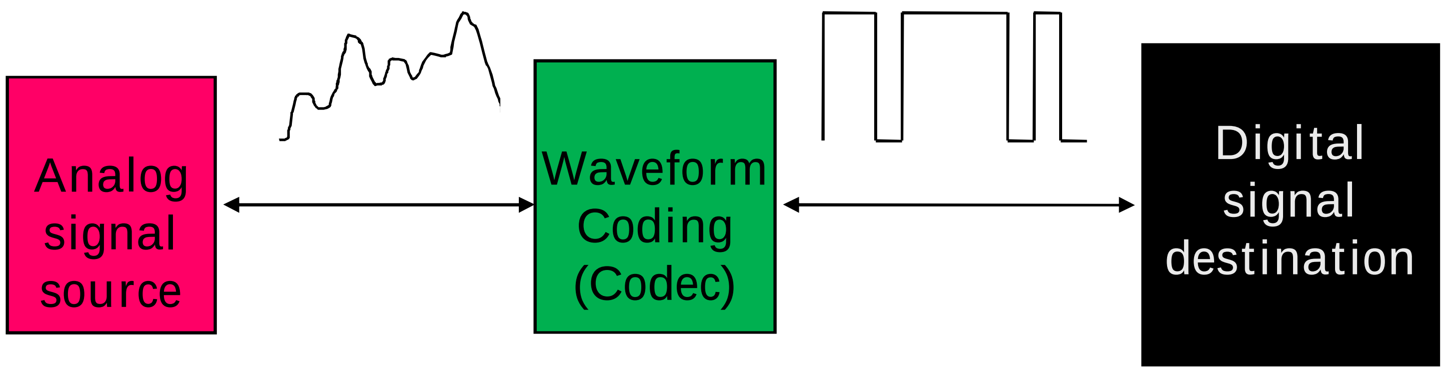
UNIT 1: Sampling and Quantization

Introduction

- Digital representation of analog signals



Analog-to-Digital Encoding

Advantages of Digital Transmissions

- Noise immunity
- Error detection and correction
- Ease of multiplexing
- Integration of analog and digital data
- Use of signal regenerators
- Data integrity and security
- Ease of evaluation and measurements
- More suitable for processing ....

Disadvantages of Digital Transmissions

- More bandwidth requirement
- Need of precise time synchronization
- Additional hardware for encoding/decoding
- Integration of analog and digital data
- Sudden degradation in QoS
- Incompatible with existing analog facilities

A Typical Digital Communication Link

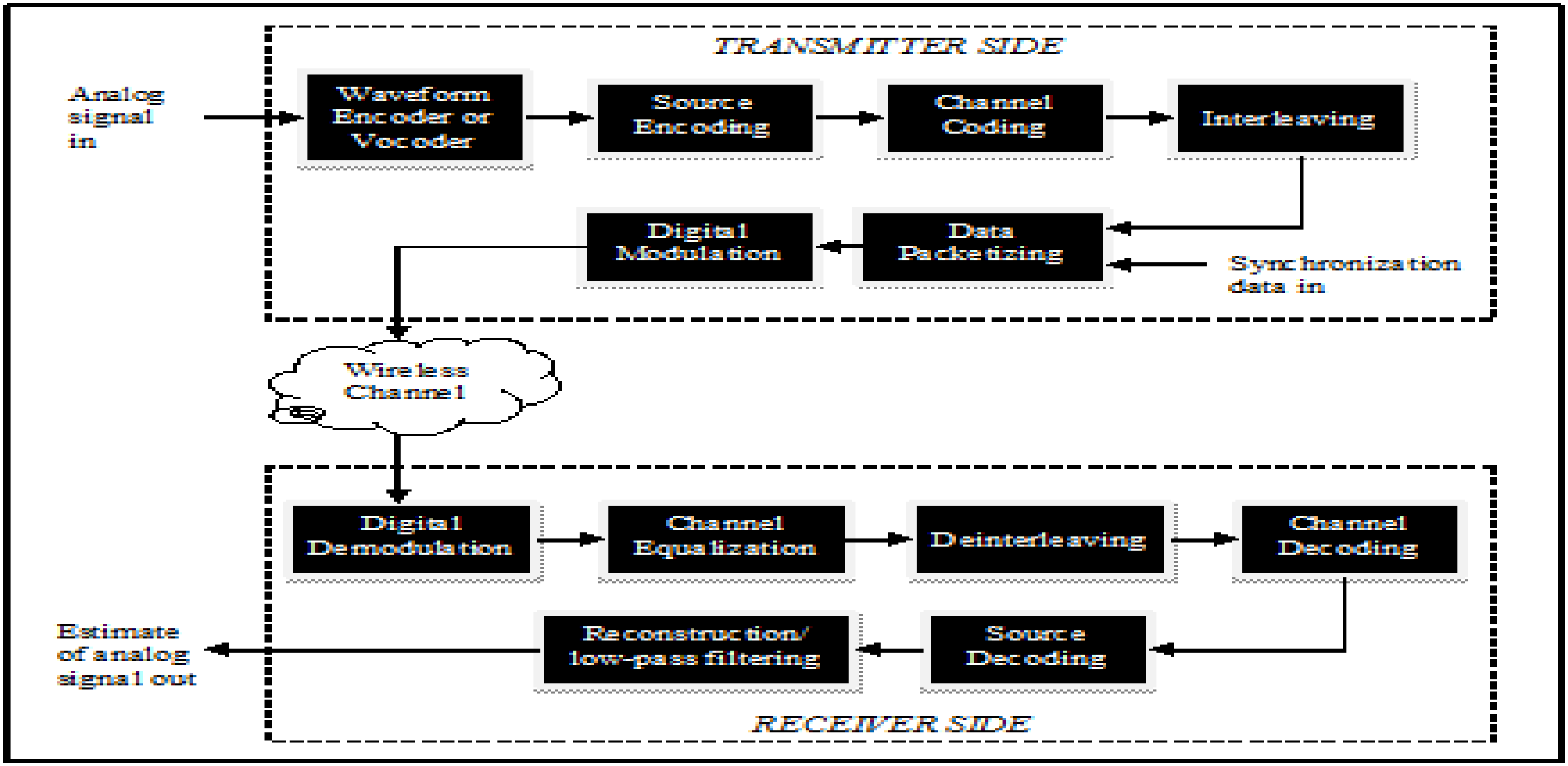


Fig. 2 Block Diagram

Formatting

aracter coding
mpling
antization
se code modulation
PCM)

Source Coding

Predictive coding
Block coding
Variable length coding
Synthesis/analysis coding
Lossless compression
Lossy compression

Baseband Signaling

PCM waveforms (line codes)
- Nonreturn-to-zero (NRZ)
- Return-to-zero (RZ)
- Phase encoded
- Multilevel binary

M-ary pulse modulation
- PAM, PPM, PDM

Equalization

Maximum-likelihood sequence estimation (MLSE)

Equalization with filters
- Transversal or decision feedback
- Preset or Adaptive
- Symbol spaced or fractionally spaced

Bandpass Signaling

Coherent

e shift keying (PSK)
uency shift keying (FSK)
litude shift keying (ASK)
inuous phase modulation (CPM)
ids

Noncoherent

Differential phase shift keying (DPSK)
Frequency shift keying (FSK)
Amplitude shift keying (ASK)
Continuous phase modulation (CPM)
Hybrids

Channel Coding

Waveform

M-ary signaling
Antipodal
Orthogonal
Trellis-coded modulation

Structurec Sequences

Block
Convolution
Turbo

Synchronization

Frequency synchronization
Phase synchronization
Symbol synchronization
Frame synchronization
Network synchronization

Multiplexing/Multiple Access

Frequency division (FDM/FDMA)
Time division (TDM/TDMA)
Code division (CDM/CDMA)
Space division (SDMA)
Polarization division (PDMA)

Spreading

Direct sequencing (DS)
Frequency hopping (FH)
Time hopping (TH)
Hybrids

Encryption

Block
Data stream

Figure 1.3 Basic digital communication transformations.

