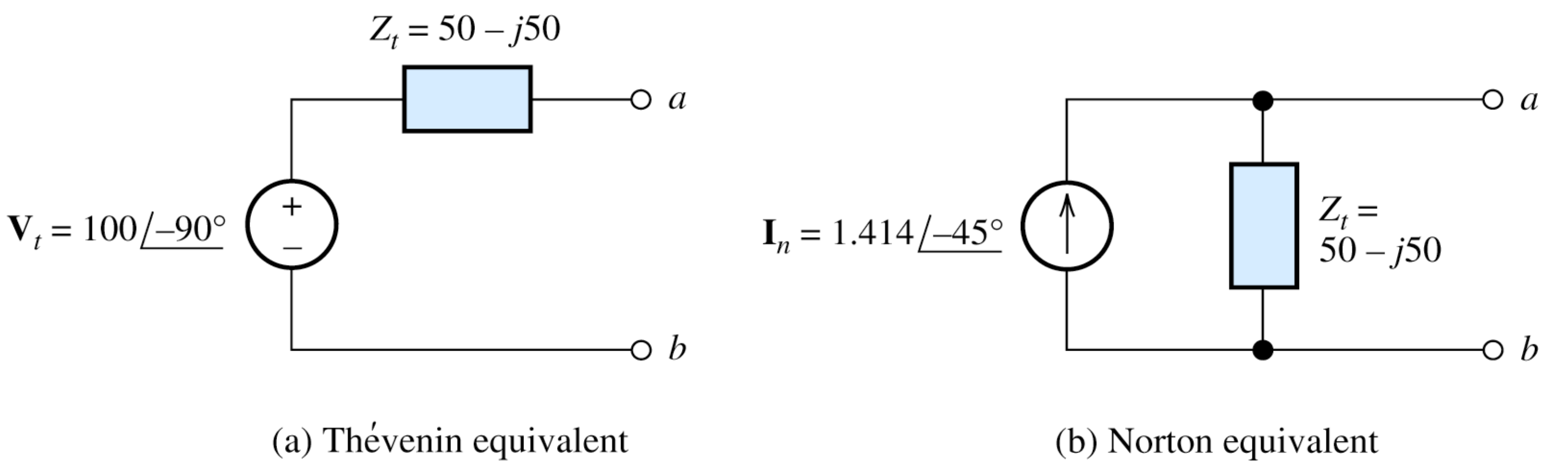


Figure 5.32 Thévenin and Norton equivalents for the circuit of Figure 5.31a.

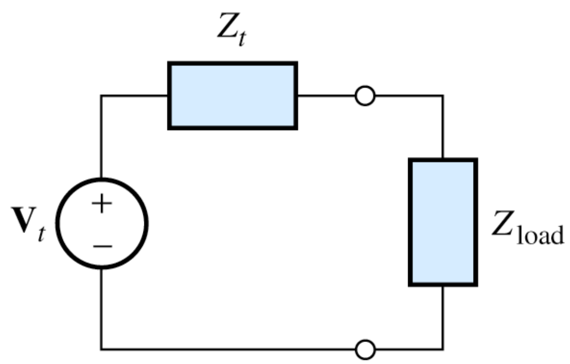


Figure 5.33 The Thévenin equivalent of a two-terminal circuit delivering power to a load impedance.

Maximum Power Transfer

- If the load can take on any complex value, maximum power transfer is attained for a load impedance equal to the complex conjugate of the Thévenin impedance.
  - If the load is required to be a pure resistance, maximum power transfer is attained for a load resistance equal to the magnitude of the Thévenin impedance.

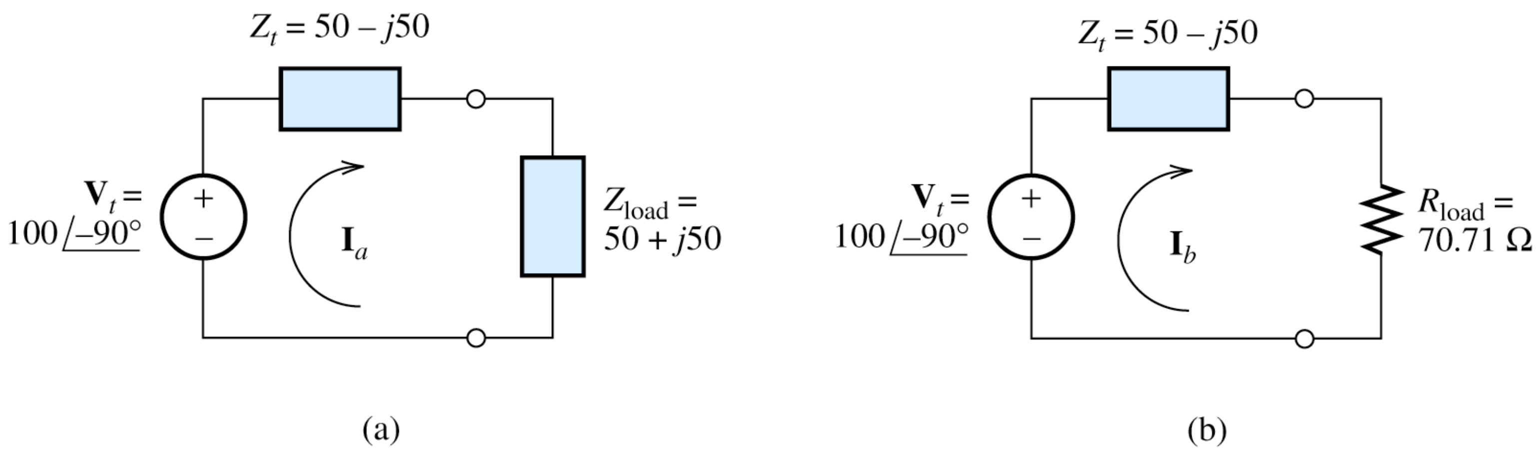


Figure 5.34 Thévenin equivalent circuit and loads of Example 5.10.

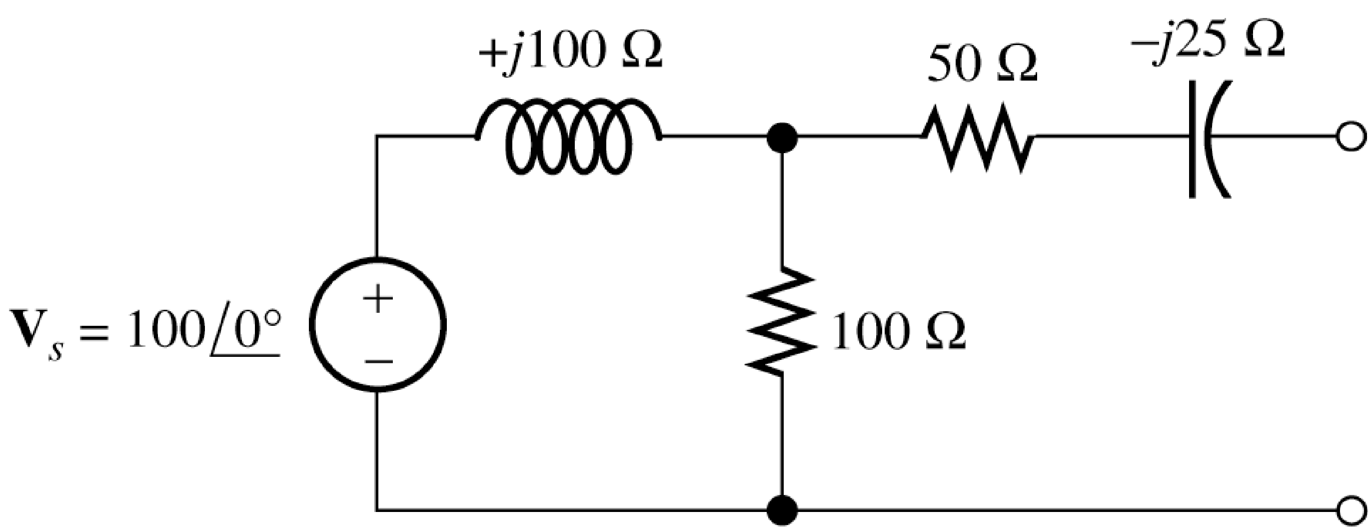


Figure 5.35 Circuit of Exercises 5.14 and 5.15.

UNIT III–NETWORK THEOREMS (BOTH AC AND DC CIRCUITS) **9**

– Superposition theorem
– The venin's theorem
- Norton's theorem
-Reciprocity theorem
- Maximum power transfer theorem.

9.1 – Introduction

- This chapter introduces important fundamental theorems of network analysis. They are the
  - **Superposition theorem**
  - **Thévenin’s theorem**
  - **Norton’s theorem**
  - **Maximum power transfer theorem**
  - **Substitution Theorem**
  - **Millman’s theorem**
  - **Reciprocity theorem**

9.2 – Superposition Theorem

Used to find the solution to networks with two or more sources that are not in series or parallel.

The current through, or voltage across, an element in a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

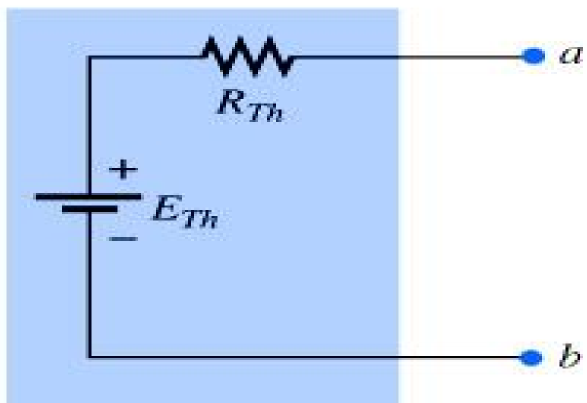
Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

Superposition Theorem

The total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

9.3 – Thévenin's Theorem

- Any two-terminal dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

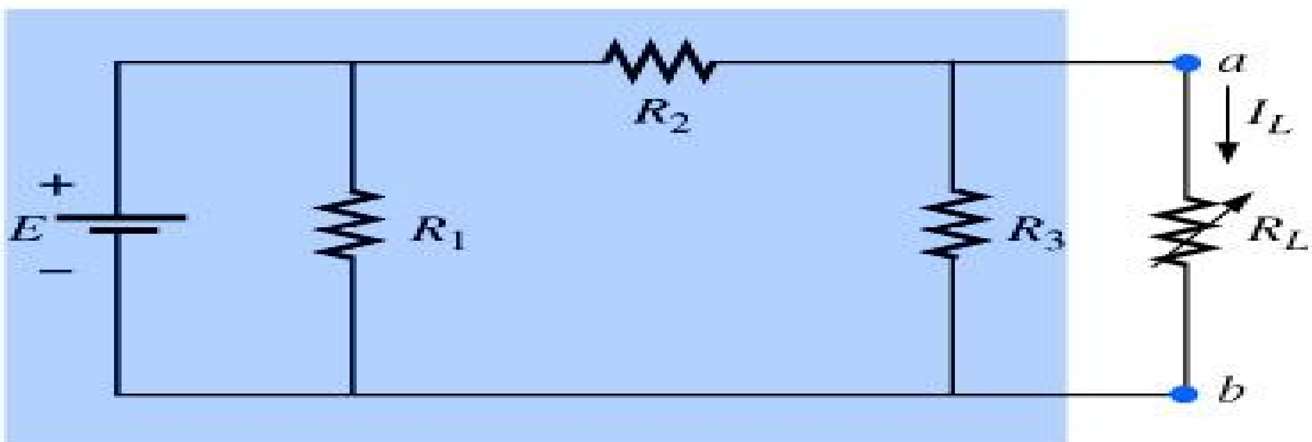


Thévenin's Theorem

- Thévenin's theorem can be used to:
  - Analyze networks with sources that are not in series or parallel.
  - Reduce the number of components required to establish the same characteristics at the output terminals.
  - Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.

Thévenin's Theorem

- Procedure to determine the proper values of R_{Th} and E_{Th}
- Preliminary
  1. **Remove that portion of the network across which the Thévenin equation circuit is to be found. In the figure below, this requires that the load resistor R_L be**



Thévenin's Theorem

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

R_{Th}:

3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

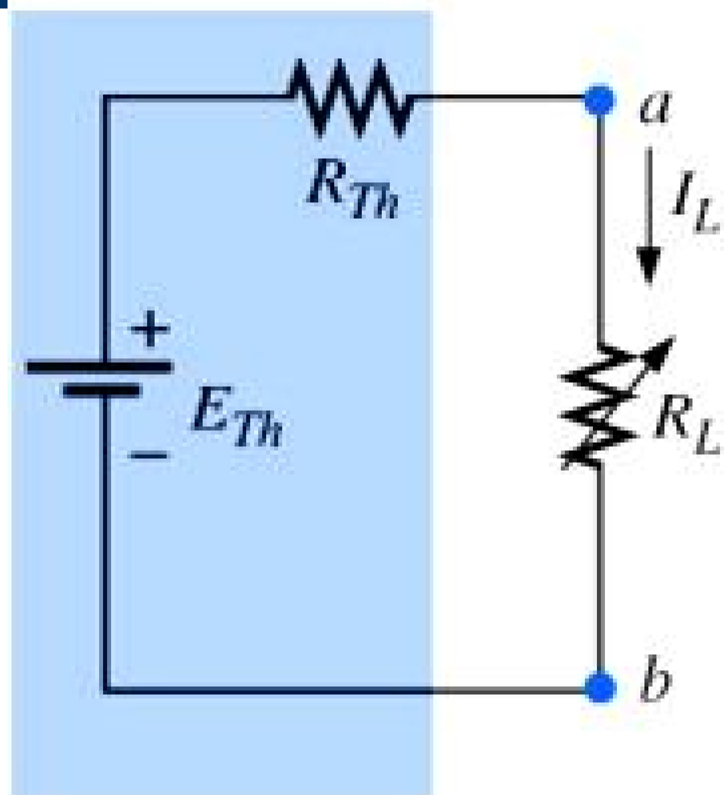
Thévenin's Theorem

E_{Th}:

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)

’s Theorem

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit.