Basic Digital Communication Transformations

- Formatting/Source Coding
- Transforms source info into digital symbols (digitization)
- Selects compatible waveforms (matching function)
- Introduces redundancy which facilitates accurate decoding despite errors

It is essential for reliable communication

- Modulation/Demodulation
- Modulation is the process of modifying the info signal to facilitate transmission
- Demodulation reverses the process of modulation. It involves the detection and retrie
  of the info signal
  - Types
  - Coherent: Requires a reference info for detection
  - Noncoherent: Does not require reference phase information

Basic Digital Communication Transformations

– Coding/Decoding

Translating info bits to transmitter data symbols

Techniques used to enhance info signal so that they are less vulnerable to channel impairment (e.g. noise, fading, jamming, interference)

- Two Categories
  - *Waveform Coding*
- Produces new waveforms with better performance
  - *Structured Sequences*

Involves the use of redundant bits to determine the occurrence of error (an sometimes correct it)

– Multiplexing/Multiple Access Is synonymous with resource sharing with other users

– Frequency Division Multiplexing/Multiple Access (FDM/FDMA

Practical Aspects of Sampling

1. Sampling Theorem

2 .Methods of Sampling

3. Significance of Sampling Rate

4. Anti-aliasing Filter

5 . Applications of Sampling Theorem – PAM/TDM

Sampling

Sampling is the processes of converting continuous-time analog signal, $x_a(t)$, into a discrete-time s
by taking the “samples” at discrete-time intervals

- Sampling analog signals makes them discrete in time but still continuous valued
- If done properly (***Nyquist theorem*** is satisfied), sampling does not introduce distortion

Sampled values:

- The value of the function at the sampling points

Sampling interval:

- The time that separates sampling points (interval b/w samples), T_s
- If the signal is slowly varying, then fewer samples per second will be required than if the wavef is rapidly varying
- So, the optimum sampling rate depends on the *maximum frequency* component present in the signal

Analog-to-digital conversion is (basically) a 2 step process:

- Sampling
  - Convert from continuous-time analog signal $x_a(t)$ to discrete-time continuous value signal
- Is obtained by taking the "samples" of $x_a(t)$ at discrete-time intervals, T_s

Quantization

- Convert from discrete-time continuous valued signal to discrete time discrete valued signal

Sampling

ling Rate (or sampling frequency f_s):

he rate at which the signal is sampled, expressed as the number of samples per second eciprocal of the sampling interval), $\boldsymbol{1/T_s = f_s}$

st Sampling Theorem (or Nyquist Criterion):

the sampling is performed at a proper rate, no info is lost about the original signal and it can be operly reconstructed later on

atement:

"If a signal is sampled at a rate at least, but not exactly equal to twice the max frequency onent of the waveform, then the waveform can be exactly reconstructed from the samples without any distortion"

$$f_s \geq 2 f_{\max}$$

….. Sampling Theorem

Sampling Theorem for Bandpass Signal - If an analog information signal containing no frequency outside the specified bandwidth W Hz, it may be reconstructed from its samples at a sequence of points spaced 1/(2W) seconds apart with zero-mean squared error.

The minimum sampling rate of (2W) samples per second, for an analog signal bandwidth of W Hz, is called the **Nyquist rate**.

The reciprocal of Nyquist rate, 1/(2W), is called the **Nyquist interval**, that is, $T_s = 1/(2W)$.

The phenomenon of the presence of high-frequency component in the spectrum of the original analog signal is called aliasing or simply foldover.

Sampling Theorem

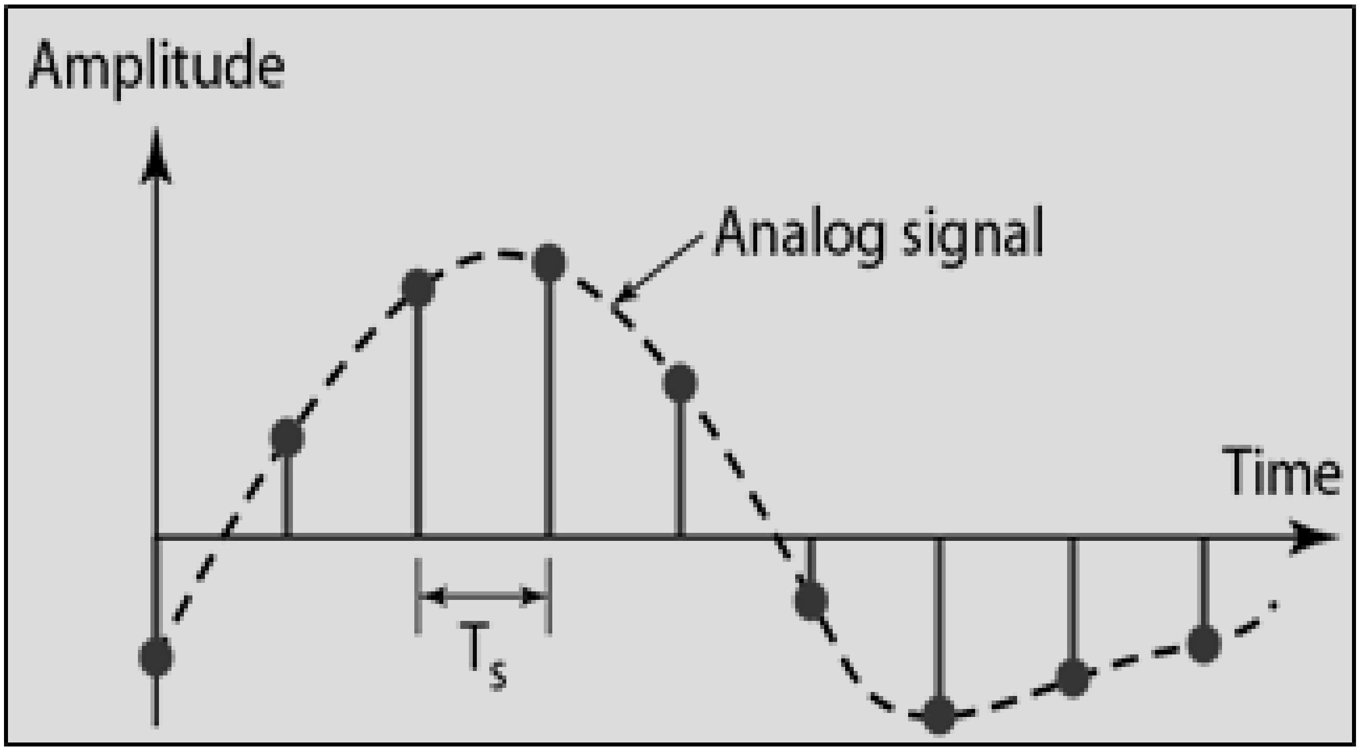
Sampling Theorem for Baseband Signal - A baseband signal having no frequency components higher than f_m Hz may be completely recovered from the knowledge of its samples taken at a rate of at least $2\,f_m$ samples per second, that is, sampling frequency $\boldsymbol{f_s \geq 2\,f_m}$.

The minimum sampling rate $f_s = 2\,f_m$ samples per second is called the Nyquist sampling rate.

A baseband signal having no frequency components higher than f_m Hz is completely described by its sample values at uniform intervals less than or equal to $1/(2f_m)$ seconds apart, that is, the sampling interval $T_s \leq 1/(2f_m)$ seconds.

Methods of Sampling

Ideal sampling - an impulse at each sampling instant

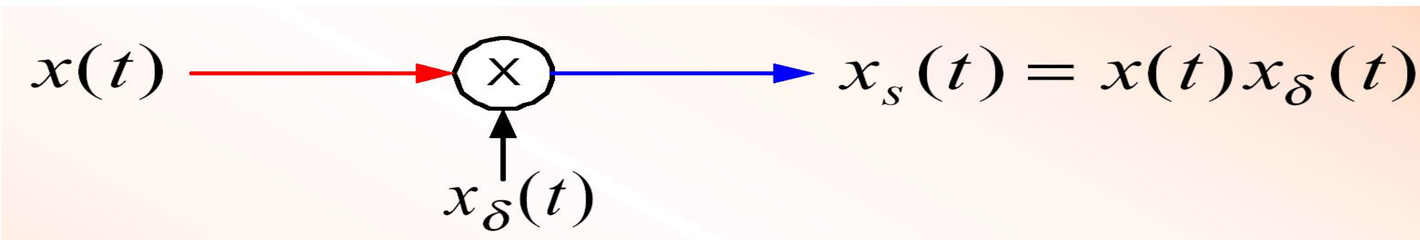


Ideal Sampling

Ideal Sampling (or Impulse Sampling)

Is accomplished by the multiplication of the signal *x(t)* by the uniform train of impulses (comb function)