Experimental Procedures

- popular experimental procedures for determining the parameters of the Thévenin equivalent network:
  - Direct Measurement of E_{Th} and R_{Th}
    - For any physical network, the value of E_{Th} can be determined experimentally by measuring the open-circuit voltage across the load terminals.
    - The value of R_{Th} can then be determined by completing the network with a variable resistance R_L.

Thévenin's Theorem

Measuring V_{OC} and I_{SC}

- The Thévenin voltage is again determined by measuring the open-circuit voltage across the terminals of interest; that is, $E_{Th} = V_{OC}$. To determine R_{Th}, a short-circuit condition is established across the terminals of interest and the current through the short circuit (I_{sc}) is measured with an ammeter.
- Using Ohm's law:

$$R_{Th} = V_{oc} / I_{sc}$$

Vth

the Thévenin voltage is determined by connecting a voltmeter to the output terminals of the network. Be sure the internal resistance of the voltmeter is significantly more than the expected level of R_{Th}.

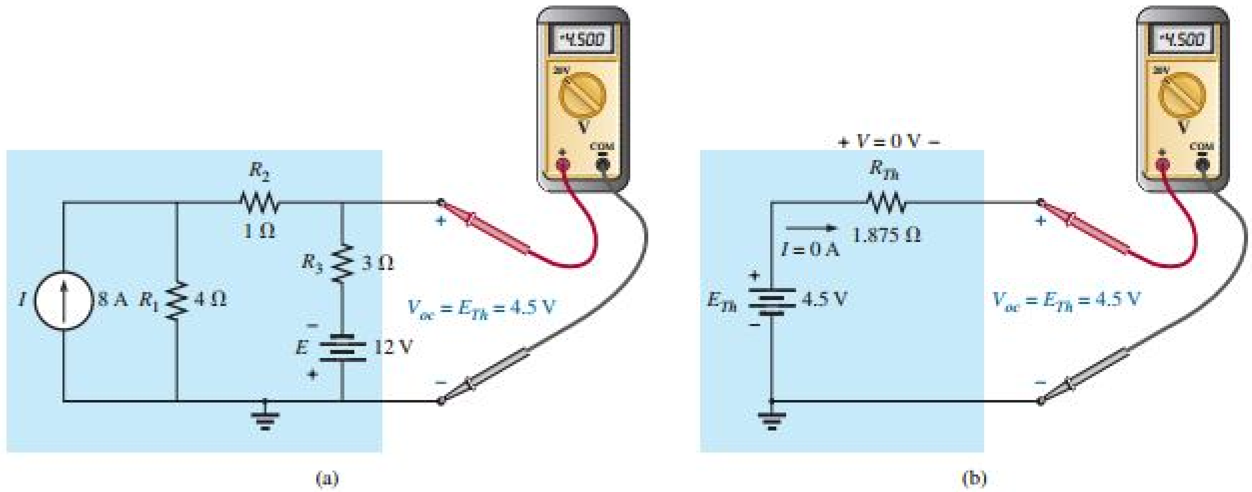


FIG. 9.60

Measuring the Thévenin voltage with a voltmeter: (a) actual network; (b) Thévenin equivalent.

Rth

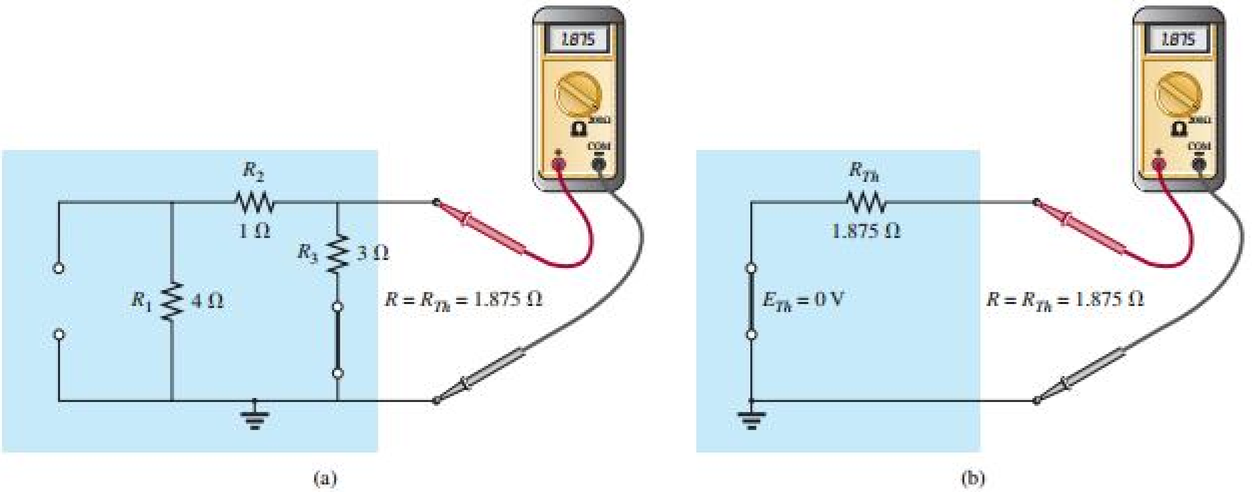


FIG. 9.61

Measuring R_{Th} with an ohmmeter: (a) actual network; (b) Thévenin equivalent.

Norton's Theorem

Norton's theorem states the following:

- Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.

The steps leading to the proper values of I_N and R_N.

Preliminary steps:

1. Remove that portion of the network across which the Norton equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network.

Norton's Theorem

Finding R_N:

3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$ the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

Norton's Theorem

Finding I_N :

4. [redacted] their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

Conclusion:

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

9.4 NORTON'S THEOREM

In Section 8.2, we learned that every voltage source with a series internal resistance has a current source equivalent. The current source equivalent can be determined by **Norton's theorem** (Fig. 9.64). It can also be found through the conversions of Section 8.2.

The theorem states the following:

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig. 9.65.

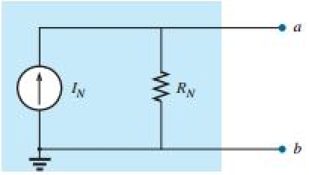


FIG. 9.65
Norton equivalent circuit.

The discussion of Thévenin's theorem with respect to the equivalent circuit can also be applied to the Norton equivalent circuit. The steps leading to the proper values of I_N and R_N are now listed.

Norton's Theorem Procedure

Preliminary:

1. ***Remove that portion of the network across which the Norton equivalent circuit is found.***
2. ***Mark the terminals of the remaining two-terminal network.***

FIG. 9.64
Edward L. Norton.
Reprinted with the permission of Lucent Technologies, Inc./Bell Labs.

American (Rockland, Maine; Summit, New Jersey)
1898–1983
Electrical Engineer, Scientist, Inventor
Department Head: Bell Laboratories
Fellow: Acoustical Society and Institute of Radio Engineers

Although interested primarily in communications circuit theory and the transmission of data at high speeds over telephone lines, Edward L. Norton is best remembered for development of the dual of Thévenin equivalent circuit, currently referred to as *Norton's equivalent circuit.* In fact, Norton and his associates at AT&T in the early 1920s are recognized as being among the first to perform work applying Thévenin's equivalent circuit and referring to this concept simply as Thévenin's theorem. In 1926, he proposed the equivalent circuit using a current source and parallel resistor to assist in the design of recording instrumentation that was primarily current driven. He began his telephone career in 1922 with the Western Electric Company's Engineering Department, which later became Bell Laboratories. His areas of active research included network theory, acoustical systems, electromagnetic apparatus, and data transmission. A graduate of MIT and Columbia University, he held nineteen patents on his work.

Maximum Power Transfer Theorem

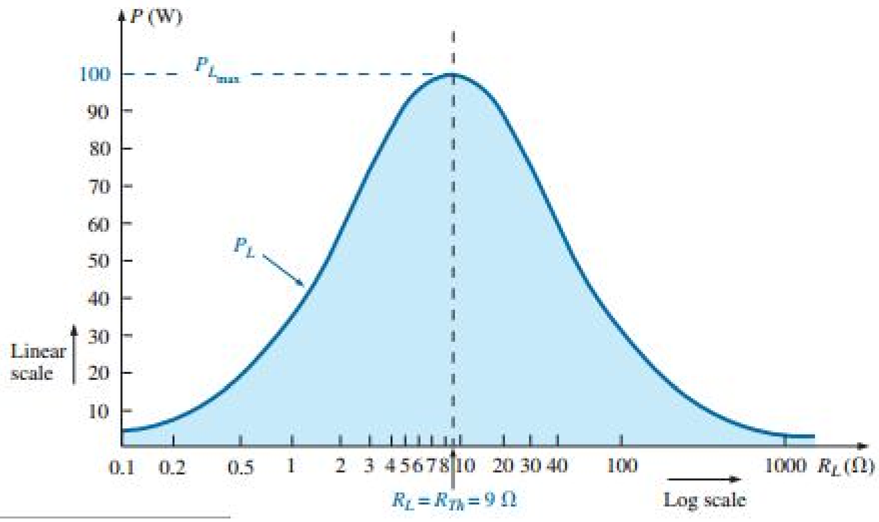
- The maximum power transfer theorem states the following:

  A load will receive maximum power from a network when its total resistive value is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$

Maximum Power Transfer Theorem

For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when: $R_L = R_{int}$



Reciprocity Theorem

- The reciprocity theorem is applicable only to single-source networks and states the following:
  - The current ***I*** in any branch of a network, due to a single voltage source ***E*** anywhere in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current ***I*** was originally measured.
    - The location of the voltage source and the resulting current may be interchanged without a change in current

UNIT IV - TRANSIENT RESPONSE FOR DC CIRCUITS

–– Transient response of RL, RC and RLC

–– Laplace transform for DC input with sinusoidal input.

Solution to First Order Differential Equation

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Let the initial condition be $x(t = 0) = x(0)$, then we solve the differential equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete solution consists of two parts:
- the homogeneous solution (natural solution)
- the particular solution (forced solution)

The Natural Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

Setting the excitation *f* (*t*) equal to zero,

$$\tau \frac{dx_N(t)}{dt} + x_N(t) = 0 \ or \quad \frac{dx_N(t)}{dt} = -\frac{x_N(t)}{\tau}, \ \frac{dx_N(t)}{x_N(t)} = -\frac{dt}{\tau}$$

$$\int \frac{dx_N(t)}{x_N(t)} = \int -\frac{dt}{\tau}, \qquad x_N(t) = \alpha\, e^{-t/\tau}$$

It is called the natural response.

The Forced Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation *f* (*t*) equal to F, a constant for *t* ≥0

$$\tau \frac{dx_F(t)}{dt} + x_F(t) = K_S F$$

$$x_F(t) = K_S F \quad for \ t \geq 0$$

It is called the forced response.

The Complete Response

Solve for α,

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete response is:

- **the natural response** +
- **the forced response**

$$x = x_N(t) + x_F(t)$$

$$= \alpha e^{-t/\tau} + K_S F$$

$$= \alpha e^{-t/\tau} + x(\infty)$$

$$for\ \ t = 0$$

$$x(t=0) = x(0) = \alpha + x(\infty)$$

$$\alpha = x(0) - x(\infty)$$

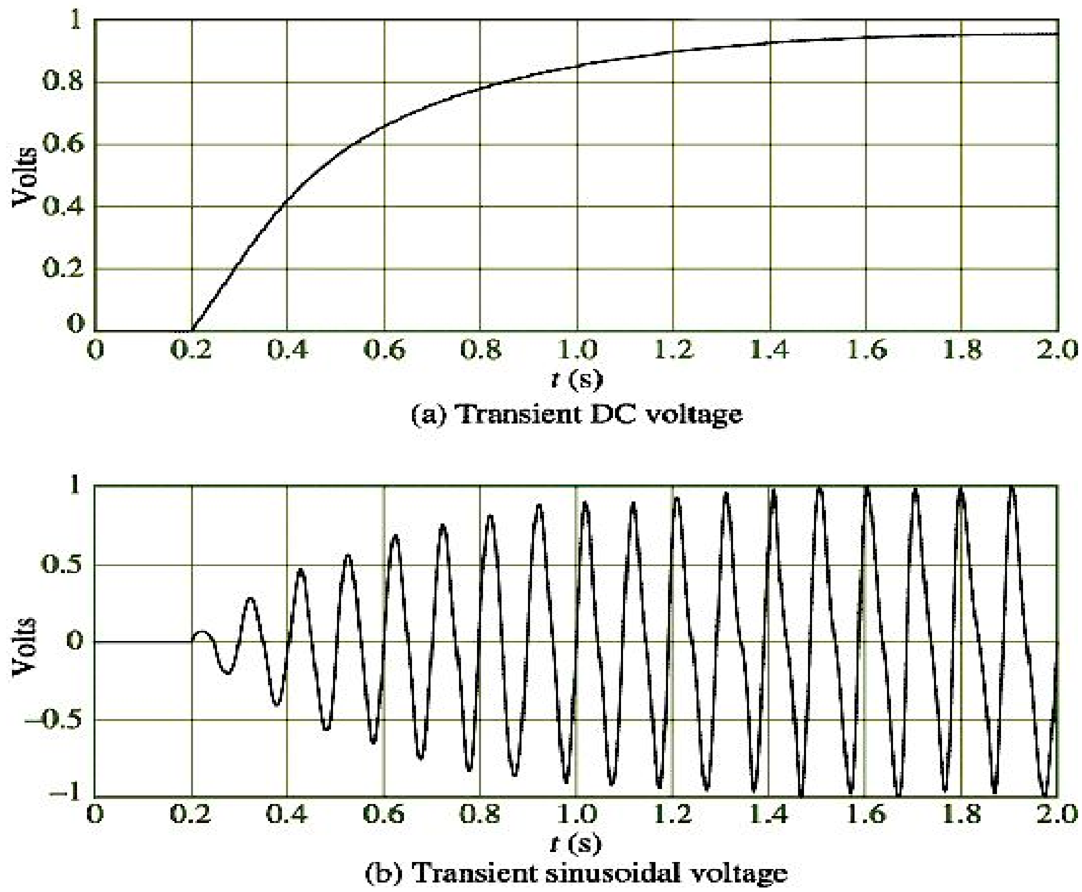
The Complete solution:

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

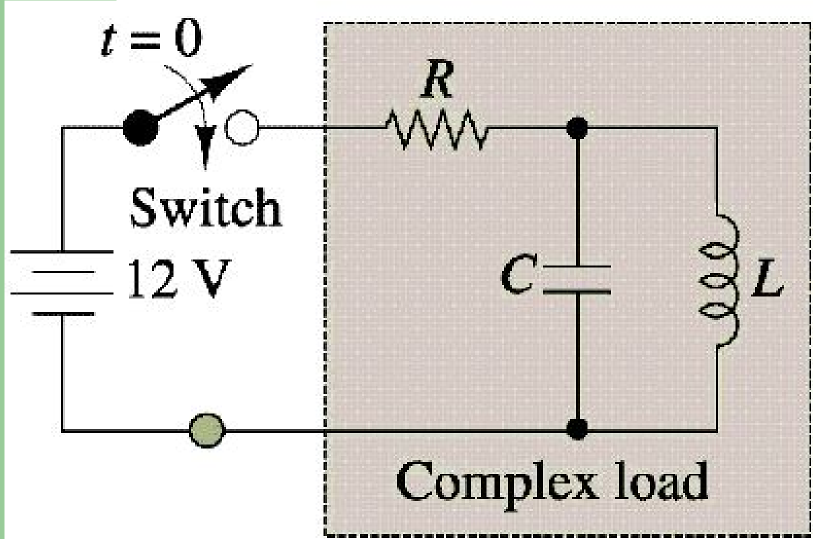
$[x(0) - x(\infty)]e^{-t/\tau}$ **called transient response**

$x(\infty)$ **called steady state response**

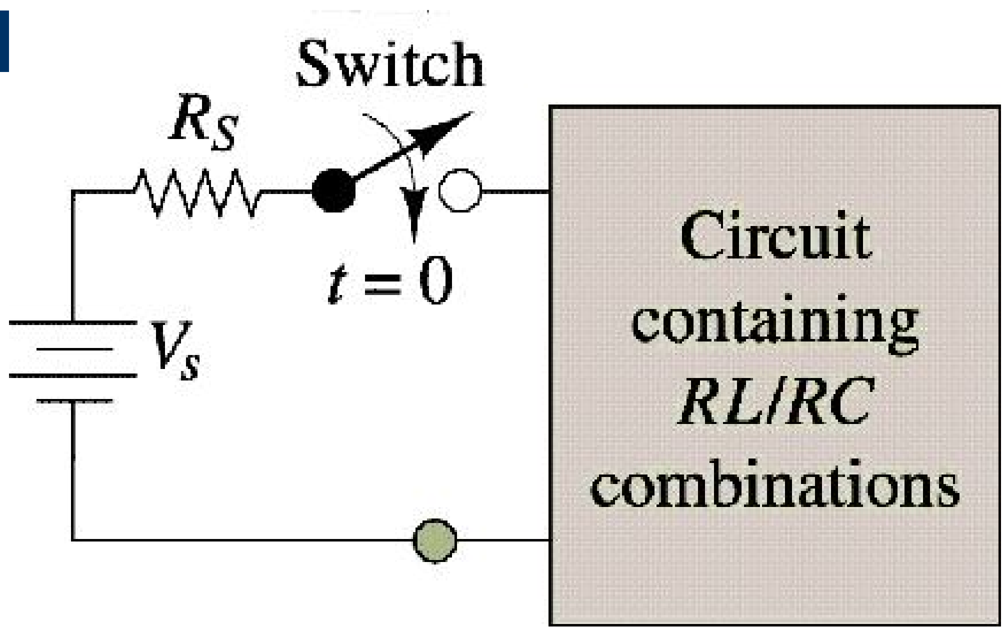
TRANSIENT RESPONSE



Circuit with switched DC excitation



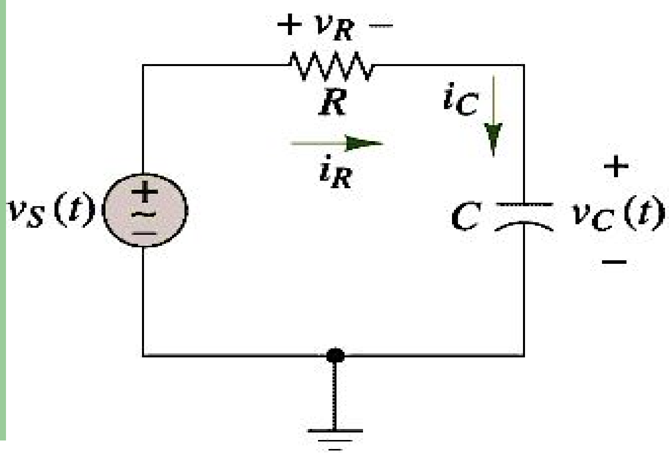
A general model of the transient analysis problem

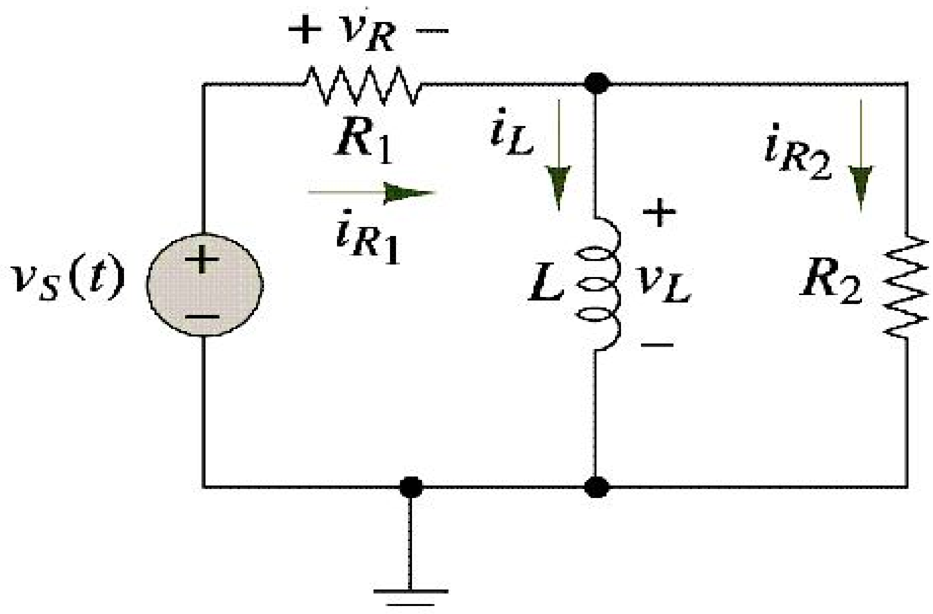


For a circuit containing energy storage element

A circuit containing energy-storage elements is described by a differential equation. The differential equation describing the series *RC* circuit shown is

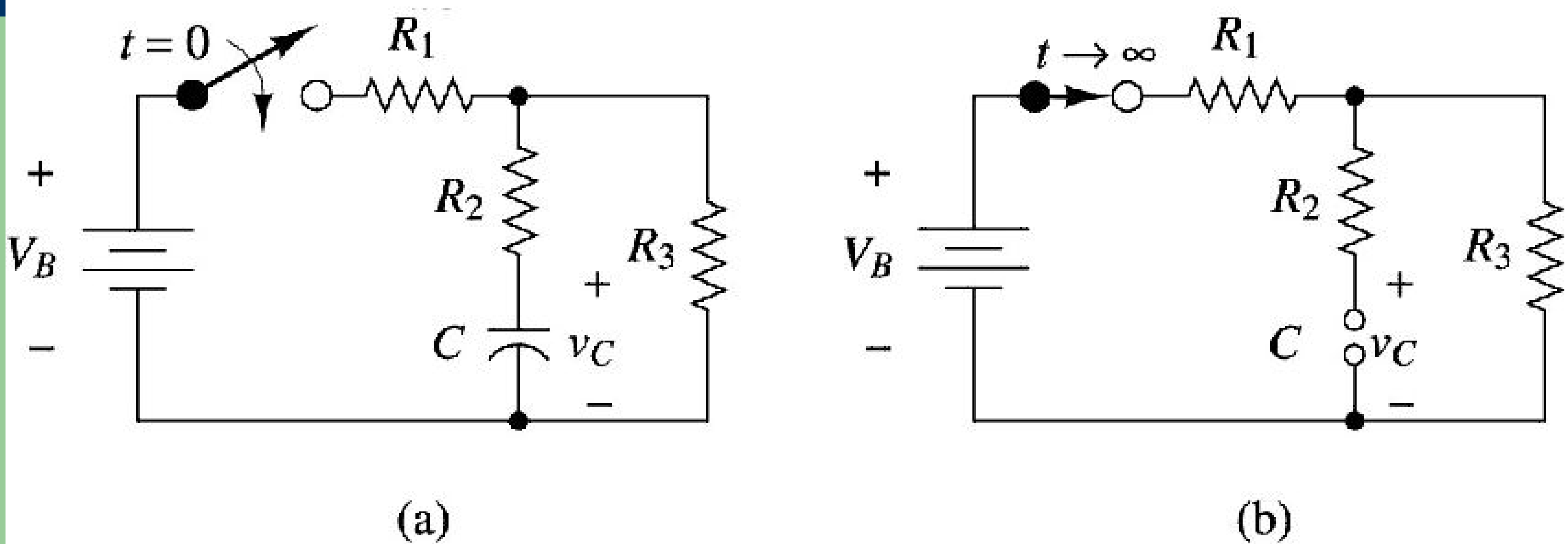
$$\frac{di_C}{dt} + \frac{1}{RC} i_C = \frac{dv_S}{dt}$$





(a) Circuit at t = 0

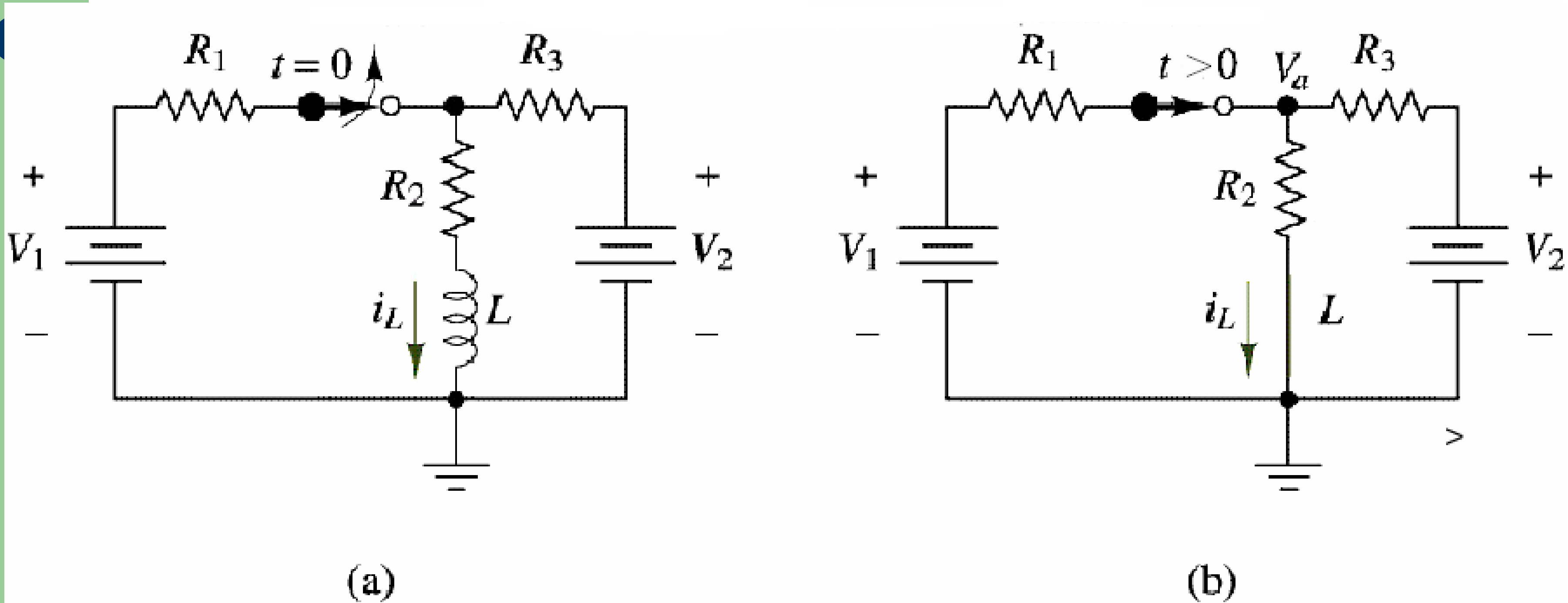
(b) Same circuit a long time after the switch is closed



The capacitor acts as open circuit for the steady state condition (a long time after the switch is closed).

(a) Circuit for t = 0

(b) Same circuit a long time before the switch is opened



The inductor acts as short circuit for the steady state condition
(a long time after the switch is closed).

Reason for transient response

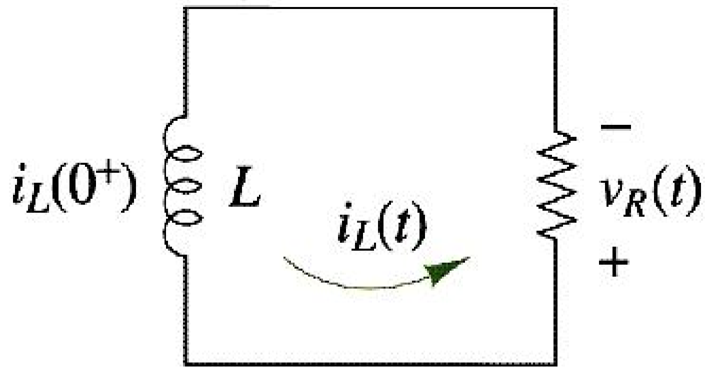
- The voltage across a capacitor cannot be changed instantaneously.