Consider the instantaneous sampling of the analog signal *x(t)*



Train of impulse functions select sample values at regular intervals

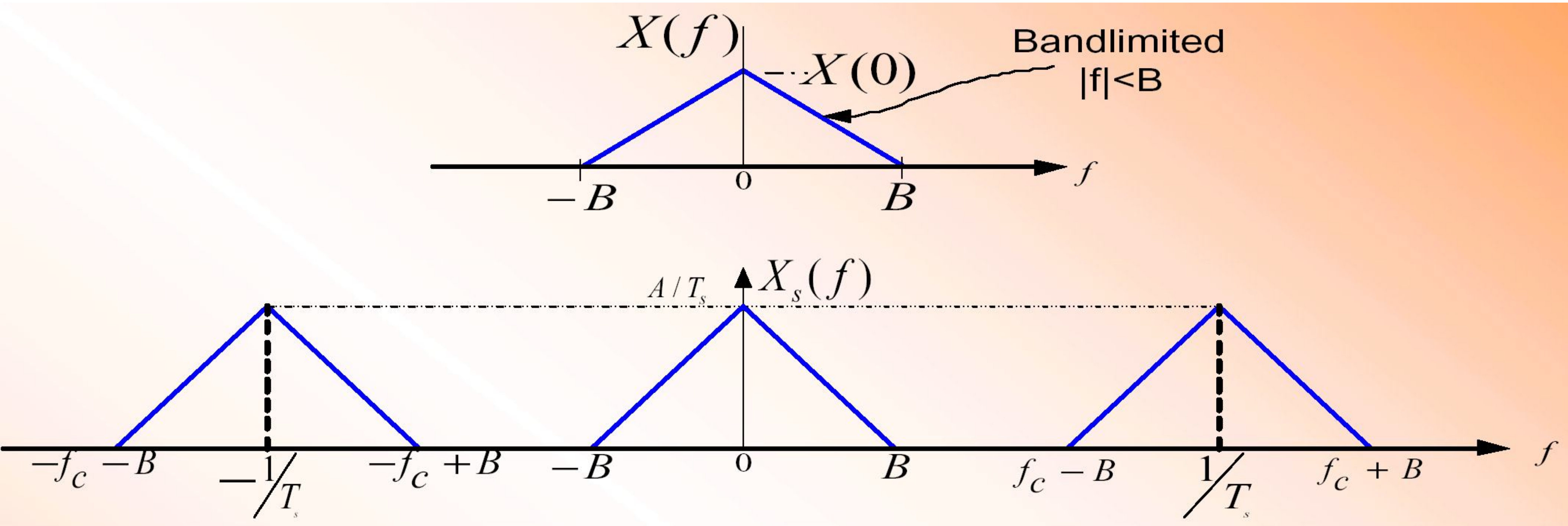
$$x_s(t) = x(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Fourier Series representation:

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} e^{jn\omega_s t}, \qquad \omega_s = \frac{2\pi}{T_s}$$

Ideal Sampling (or Impulse Sampling)

is shows that the Fourier Transform of the sampled signal is the Fourier Transform of the original nal at rate of $1/T_s$



Ideal Sampling (or Impulse Sampling)

s long as $f_s > 2f_m$, no overlap of repeated replicas $X(f - n/T_s)$ will occur in $X_s(f)$

Minimum Sampling Condition:

$$\boxed{f_s - f_m > f_m \quad \Rightarrow \quad f_s > 2 f_m}$$

Sampling Theorem: A finite energy function $x(t)$ can be completely ***reconstructed*** from ampled value $x(nTs)$ with

$$(t) = \sum_{n=-\infty}^{\infty} T_s x(nT_s) \left\{ \frac{\sin\left[\dfrac{2\pi f(t - nT_s)}{2T_s}\right]}{\pi(t - nT_s)} \right\}$$

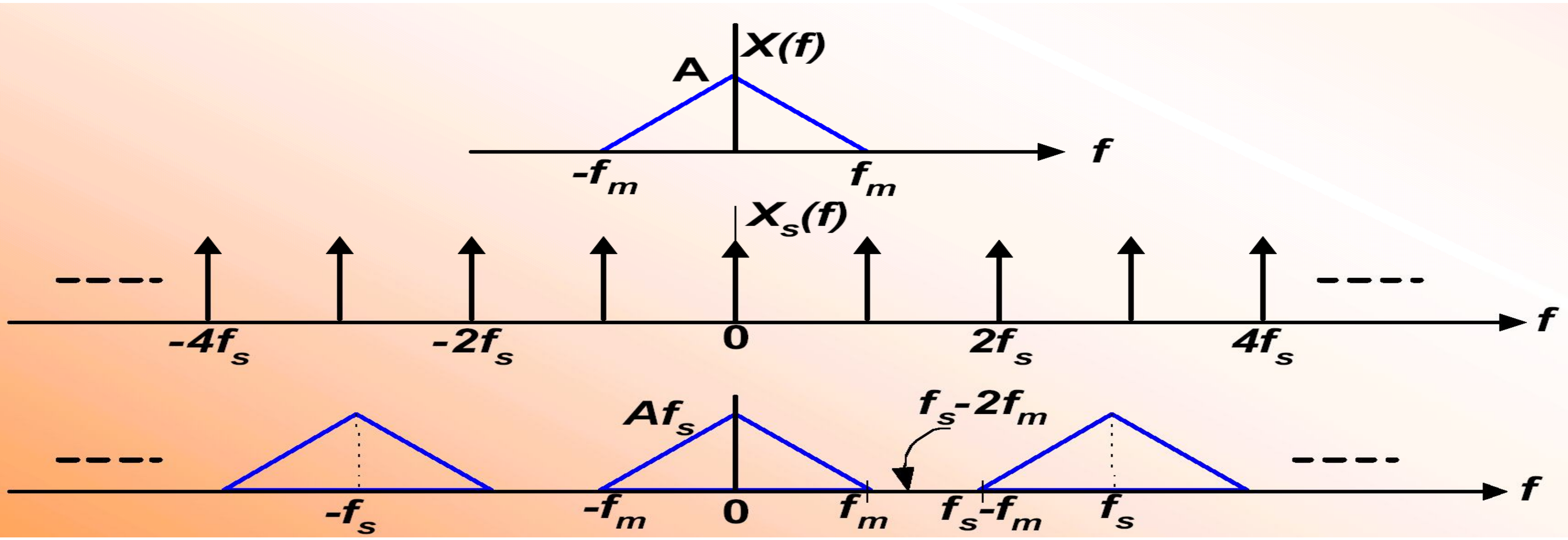
$$= \sum_{n=-\infty}^{\infty} T_s \, x(nT_s) \, \sin c(2 f_s (t - nT_s))$$

provided that =>

$$\boxed{\frac{1}{f_s} = T_s \leq \frac{1}{2 f_m}}$$

Ideal Sampling (or Impulse Sampling)

is means that the output is simply the replication of the original signal at discrete intervals, e.g



T_s is called the ***Nyquist interval:*** It is the longest time interval that can be used for sampling a bar
signal and still allow reconstruction of the signal at the receiver without distortion