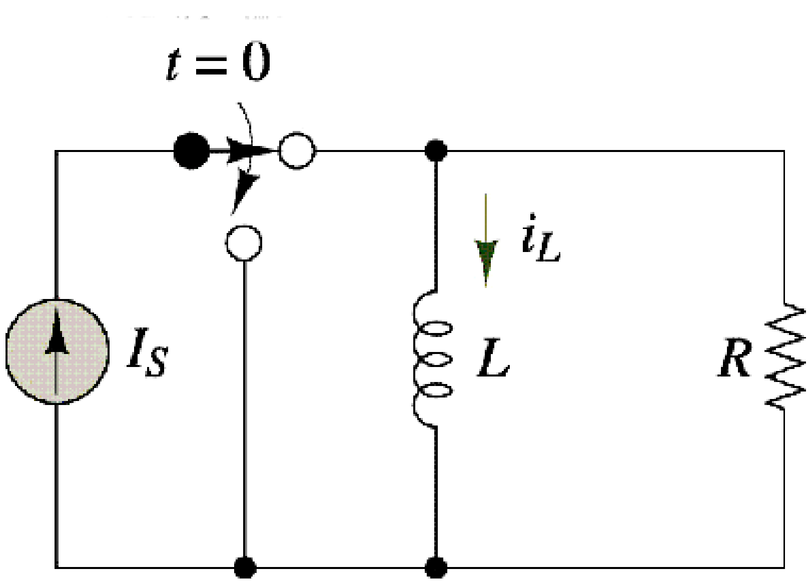
$$V_C(0^-) = V_C(0^+)$$

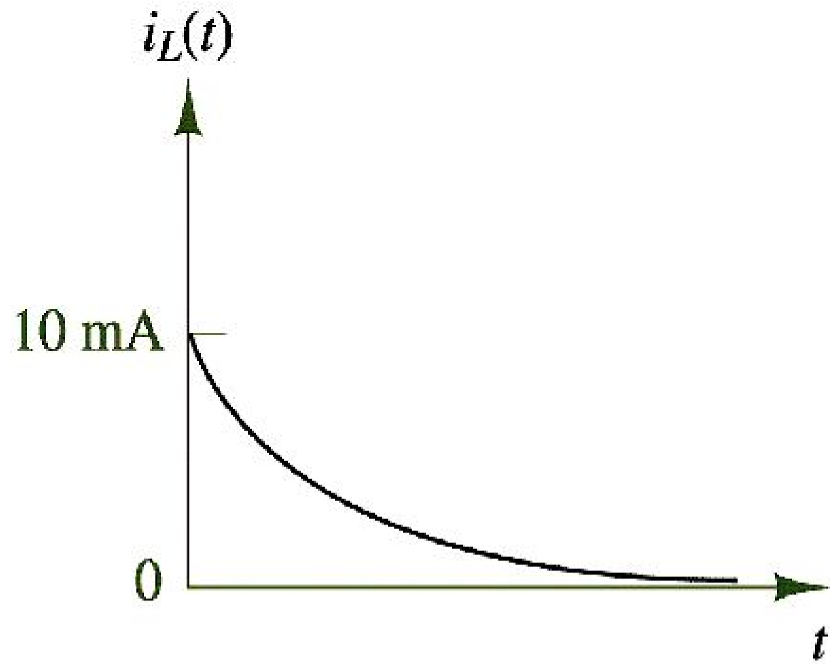
- The current across an inductor cannot be changed instantaneously.

$$I_L(0^-) = I_L(0^+)$$

Example







Transients Analysis

1. Solve first-order *RC* or *RL* circuits.

2. Understand the concepts of transient response and steady-state response.

3. Relate the transient response of first-order
circuits to the time constant.

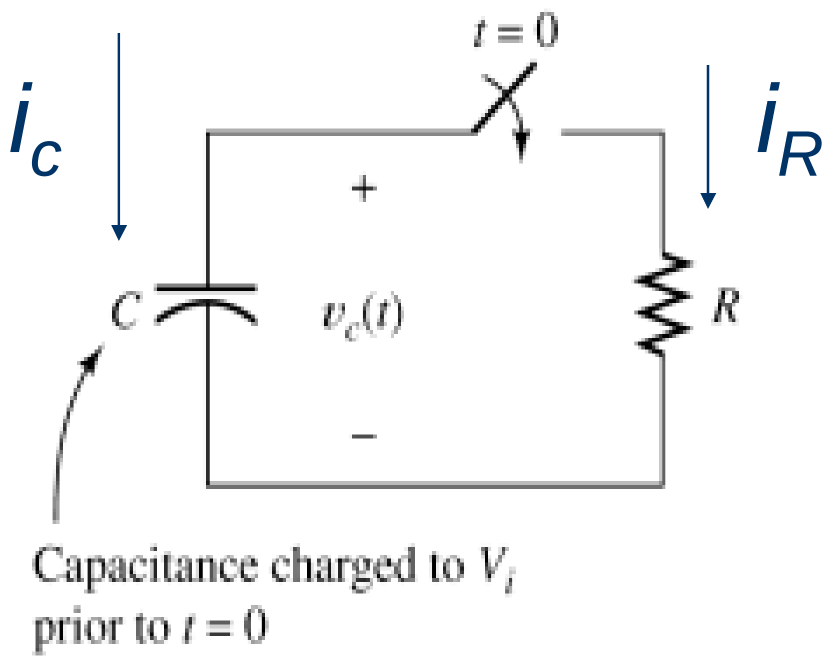
Transients

The solution of the differential equation represents are response of the circuit. It is called *natural response.*

The response must eventually die out, and therefore referred to as *transient response.*
(source free response)

Discharge of a Capacitance through a Resistance

$$\sum i = 0, \quad i_C + i_R = 0$$



$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

Solving the above equation with the initial condition $V_c(0) = V_i$

Discharge of a Capacitance through a Resistance

$$C\frac{dv_C(t)}{dt}+\frac{v_C(t)}{R}=0$$

$$RC\frac{dv_C(t)}{dt}+v_C(t)=0$$

$$v_C(t)=Ke^{st}$$

$$RCKse^{st}+Ke^{st}=0$$

$$s=\frac{-1}{RC}$$

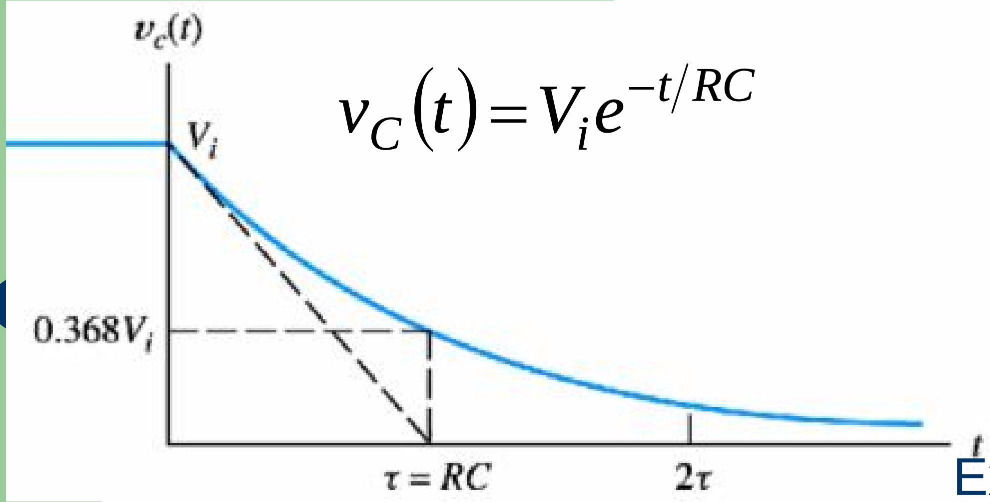
$$v_C(t)=Ke^{-t/RC}$$

$$v_C(0^+)=V_i$$

$$=Ke^{0/RC}$$

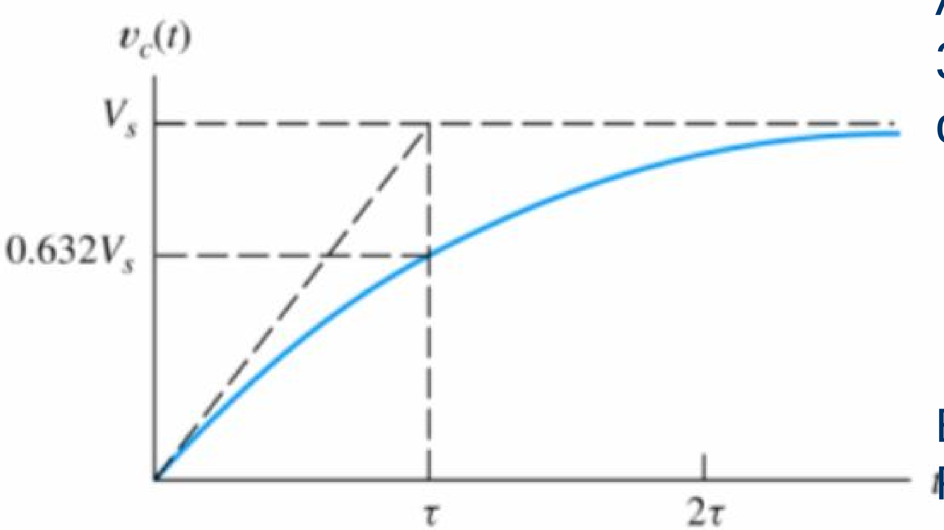
$$=K$$

$$v_C(t)=V_ie^{-t/RC}$$



$$v_C(t) = V_i e^{-t/RC}$$

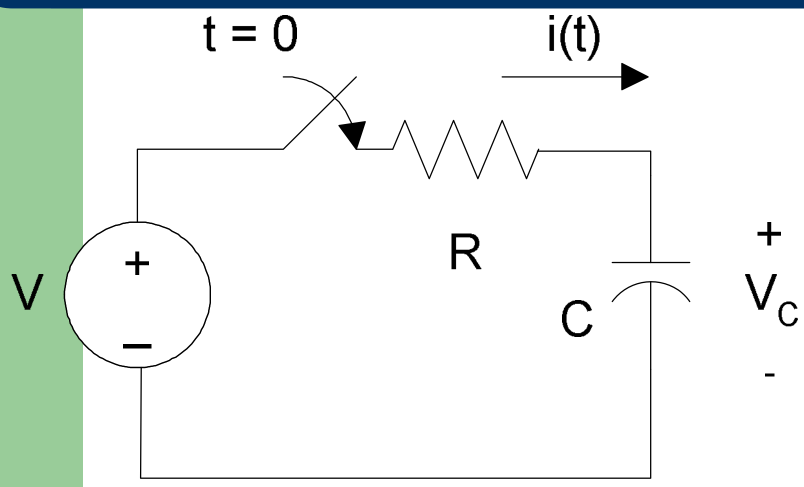
Exponential decay waveform
RC is called the time constant.
At time constant, the voltage is 36.8%
of the initial voltage.

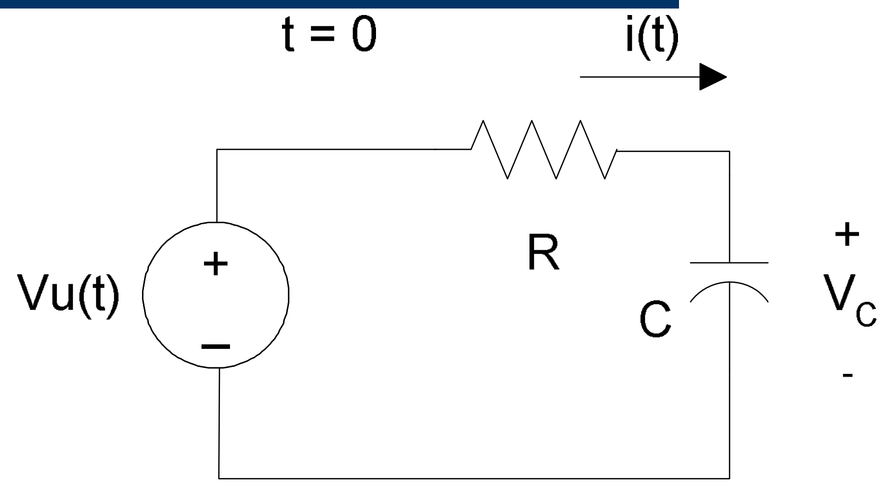


$$v_C(t) = V_i(1 - e^{-t/RC})$$

Exponential rising waveform
RC is called the time constant.
At time constant, the voltage is 63.2%
of the initial voltage.

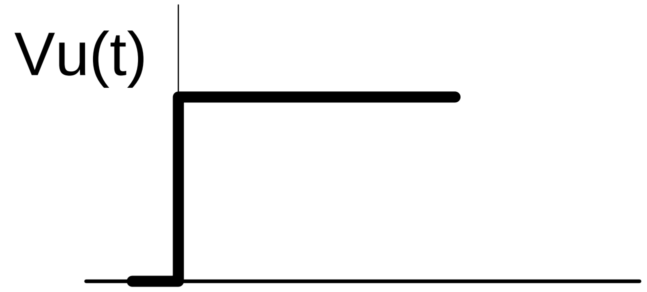
RC CIRCUIT



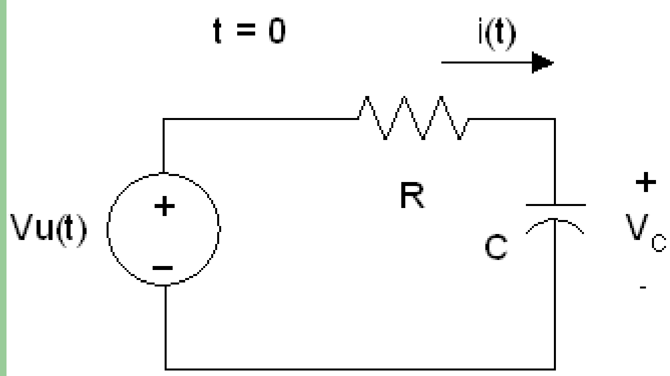


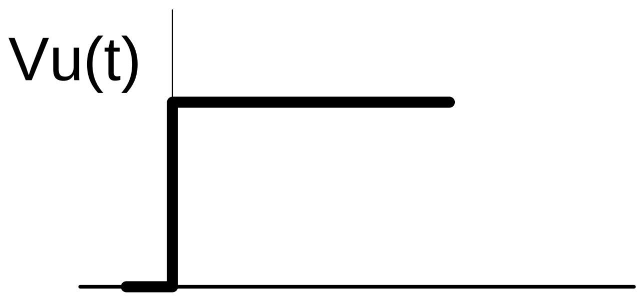
for $t = 0^{-}$, $i(t) = 0$

$u(t)$ is voltage-step function



RC CIRCUIT





$$i_R = i_C$$

$$i_R = \frac{vu(t) - v_C}{R}, \quad i_C = C\frac{dv_C}{dt}$$

$$RC\frac{dv_C}{dt} + v_C = V, \quad vu(t) = V \; for \; t \geq 0$$

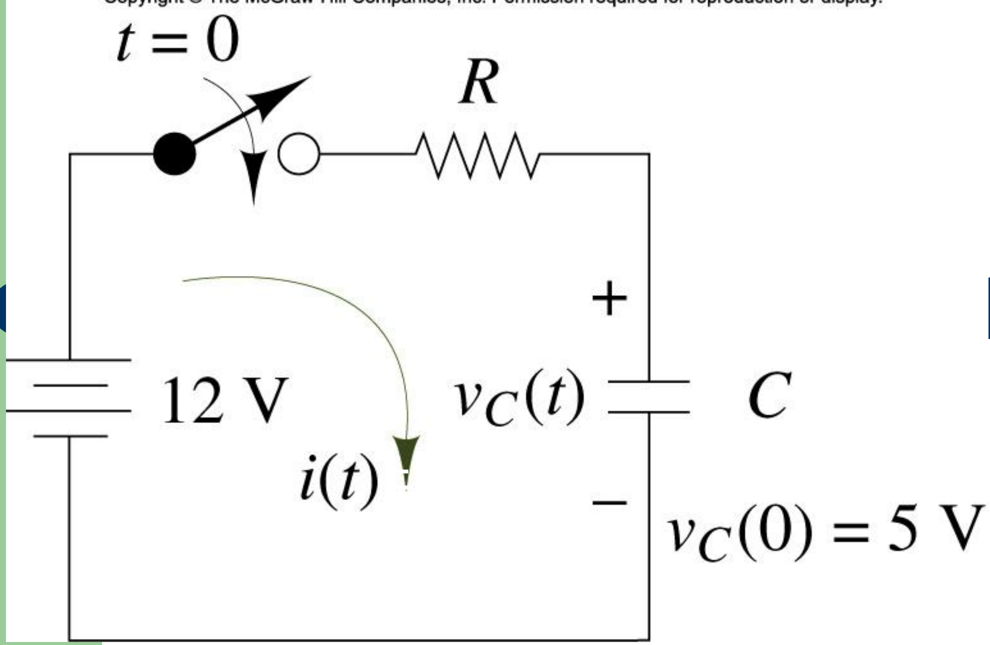
Solving the differential equation

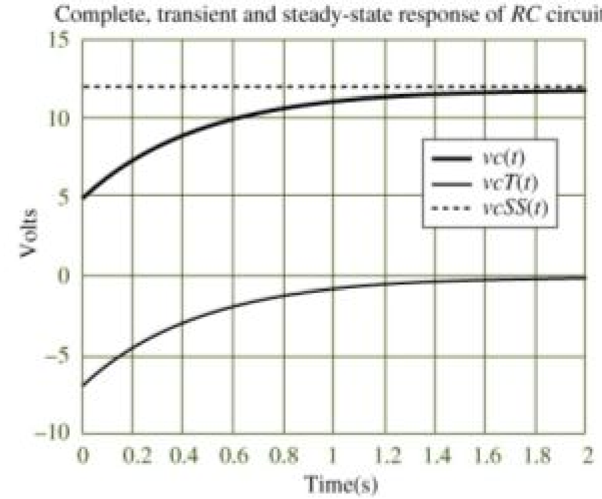
Complete Response

Complete response

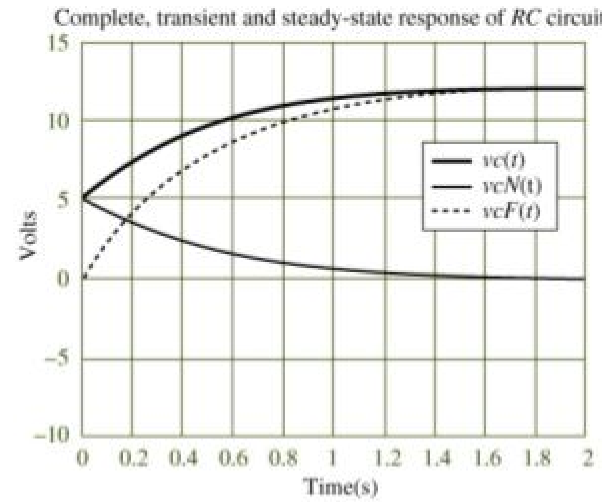
= natural response + forced response

- **Natural response (source free response) is due to the initial condition**
- **Forced response is the due to the external excitation.**





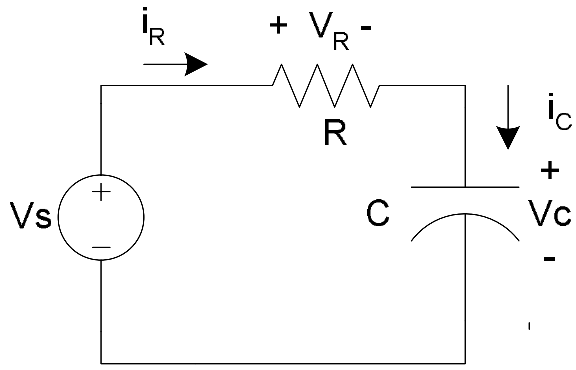
(a)



(b)

- **a).** **Complete, transient and steady state response**
- **b).** **Complete, natural, and forced responses of the circuit**

Circuit Analysis for RC Circuit



Apply KCL

$$i_R = i_C$$

$$i_R = \frac{v_s - v_R}{R}, i_C = C\frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} + \frac{1}{RC}v_R = \frac{1}{RC}v_s$$

v_s **is the source applied.**

Solution to First Order Differential Equation

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Let the initial condition be $x(t = 0) = x(0)$, then we solve the differential equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete solution consist of two parts:

- the homogeneous solution (natural solution)
- the particular solution (forced solution)

The Natural Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation *f* (*t*) equal to zero,

$$\tau \frac{dx_N(t)}{dt} + x_N(t) = 0 \;\; or \quad \frac{dx_N(t)}{dt} = -\frac{x_N(t)}{\tau}$$

$$x_N(t) = \alpha\, e^{-t/\tau}$$

It is called the natural response.

The Forced Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation *f* (*t*) equal to F, a constant for *t* ≥0

$$\tau \frac{dx_F(t)}{dt} + x_F(t) = K_S F$$

$$x_F(t) = K_S F \quad for \ t \geq 0$$

It is called the forced response.

The Complete Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete response is:
• the natural response +
• the forced response

$$x = x_N(t) + x_F(t)$$
$$= \alpha e^{-t/\tau} + K_S F$$
$$= \alpha e^{-t/\tau} + x(\infty)$$

Solve for α,

$$for\ t = 0$$
$$x(t=0) = x(0) = \alpha + x(\infty)$$
$$\alpha = x(0) - x(\infty)$$

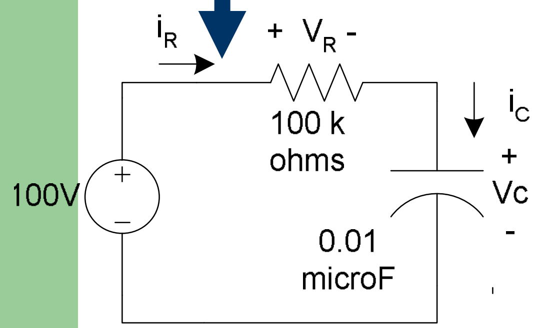
The Complete solution:

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

$$[x(0) - x(\infty)]e^{-t/\tau}$$ **called transient response**

$$x(\infty)$$ **called steady state response**

Example



Initial condition *Vc*(0) = 0V

$$i_R = i_C$$

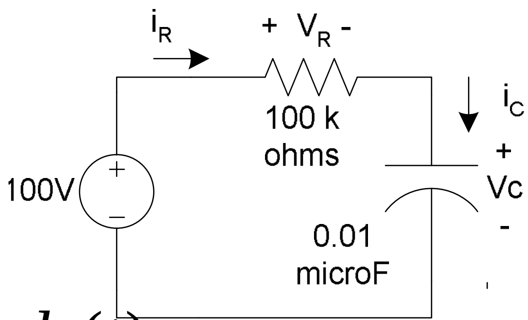
$$i_R = \frac{v_s - v_C}{R}, i_C = C\frac{dv_C}{dt}$$

$$RC\frac{dv_C}{dt} + v_C = v_s$$

$$10^5 \times 0.01 \times 10^{-6}\frac{dv_C}{dt} + v_C = 100$$

$$10^{-3}\frac{dv_C}{dt} + v_C = 100$$

Example



Initial condition *Vc*(0) = 0V

$$10^{-3}\frac{dv_C}{dt}+v_C=100$$

$$\tau\frac{dx(t)}{dt}+x(t)=K_s f(t)$$

and

$$x=x_N(t)+x_F(t)$$

$$=\alpha e^{-t/\tau}+K_S F$$

$$=\alpha e^{-t/\tau}+x(\infty)$$

$$v_c=100+Ae^{-\frac{t}{10^{-3}}}$$

$$As\;\; v_c(0)=0,\; 0=100+A$$

$$A=-100$$

$$v_c=100-100e^{-\frac{t}{10^{-3}}}$$

Energy stored in capacitor

$$p = vi = Cv\frac{dv}{dt}$$

$$\int_{t_o}^{t} p\,dt = \int_{t_o}^{t} Cv\frac{dv}{dt}dt = C\int_{t_o}^{t} v\,dv$$

$$= \frac{1}{2}C\left\{[v(t)]^2 - [v(t_o)]^2\right\}$$

If the zero-energy reference is selected at t_o, implying that the capacitor voltage is also zero at that instant, then

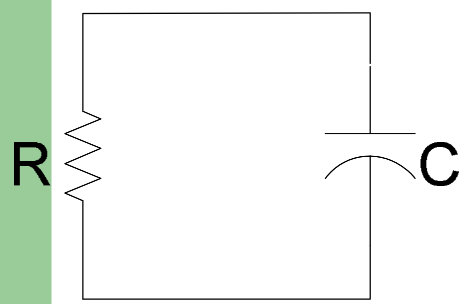
$$w_C(t) = \frac{1}{2}Cv^2$$

RC CIRCUIT

Power dissipation in the resistor is:

$p_R = V^2/R = (V_o^{\,2}/R)\, e^{-2t/RC}$

Total energy turned into heat in the resistor



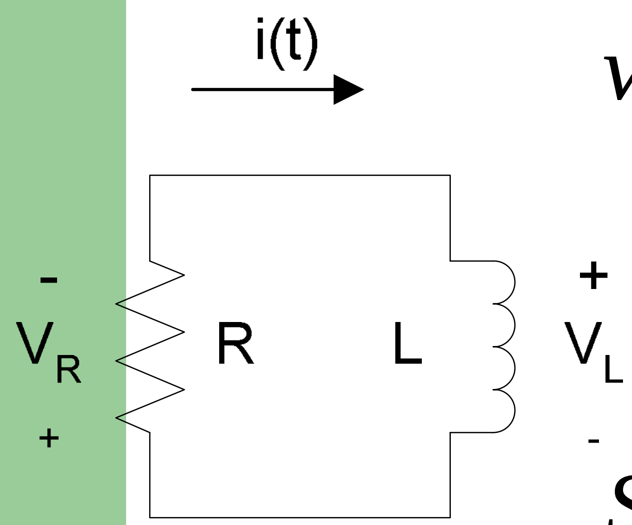
$$W_R = \int_0^\infty p_R dt = \frac{V_o^2 \int_0^\infty e^{-2t/RC} dt}{R}$$

$$= V_o^2 R\left(-\frac{1}{2RC}\right) e^{-2t/RC} \Big|_0^\infty$$

$$= \frac{1}{2} C V_o^2$$

RL CIRCUITS

Initial condition
$i(t = 0) = I_o$

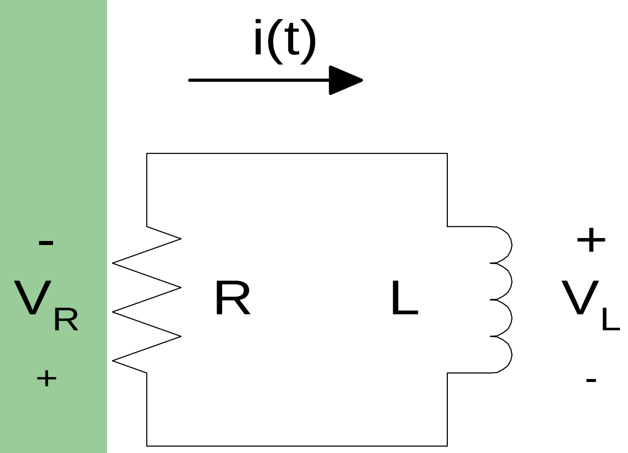


$$v_R + v_L = 0 = Ri + L\frac{di}{dt}$$

$$\frac{L}{R}\frac{di}{dt} + i = 0$$

Solving the differential equation

RL CIRCUITS



$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$\frac{di}{i} = -\frac{R}{L} dt , \qquad \int_{I_o}^{i(t)} \frac{di}{i} = \int_o^t -\frac{R}{L} dt$$

$$\ln i \,\Big|_{I_o}^{i} = -\frac{R}{L} t \,\Big|_o^t$$

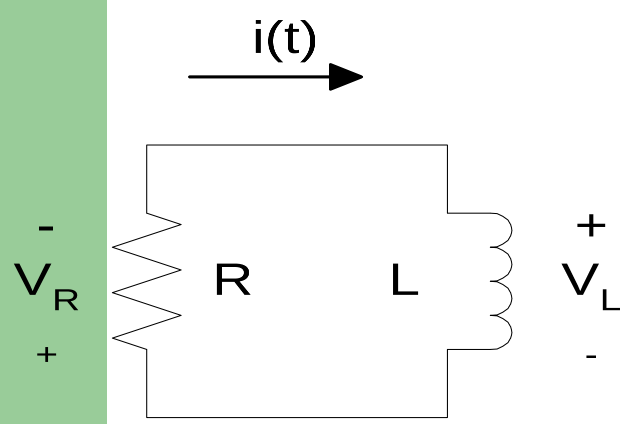
$$\ln i - \ln I_o = -\frac{R}{L} t$$

$$i(t) = I_o e^{-Rt/L}$$

Initial condition

$i(t = 0) = I_o$

RL CIRCUIT



Power dissipation in the resistor is:

$$p_R = i^2R = I_o^2 e^{-2Rt/L} R$$

Total energy turned into heat in the resistor

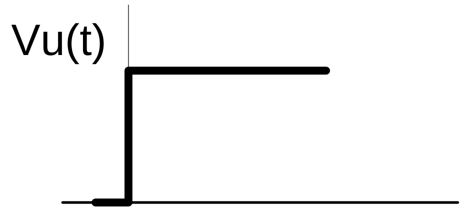
$$W_R = \int_0^\infty p_R \, dt = I_o^2 R \int_0^\infty e^{-2Rt/L} \, dt$$