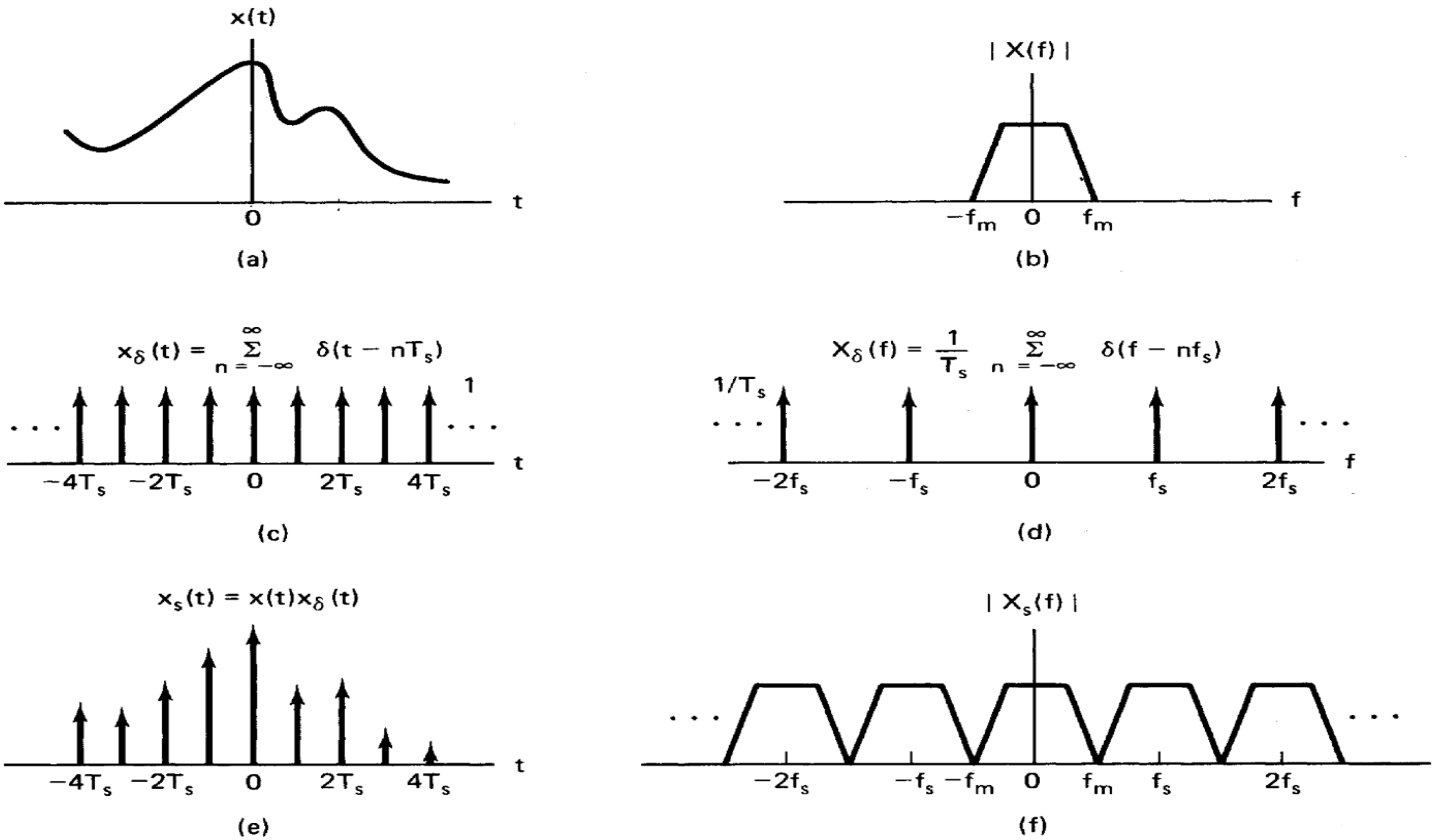
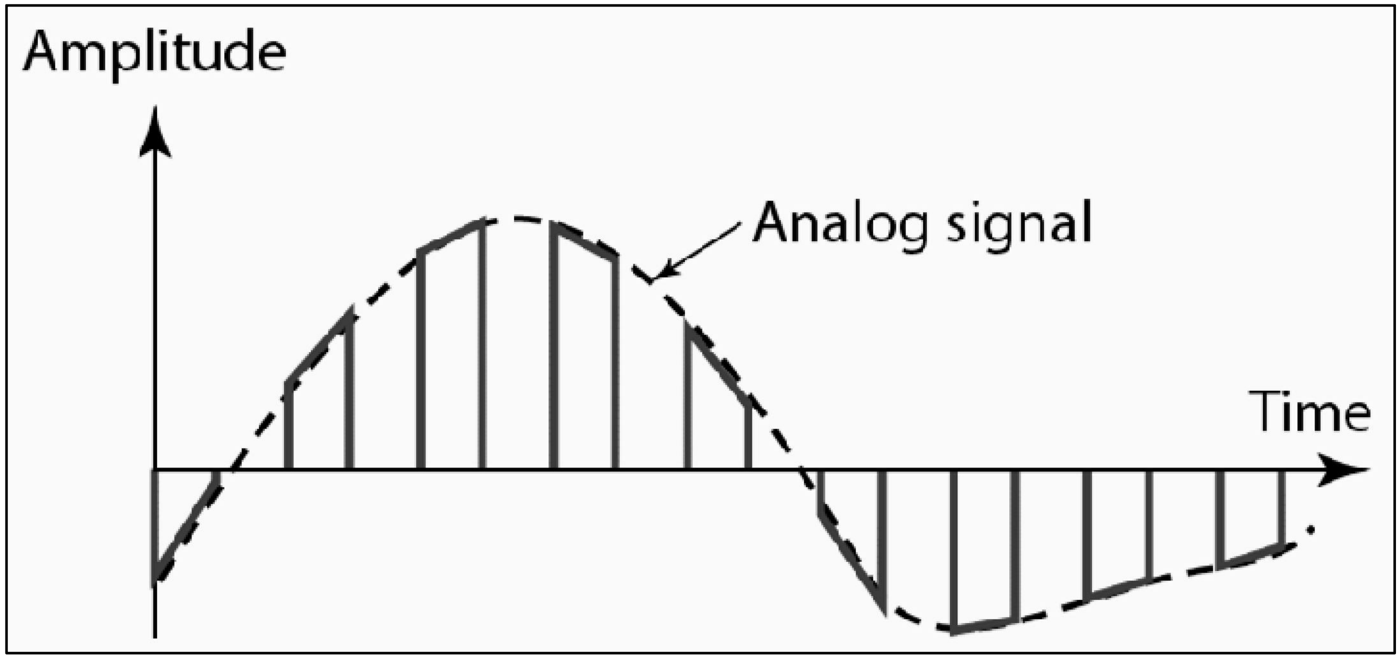


Figure 2.6 Sampling theorem using the frequency convolution property of the Fourier transform.

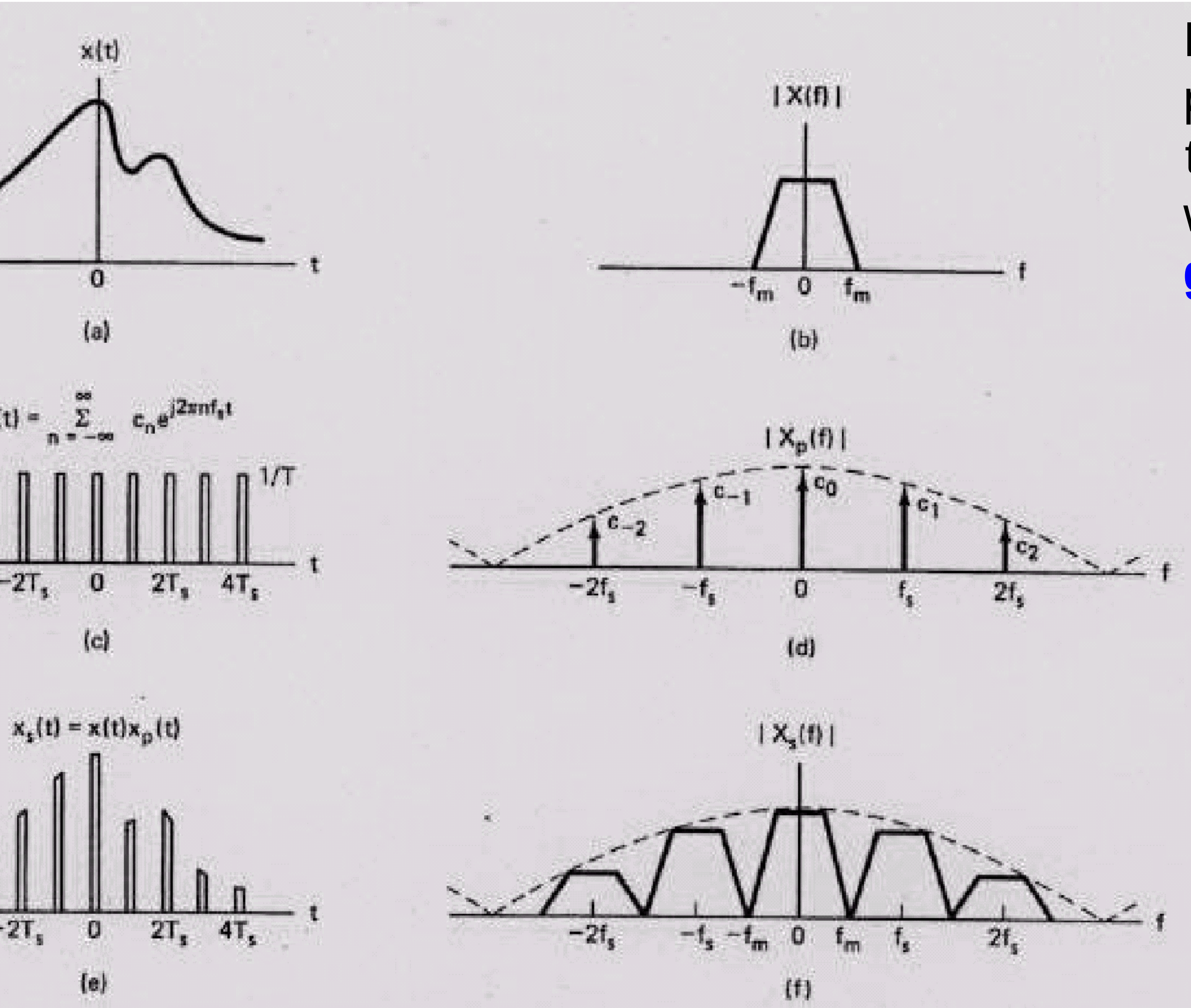
..... Methods of Sampling

Natural sampling - a pulse of short width with varying amplitude with natural tops



Natural Sampling

Natural Sampling



If we multiply *x(t)* by a train of rectang pulses $x_p(t)$, we obtain a gated wavefo that approximates the ideal sampled waveform, known as **natural samplin gating (s**ee Figure 2.8)

$$x_s(t) = x(t)\,x_p(t)$$

$$= x(t)\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s}$$

$$X_s(f) = \Im[x(t)\,x_p(t)]$$

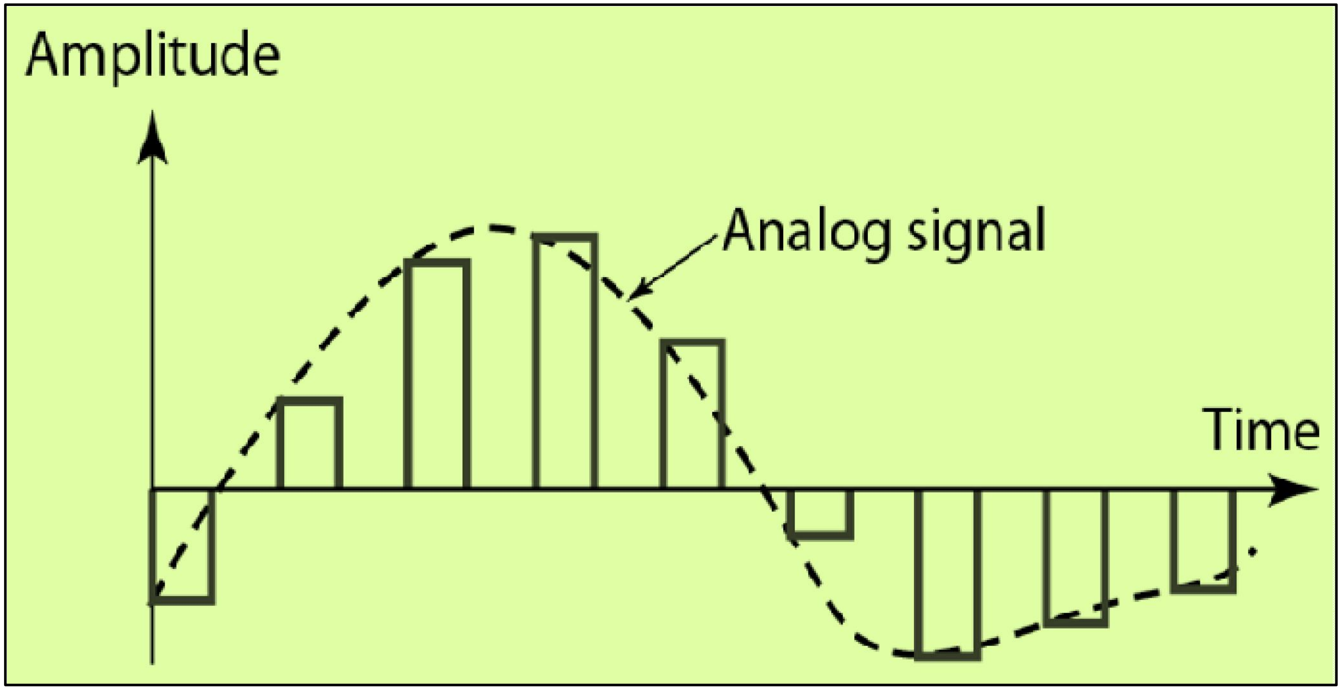
$$= \sum_{n=-\infty}^{\infty} c_n \Im[x(t)e^{j2\pi}$$

$$= \sum_{n=-\infty}^{\infty} c_n X[f - nf_s$$

- Each pulse in $x_p(t)$ has width T_s and amplitude $1/T_s$
- The top of each pulse follows the variation of the signal being sampled
- $X_s(f)$ is the replication of $X(f)$ periodically every f_s Hz
- $X_s(f)$ is weighted by $C_n \leftarrow$ Fourier Series Coeffiecient
- The problem with a natural sampled waveform is that the tops of the sample pulses are not flat
- It is not compatible with a digital system since the amplitude of each sample has infinite number of possible values
- Another technique known as ***flat top sampling*** is used to alleviate this problem

..... Methods of Sampling

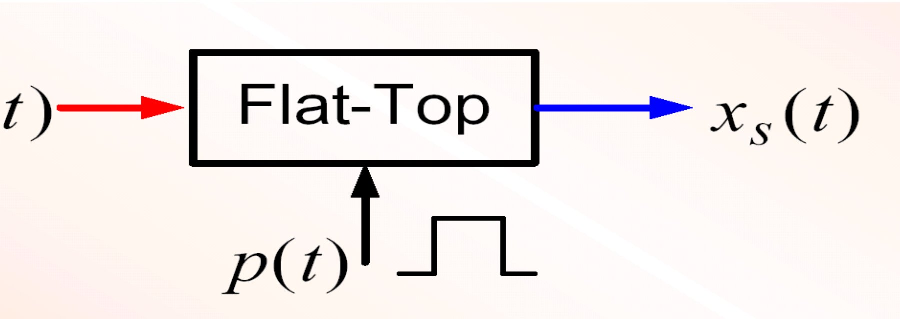
Flat-top sampling - a pulse of short width with varying amplitude with flat tops

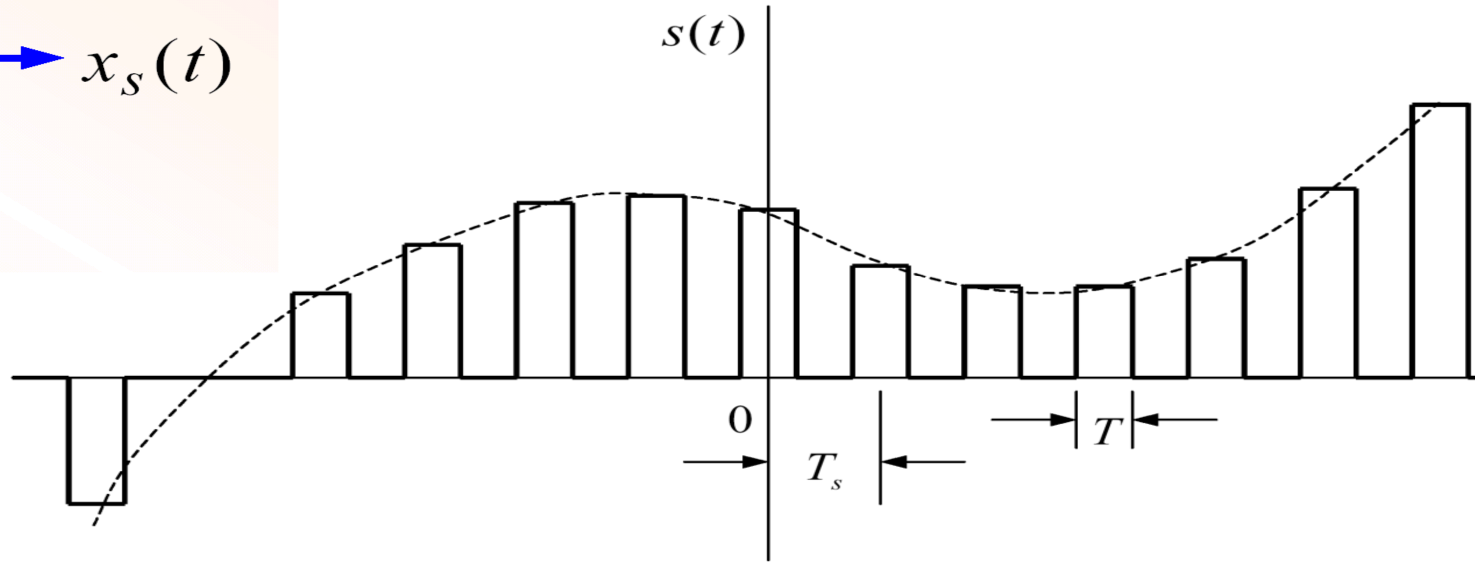


Flat-top Sampling

Flat-Top Sampling

- re, the pulse is held to a constant height for the whole sample period
- t top sampling is obtained by the convolution of the signal obtained after ideal mpling with a unity amplitude rectangular pulse, *p(t)*
- is technique is used to realize ***Sample-and-Hold*** (S/H) operation
- S/H, input signal is continuously sampled and then the value is held for as long as kes to for the A/D to acquire its value





Flat top sampling (Time Domain)

$$x'(t) = x(t)\delta(t)$$

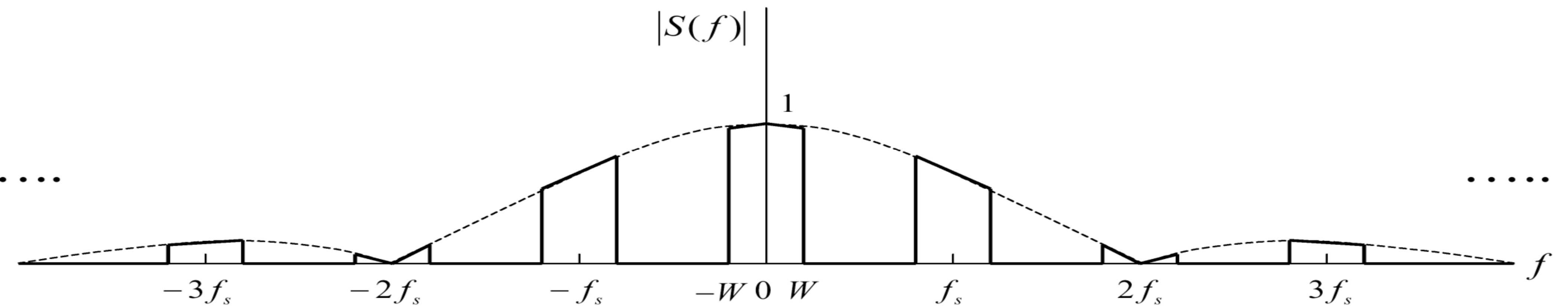
$$x_s(t) = x'(t) * p(t)$$

$$= p(t) * x(t)\delta(t) = p(t) * \left[x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right]$$