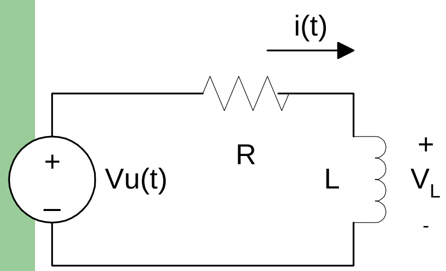
$$= I_o^2 R \left(-\frac{L}{2R}\right) e^{-2Rt/L} \Big|_0^\infty$$

$$= \frac{1}{2} L I_o^2$$

It is expected as the energy stored in the inductor is $\frac{1}{2} L I_o^2$

RL CIRCUIT





$$Ri + L\frac{di}{dt} = V$$

$$\frac{Ldi}{V - Ri} = dt$$

Integrating both sides,

$$-\frac{L}{R}\ln(V - Ri) = t + k$$

$$i(0^+) = 0, thus \quad k = -\frac{L}{R}\ln V$$

$$-\frac{L}{R}[\ln(V - Ri) - \ln V] = t$$

$$\frac{V - Ri}{V} = e^{-Rt/L} \qquad or$$

$$i = \frac{V}{R} - \frac{V}{R}e^{-Rt/L}, \quad for\ t > 0$$

where *L/R* is the time constant

DC STEADY STATE

The steps in determining the forced response for *RL or RC* circuits with dc sources are:

1. Replace capacitances with open circuits.

2. Replace inductances with short circuits.

3. Solve the remaining circuit.

UNIT V RESONANCE AND COUPLED CIRCUITS

– Series and parallel resonance
– their frequency response
– Quality factor and Bandwidth
– Self and mutual inductance
– Coefficient of coupling
– Tuned circuits
– Single tuned circuits.

Resonance In Electric Circuits

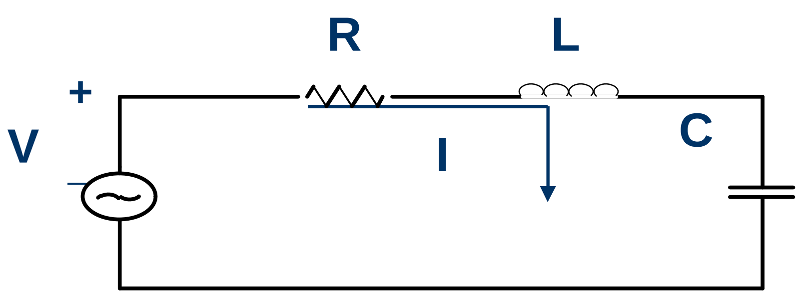
Any passive electric circuit will resonate if it has an inductor and capacitor.

Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.

In this presentation we will consider (a) series resonance, and (b) parallel resonance.

Consider the series RLC circuit shown below.

$V = V_M \angle 0$



The input impedance is given by:

$$Z = R + j(wL - \frac{1}{wC})$$

The magnitude of the circuit current is;

$$I = |\bar{I}| = \frac{V_m}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

Resonance occurs when,

$$wL = \frac{1}{wC}$$

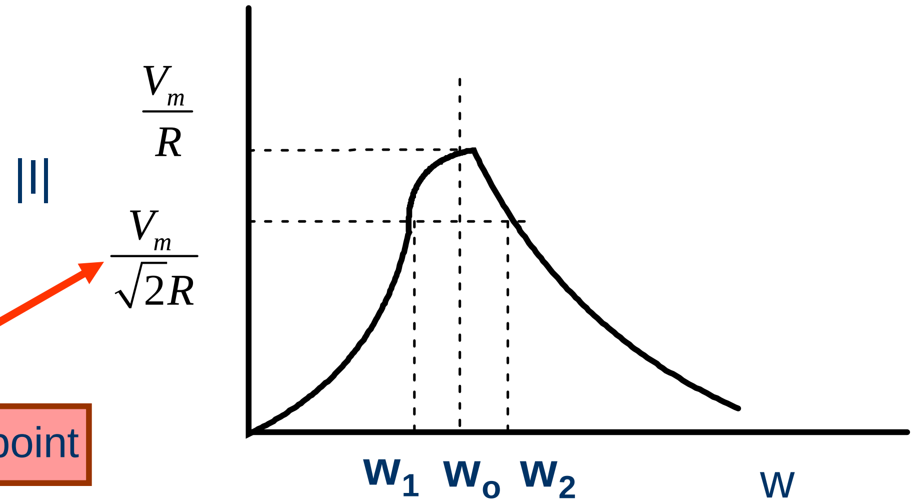
At resonance we designate w as w_o and write;

$$w_o = \frac{1}{\sqrt{LC}}$$

This is *an important equation* to remember. It applies to both series And parallel resonant circuits.

Series Resonanc

The magnitude of the current response for the series resonance circuit is as shown below.



Half power point

Bandwidth:

$BW = w_{BW} = w_2 - w_1$

Series Resonance

The <u>peak power</u> delivered to the circuit is;

$$P = \frac{V_m^2}{R}$$

The so-called half-power is given when

$$I = \frac{V_m}{\sqrt{2}R}$$

We find the frequencies, w_1 and w_2, at which this half-power occurs by using;

$$\sqrt{2}R = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

After some insightful algebra one will find two frequencies at which the previous equation is satisfied, they are:

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

an
d

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The two half-power frequencies are related to the resonant frequency by

$$w_o = \sqrt{w_1 w_2}$$

The bandwidth of the series resonant circuit is given by;

$$BW = w_b = w_2 - w_1 = \frac{R}{L}$$

We define the Q (quality factor) of the circuit as;

$$Q = \frac{w_o L}{R} = \frac{1}{w_o RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

Using Q, we can write the bandwidth as;

$$BW = \frac{w_o}{Q}$$

These are all important relationships.

An Observation:

If Q > 10, one can safely use the approximation;

$$w_1 = w_o - \frac{BW}{2} \qquad and \qquad w_2 = w_o + \frac{BW}{2}$$

These are useful approximations.

By using $Q = w_oL/R$ in the equations for w_1 and w_2 we have;

$$w_1 = w_o\left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

and

$$w_2 = w_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

An Example Illustrating Resonance:

Case 1:

$$\frac{ks}{s^2 + 2s + 400}$$

Case 2:

$$\frac{ks}{s^2 + 5s + 400}$$

Case 3:

$$\frac{ks}{s^2 + 10s + 400}$$

An Example Illustrating Resonance:

[redacted]

Case 1:

$$s^2 + 2s + 400 = (s + 1 + j19.97)(s + 1 - j19.97)$$

Case 2:

$$s^2 + 5s + 400 = (s + 2.5 + j19.84)(s + 2.5 - j19.84)$$

Case 3:

$$s^2 + 10s + 400 = (s + 5 + j19.36)(s + 5 - j19.36)$$

Comments:

Observe the denominator of the CE equation.

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

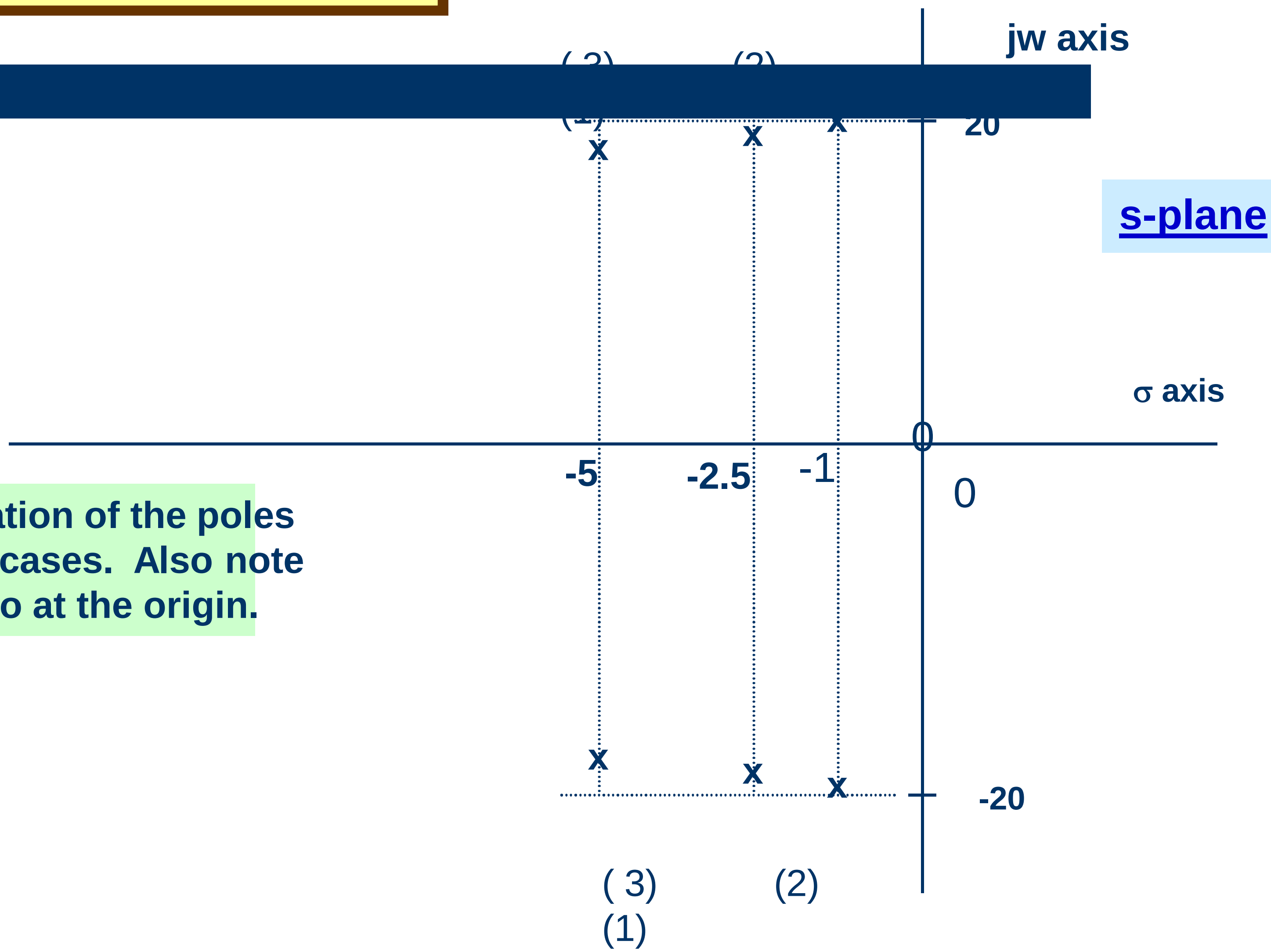
Compare to actual characteristic equation for *Case 1*:

$$s^2 + 2s + 400$$

$$w_o^{\,2} = 400 \longrightarrow w = 20$$ **rad/sec**

$$BW = \frac{R}{L} = 2$$ **rad/sec** $$\longrightarrow Q = \frac{w_o}{BW} = 10$$

Poles and Zeros In the s-plane:

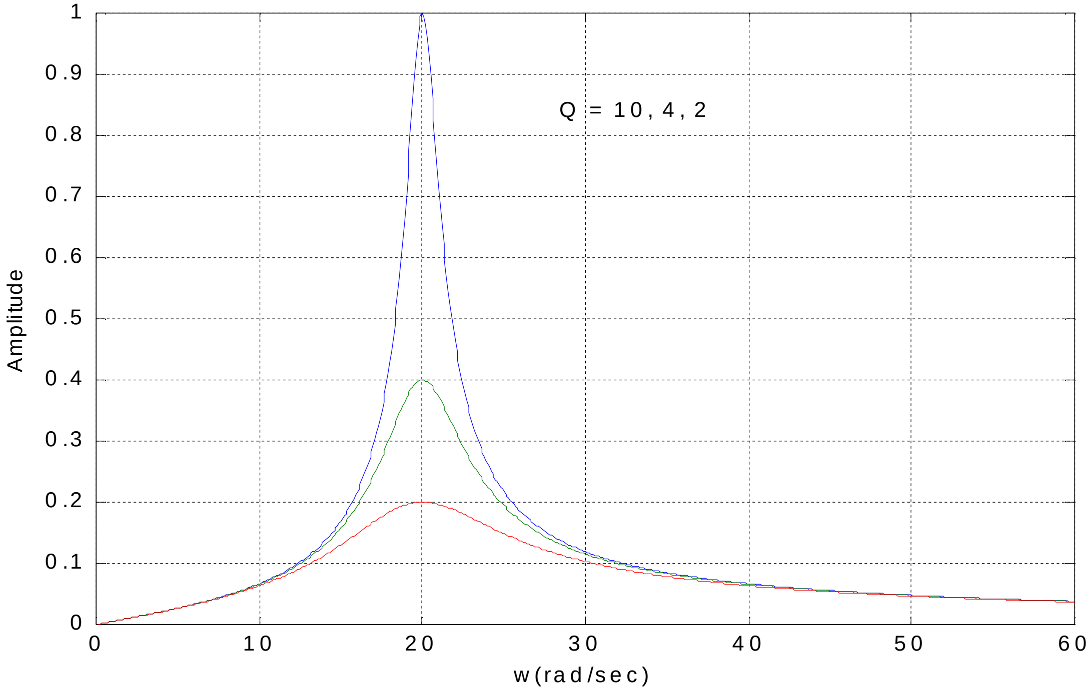


Note the location of the poles for the three cases. Also note there is a zero at the origin.