Taking the Fourier Transform will result to

$$
\begin{aligned}
X_s(f) &= \Im[x_s(t)] \\
&= P(f)\,\Im\left[x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] \\
&= P(f)\,\Im\left[X(f) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-nf_s)\right] \\
&= P(f)\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(f-nf_s)
\end{aligned}
$$

where *P(f)* is a *sinc* function

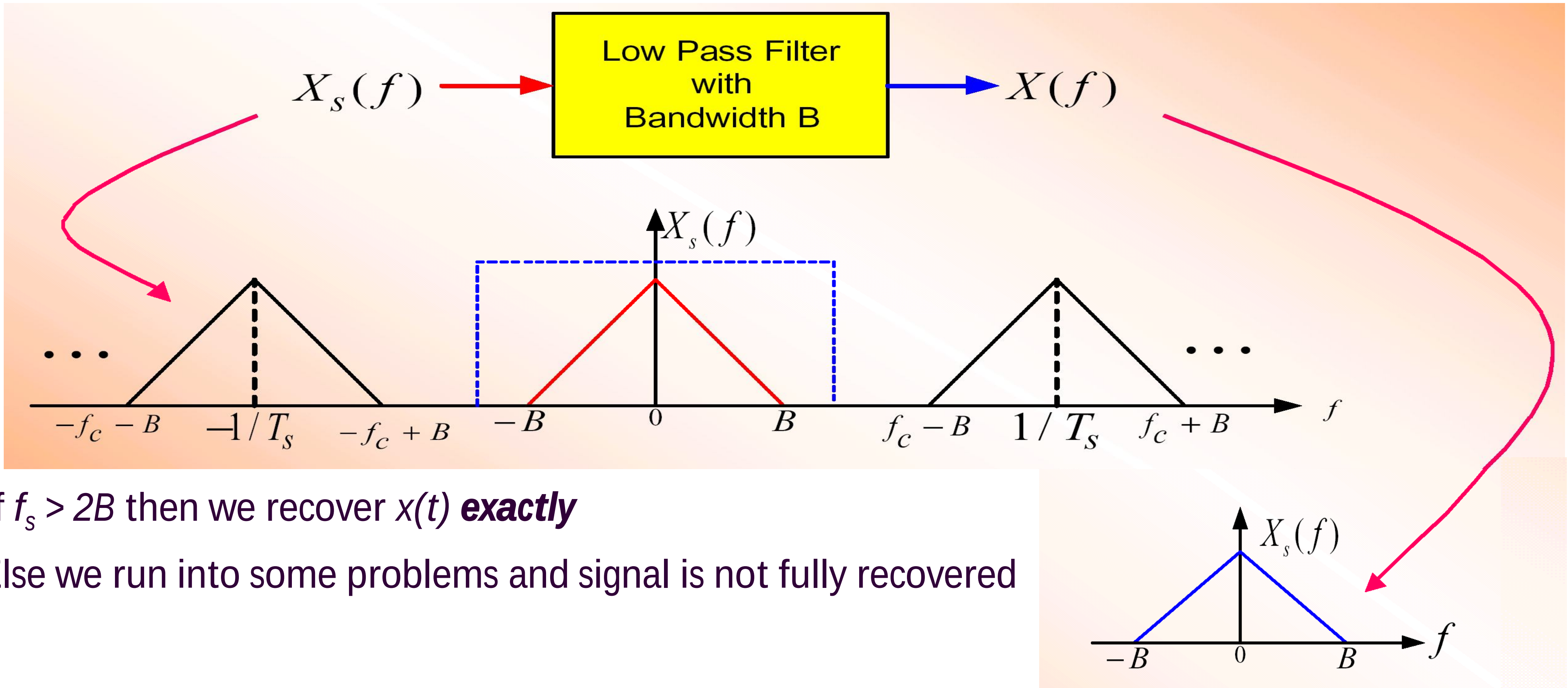


Flat top sampling (Frequency Domain)

- Flat top sampling becomes identical to ideal sampling as the width of the pulses become shorter

Recovering the Analog Signal

One way of recovering the original signal from sampled signal $X_s(f)$ is to pass it through a Low Pass Filter (LPF) as shown below

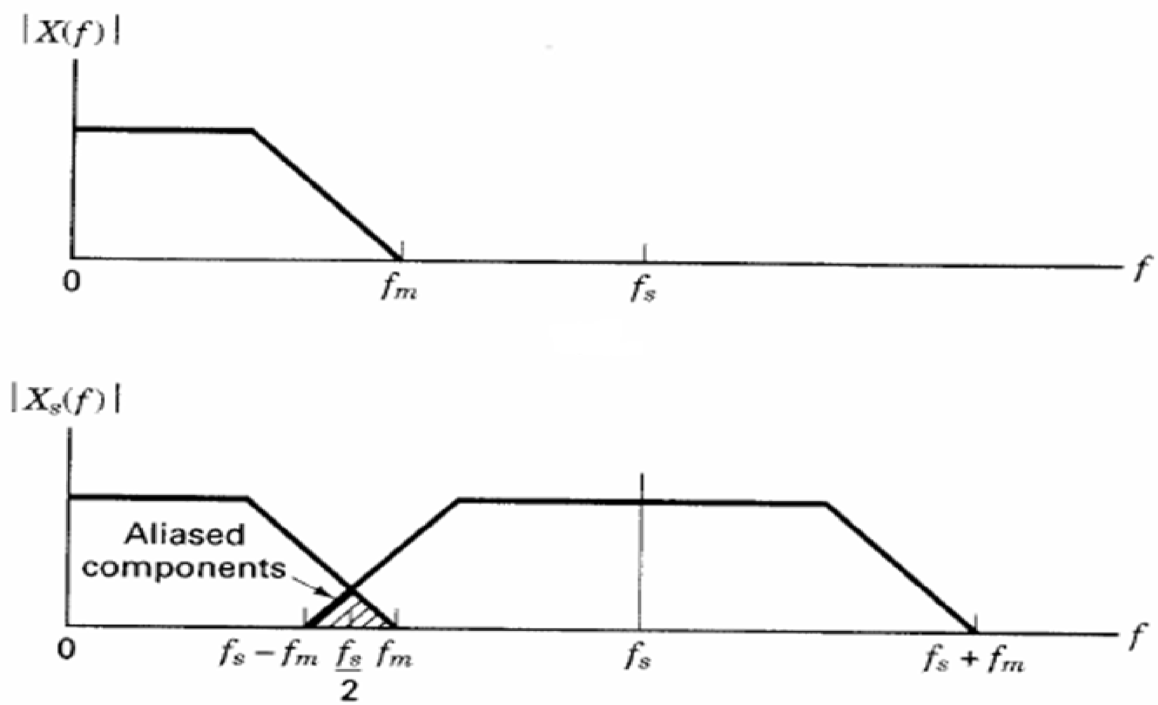


If $f_s > 2B$ then we recover $x(t)$ ***exactly***

Else we run into some problems and signal is not fully recovered

Significance of Sampling Rate

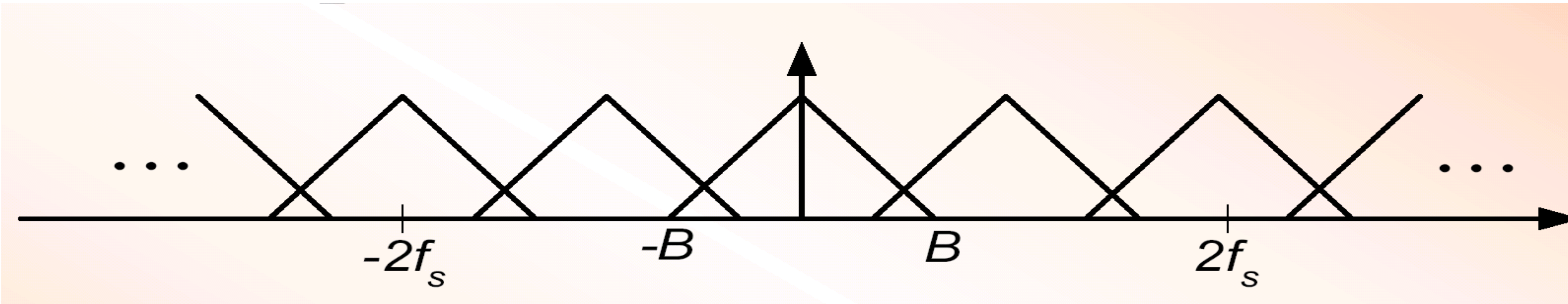
When $f_s < 2f_m$, spectral components of adjacent samples will overlap, known as aliasing



An Illustration of Aliasing

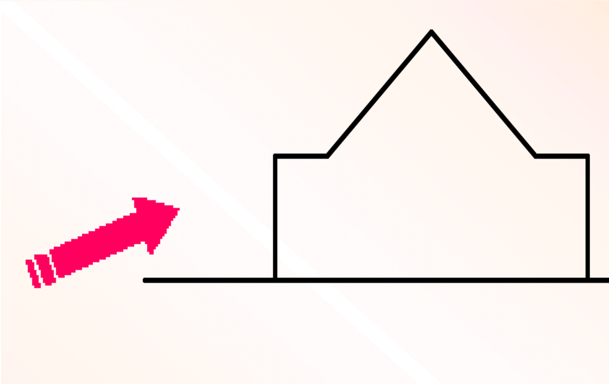
Undersampling and Aliasing

– If the waveform is ***undersampled*** (i.e. *fs* < *2B*) then there will be ***spectral overlap*** in the sample
signal



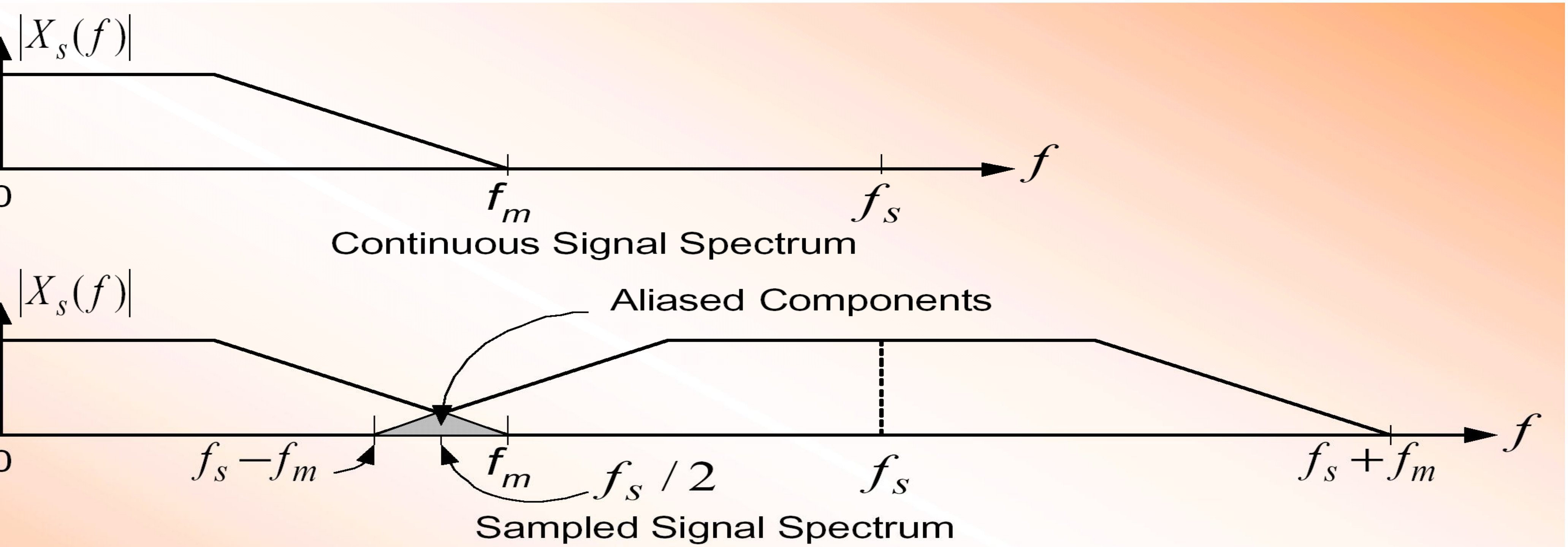
e signal at the output of the filter will be

different from the original signal spectrum



This is the outcome of ***aliasing***!

is implies that whenever the sampling condition is not met, an irreversible overlap of the spectral
licas is produced



This could be due to:

1. *x(t)* containing higher frequency than were expected
2. An error in calculating the sampling rate

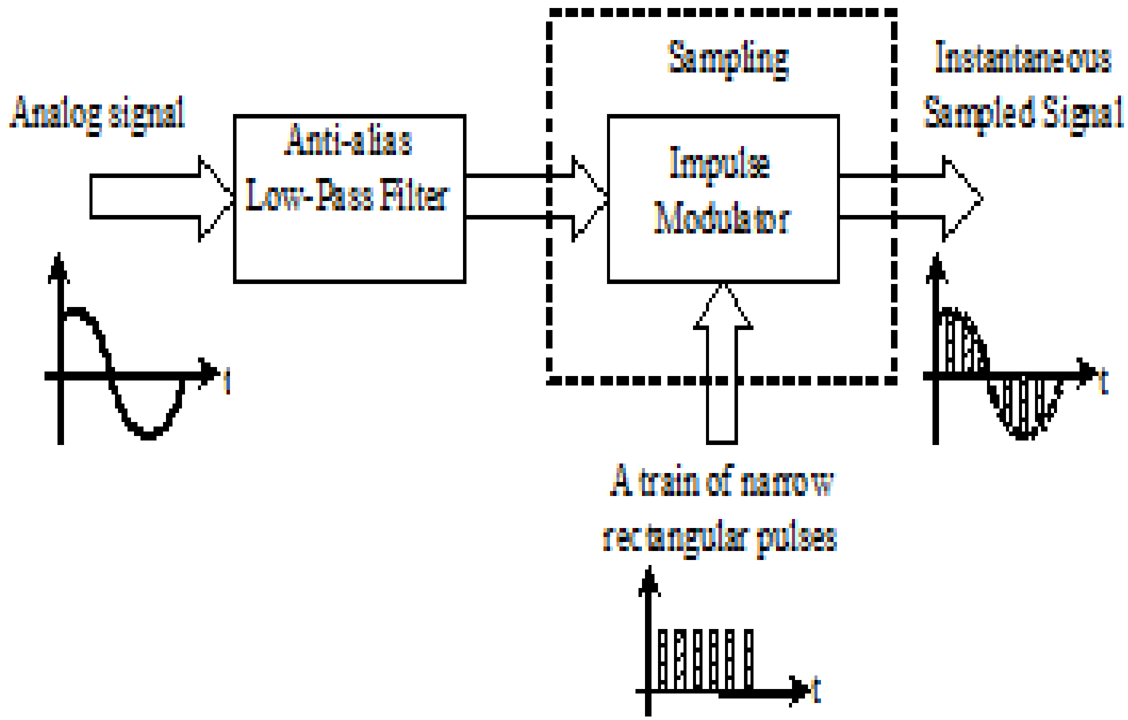
Under normal conditions, undersampling of signals causing aliasing is not recommended

Solution 1: Anti-Aliasing Analog Filter

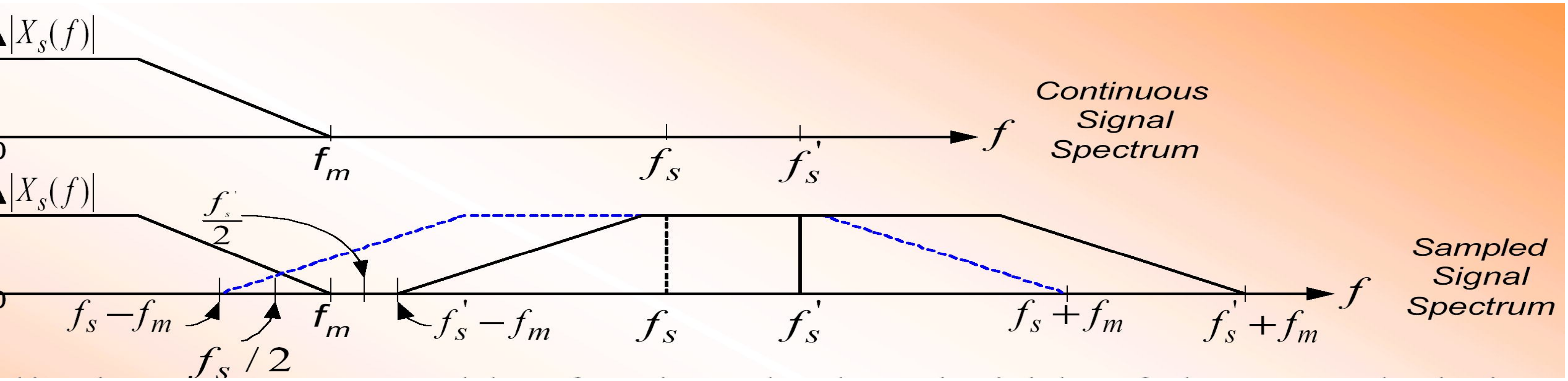
- All physically realizable signals are not completely bandlimited
- If there is a significant amount of energy in frequencies above half the sampling frequency ($f_s/2$), aliasing will occur
- Aliasing can be prevented by first passing the analog signal through an **anti-aliasing** filter (als called a **prefilter**) before sampling is performed
- The anti-aliasing filter is simply a LPF with cutoff frequency equal to half the sample rate