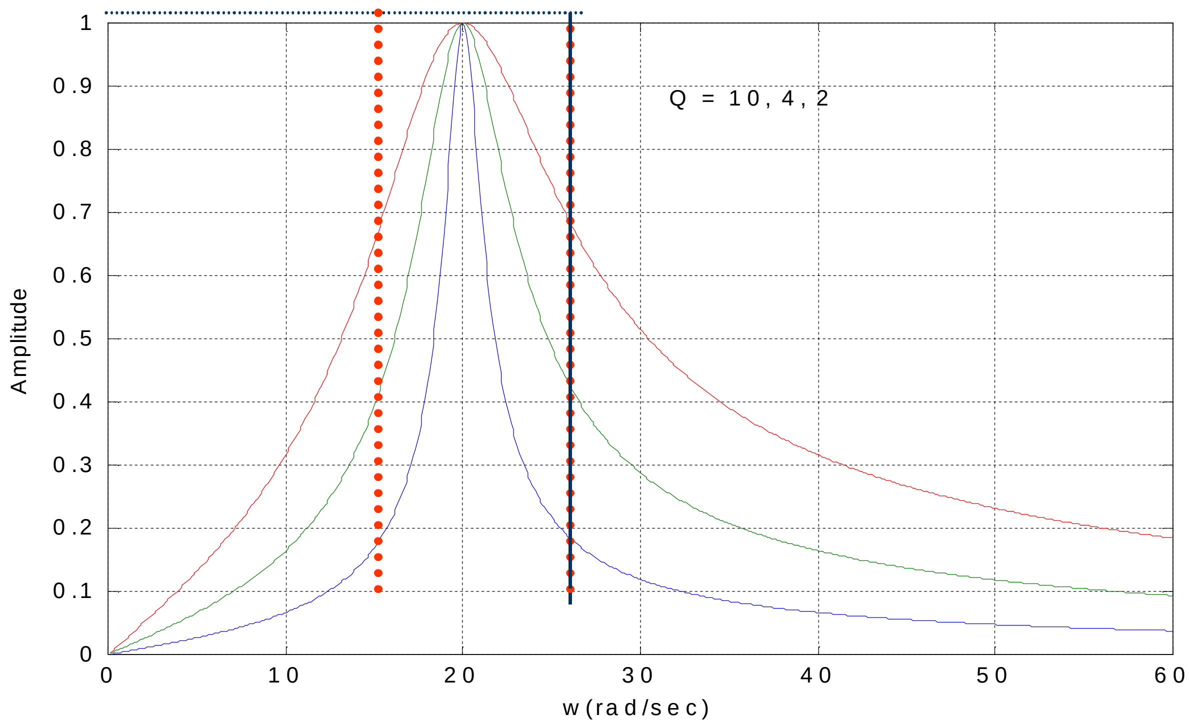
The frequency response starts at the origin in the s-plane.
At the origin the transfer function is zero because there is
a
zero at the origin.
As you get closer and closer to the complex pole, which
has a j parts in the neighborhood of 20, the response
starts
to increase.

The response continues to increase until we reach w = 20.
From there on the response decreases.

We should be able to reason through why the response
has the above characteristics, using a graphical approach.


Q = 10, 4, 2
Amplitude
w(rad/sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60

Next Case: Normalize all responses to 1 at w_o



Three dB Calculations:

Now we use the analytical expressions to calculate w_1 and w_2. We will then compare these values to what we find from the Matlab simulation.

Using the following equations with Q = 2,

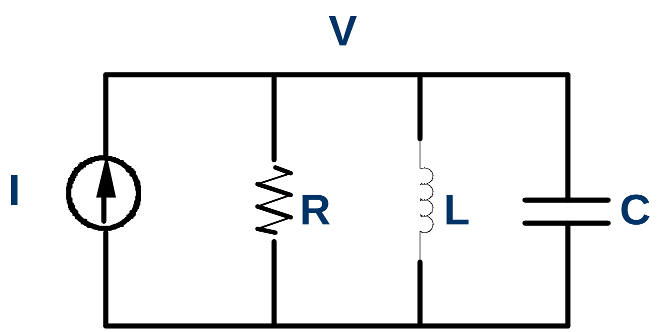
$$w_1, w_2 = w_o\left[\frac{\mp 1}{2Q} + w_o\sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

we find,

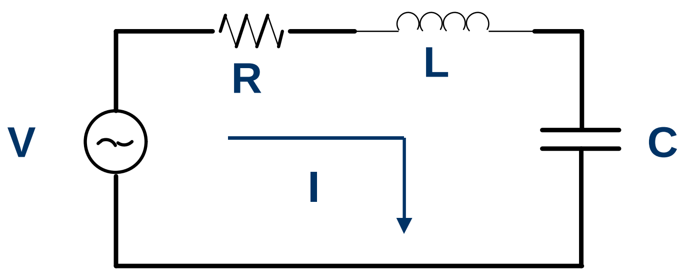
w_1 = 15.62 rad/sec

w_2 = 21.62 rad/sec

Parallel Resonanc



$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right]$$



$$V = I\left[R + jwL + \frac{1}{jwC}\right]$$

Duality

$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right] \qquad V = I\left[R + jwL + \frac{1}{jwC}\right]$$

We notice the above equations are the same provided:

$$I \longleftrightarrow V$$

$$R \longleftrightarrow \frac{1}{R}$$

$$L \longleftrightarrow C$$

If we make the inner-change, then one equation becomes the same as the other.

For such case, we say the one circuit is the dual of the other.

What this means is that for all the equations we have derived for the parallel resonant circuit, we can use for the series resonant circuit provided we make the substitutions:

$$R \quad \text{replaced be} \quad \frac{1}{R}$$

$$L \quad \text{replaced by} \quad C$$

$$C \quad \text{replaced by} \quad L$$

Parallel Resonance

$$w_o = \frac{1}{\sqrt{LC}} \qquad Q = \frac{w_o L}{R}$$

$$BW = (w_2 - w_1) = w_{BW} = \frac{R}{L}$$

$$w_1, w_2 = \left[\frac{\mp R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

w_1, w_2

Series Resonance

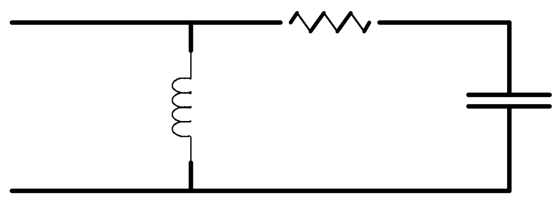
$$w_o = \frac{1}{\sqrt{LC}} \qquad Q = w_o RC$$

$$BW = w_{BW} = \frac{1}{RC}$$

$$w_1, w_2 = \left[\frac{\mp 1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

Example 1: **Determine the resonant frequency for the circuit below.**



$$Z_{IN} = \frac{jwL(R + \frac{1}{jwC})}{R + jwL + \frac{1}{jwC}} = \frac{(-w^2 LRC + jwL)}{(1 - w^2 LC) + jwRC}$$

At resonance, the phase angle of Z must be equal to zero.

Analysis

$$\frac{(-w^2 LRC + jwL)}{(1 - w^2 LC) + jwRC}$$

For zero phase;

$$\frac{wL}{(-w^2 LCR)} = \frac{wRC}{(1 - w^2 LC}$$

This gives;

$$w^2 LC - w^2 R^2 C^2 = 1$$

or

$$w_o = \frac{1}{\sqrt{(LC - R^2 C^2)}}$$

Parallel Resonance

Example 2:

A parallel RLC resonant circuit has a resonant frequency admittance of

A series RLC resonant circuit has a resonant frequency admittance of $2x10^{-2}$ S(mohs). The Q of the circuit is 50, and the resonant frequency is 10,000 rad/sec. Calculate the values of R, L, and C. Find the half-power frequencies and the bandwidth.

First, R = 1/G = 1/(0.02) = 50 ohms.

Second, from $Q = \frac{w_o L}{R}$, we solve for L, knowing Q, R, and w_o to find L = 0.25 H.

Third, we can use $$C = \frac{Q}{w_o R} = \frac{50}{10{,}000 x 50} = 100\ \mu F$$

Parallel Resonance

Example 2: (continued)

Fourth: We can use $$w_{BW} = \frac{w_o}{Q} = \frac{1x10^4}{50} = 200\, rad/\sec$$

and

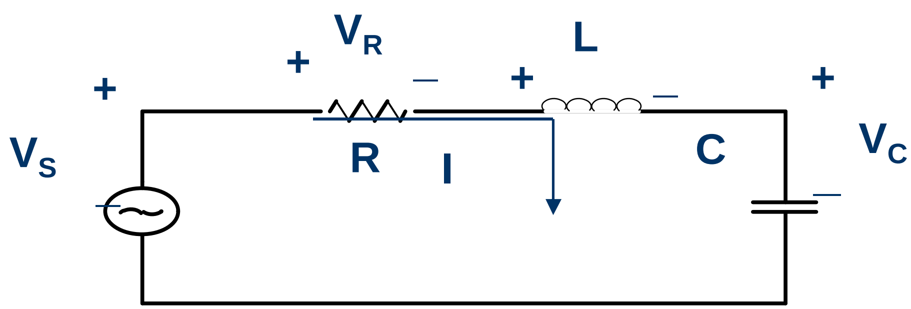
Fifth: Use the approximations;

$w_1 = w_o - 0.5w_{BW} = 10{,}000 – 100 = 9{,}900$ rad/sec

$w_2 = w_o - 0.5w_{BW} = 10{,}000 + 100 = 10{,}100$ rad/sec

Extension of Series Resonance

Peak Voltages and Resonance:



We know the following:

- When w = $w_o = \frac{1}{\sqrt{LC}}$, V_S and I are in phase, the driving point impedance is purely real and equal to R.

- A plot of |I| shows that it is maximum at w = w_o. We know the standard equations for series resonance applies: Q, w_{BW}, etc.

Reflection:

[redacted] little reflection shows that V_R is a peak value at w_o. But we are not sure about the other two voltages. We know that at resonance they are equal and they have a magnitude of QxV_S.

- Irwin shows that the frequency at which the voltage across the capacitor is a maximum is given by;

$$w_{max} = w_o \sqrt{1 - \frac{1}{2Q^2}}$$

- The above being true, we might ask, what is the frequency at which the voltage across the inductor is a maximum?

We answer this question by simulation

Series RLC Transfer Functions:

The following transfer functions apply to the series RLC circuit.

$$\frac{V_C(s)}{V_S(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{V_L(s)}{V_S(s)} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{V_R(s)}{V_S(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Parameter Selection:

We select values of R, L. and C for this first case so that Q = 2 and [redacted] C = 5μF. The transfer functions become as follows:

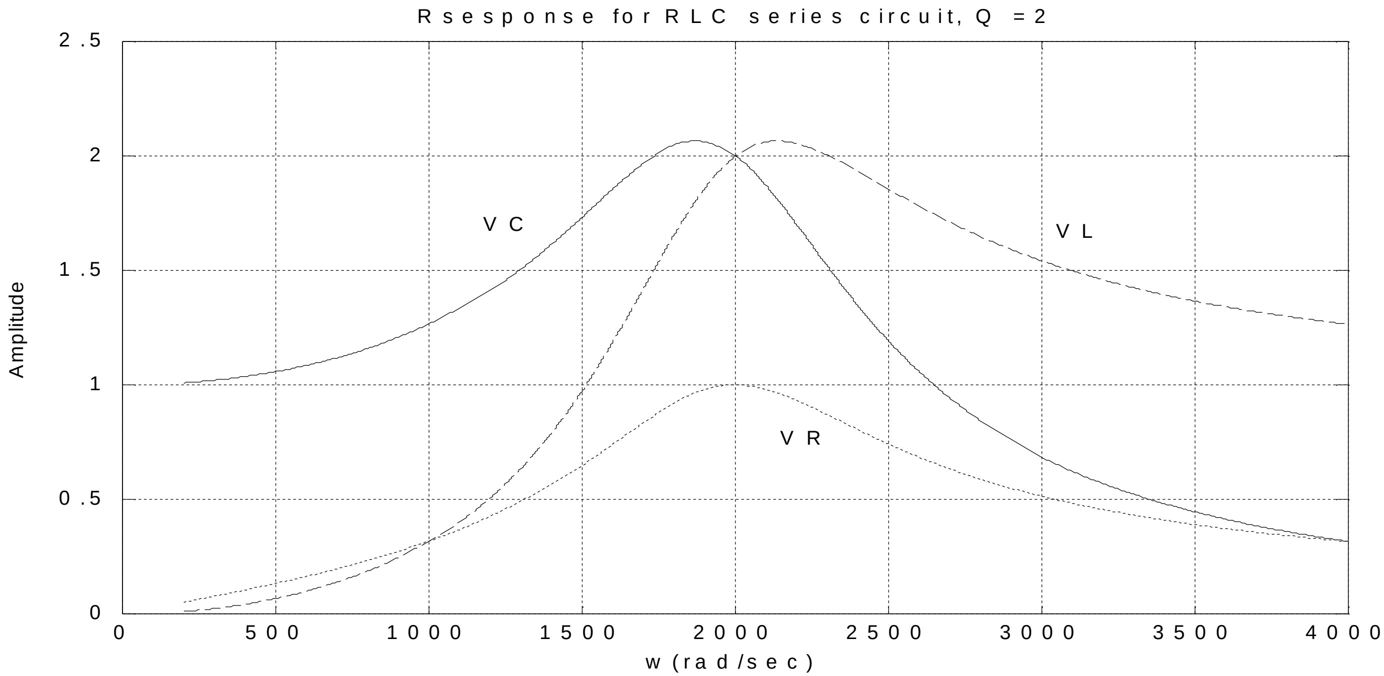
$$\frac{V_C}{V_S} = \frac{4x10^6}{s^2 + 1000s + 4x10^6}$$

$$\frac{V_L}{V_S} = \frac{s^2}{s^2 + 1000s + 4x10^6}$$

$$\frac{V_R}{V_S} = \frac{1000s}{s^2 + 1000s + 4x10^6}$$

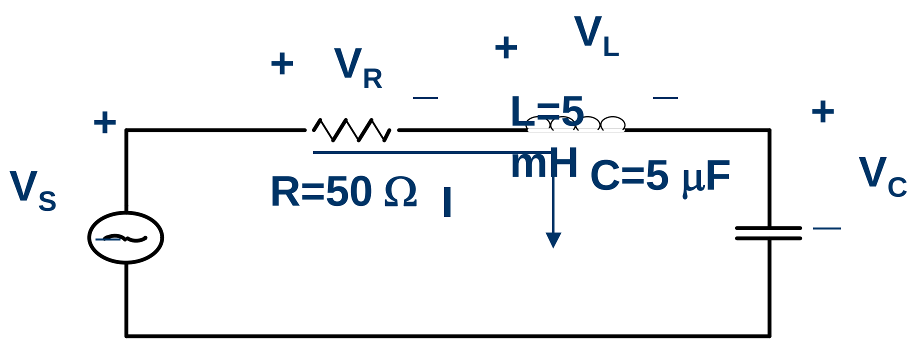
Simulation Results

Q = 2



Analysis of the problem:

Given the previous circuit. Find Q, w_0, w_{max}, $|V_c|$ at w_o, and $|V_c|$ at w_{max}



Solution:

$$w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50x10^{-2}\, x5x10^{-6}}} = 2000\, rad/\sec$$

$$Q = \frac{w_o L}{R} = \frac{2x10^{3}\, x5x10^{-2}}{50} = 2$$

Problem Solution:

$$w_{MAX} = w_O\sqrt{1-\frac{1}{2Q^2}} = 0.9354w_o$$

$$|V_R| \, at \, w_O = Q|V_S| = 2x1 = 2volts(peak)$$

$$|V_C| \, at \, w_{MAX} = \frac{Qx|V_S|}{\sqrt{1-\frac{1}{4Q^2}}} = \frac{2}{0.968} = 2.066volts(peak))$$

Now check the computer printout.

Exnsion of Series Resonance

Problem Solution (Simulation):

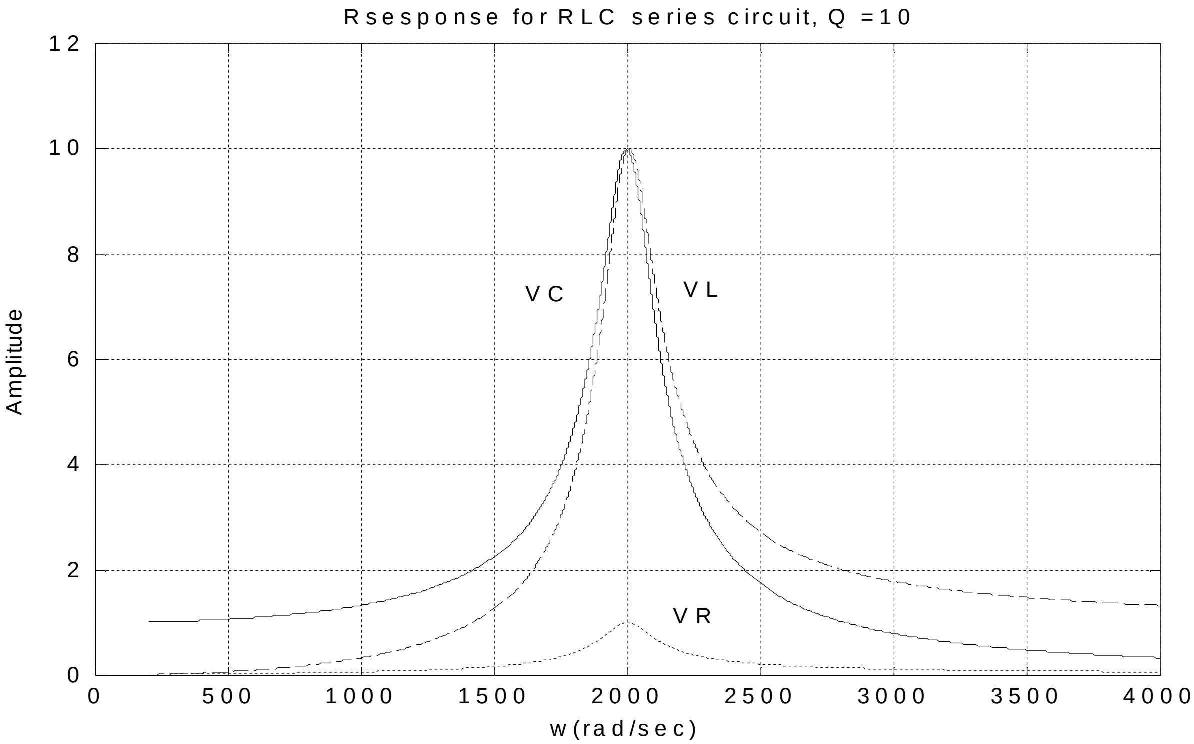
1.0e+003 *

1.8600000	0.002065141
1.8620000	0.002065292
1.8640000	0.002065411
1.8660000	0.002065501
1.8680000	0.002065560
1.8700000	0.002065588
1.8720000	0.002065585
1.8740000	0.002065552
1.8760000	0.002065487
1.8780000	0.002065392
1.8800000	0.002065265
1.8820000	0.002065107
1.8840000	0.002064917

Maximum

Simulation Results:

Q=10

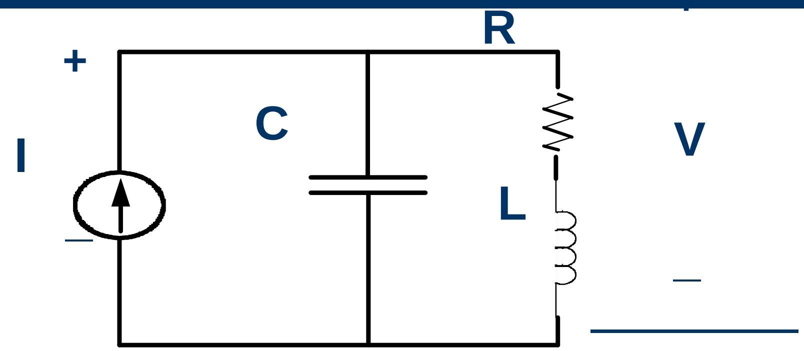


Observations From The Study:

- The voltage across the capacitor and inductor for a series RLC circuit
is not at peak values at resonance for small Q (Q <3).
- Even for Q<3, the voltages across the capacitor and inductor are
equal at resonance and their values will be QxV_S.
For Q>10, the voltages across the capacitors are for all practical purposes at their peak values and will be QxV_S.
- Regardless of the value of Q, the voltage across the resistor reaches its peak value at $w = w_o$.
- For high Q, the equations discussed for series RLC resonance can be applied to any voltage in the RLC circuit. For Q<3, this is not true.

Extension of Resonant Circuits

Given the following circuit:



- We want to find the frequency, w_r, at which the transfer function for V/I will resonate.
- The transfer function will exhibit resonance when the phase angle between V and I are zero.

The desired transfer functions is;

$$\frac{V}{I} = \frac{(1/sC)(R+sL)}{R+sL+1/sC}$$

This equation can be simplified to;

$$\frac{V}{I} = \frac{R+sL}{LCs^2+RCs+1}$$

With s $\longrightarrow$ jw

$$\frac{V}{I} = \frac{R+jwL}{(1-w^2LC)+jwR}$$

Resonant Condition:

For the previous transfer function to be at a resonant point, the phase angle of the numerator must be equal to the phase angle of the denominator.

$$\angle\theta_{num} = \angle\theta_{dem}$$

or,

$$\theta_{num} = \tan^{-1}\left(\frac{wL}{R}\right), \qquad \theta_{den} = \tan^{-1}\left(\frac{wRC}{(1-w^2LC)}\right) .$$

Therefore;

$$\frac{wL}{R} = \frac{wRC}{(1-w^2LC)}$$

Resonant Condition Analysis:

[redacted],

$$L(1-w^2LC)=R^2C \quad or \quad w^2L^2C=L-R^2C$$

This gives,

$$w_r=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

Notice that if the ratio of R/L is small compared to 1/LC, we have

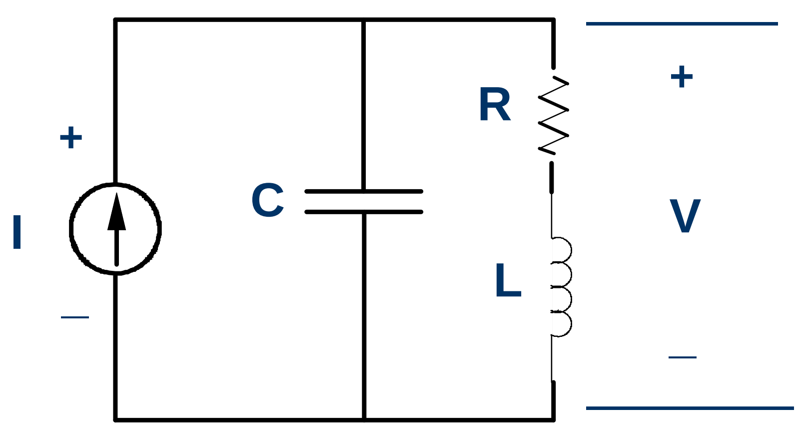
$$w_r=w_o=\frac{1}{\sqrt{LC}}$$

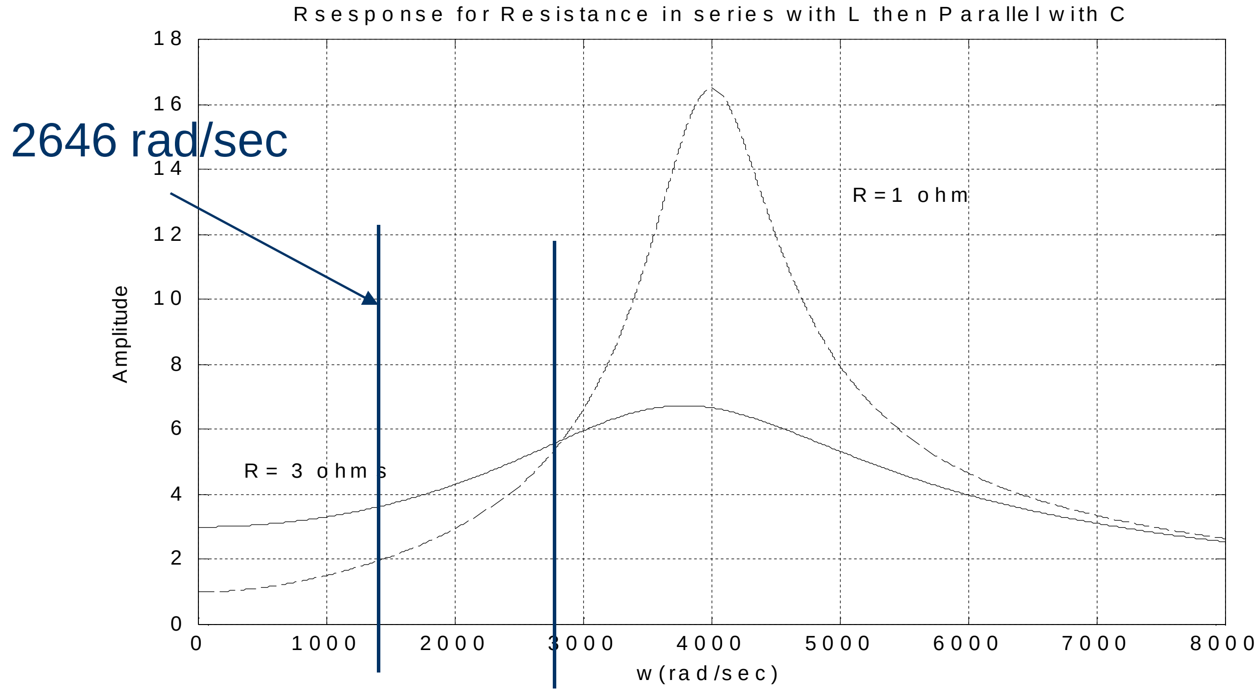
Extension of Resonant Circuits

Resonant Condition Analysis:

[redacted]ns?
[redacted]on, consider the following example.

Given the following circuit with the indicated parameters. Write a Matlab program that will determine the frequency response of the transfer function of the voltage to the current as indicated.




2646 rad/sec
Rsesponse for Resistance in series with L then Parallel with C
18
16
14
12
10
8
6
4
2
0
Amplitude
R=1 ohm
R= 3 ohms
0
1000
2000
3000
4000
5000
6000
7000
8000
w(rad/sec)