Antialiasing Filter

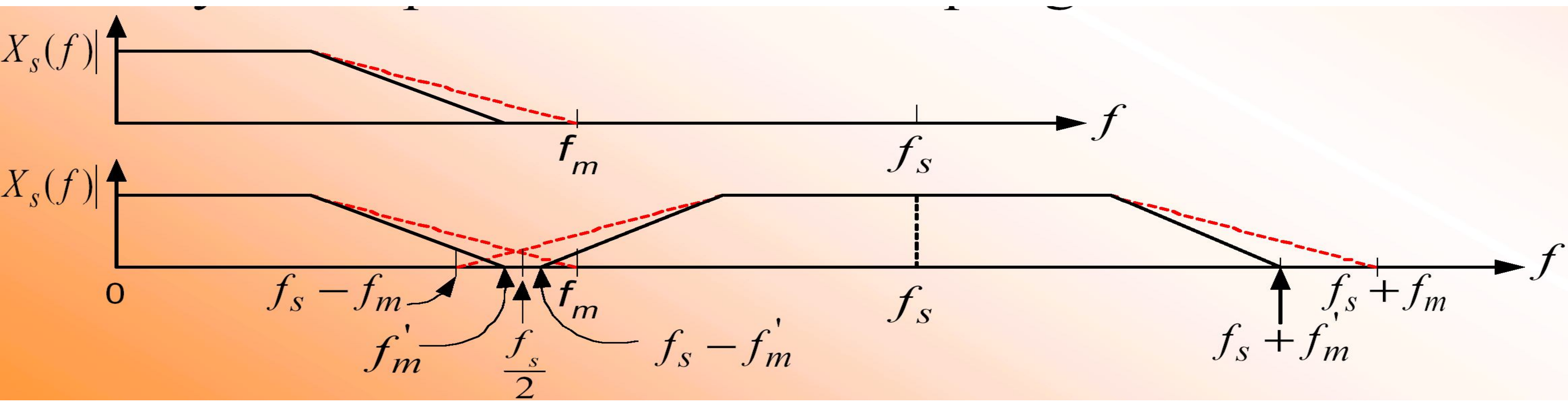
An anti-aliasing filter is a *low-pass filter* of sufficient higher order which is recommended to be used prior to sampling.



Minimizing Aliasing



- Aliasing is prevented by forcing the bandwidth of the sampled signal to satisfy the requirement of the Sampling Theorem



Solution 2: Over Sampling and Filtering in the Digital Domain

– The signal is passed through a low performance (less costly) analog low-pass filter to lim the bandwidth.
– Sample the resulting signal at a high sampling frequency.
– The digital samples are then processed by a high performance digital filter and down sample the resulting signal.

Summary Of Sampling

Ideal Sampling
(or Impulse Sampling)

$$x_s(t) = x(t)x_\delta(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT$$

Natural Sampling
(or Gating)

$$x_s(t) = x(t)x_p(t) = x(t)\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n}$$

Flat-Top Sampling

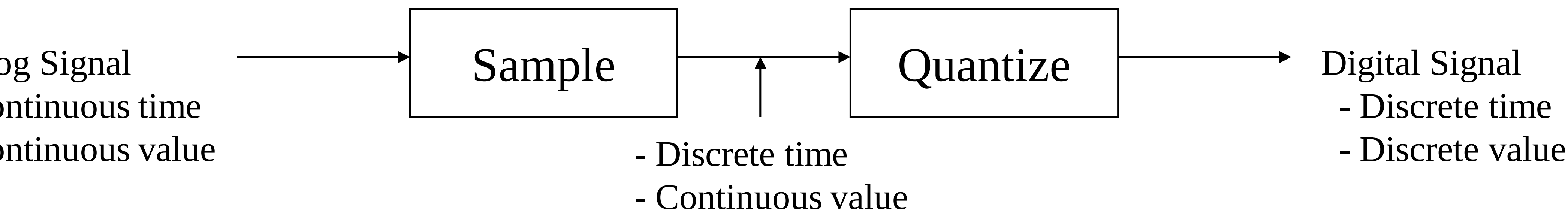
$$x_s(t) = x'(t) * p(t) = \left[x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)\right] * p$$

For all sampling techniques

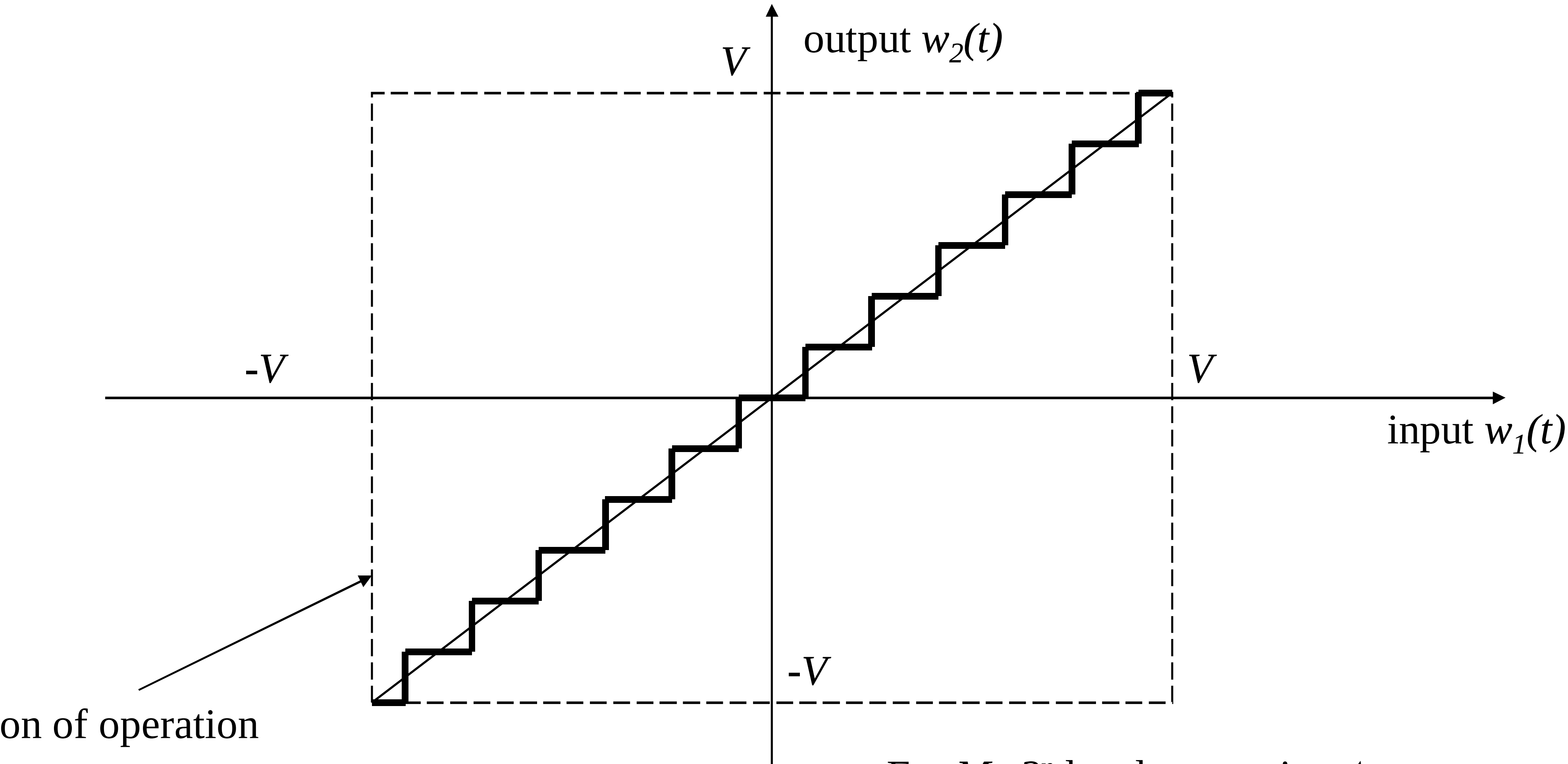
- If *fs > 2B then we can recover x(t) exactly*
- If *fs < 2B*) ***spectral overlapping*** known as **aliasing** will occur

Quantization

Quantization is a non linear transformation which maps elements from a continuous
o a finite set. It is also the second step required by A/D conversion.



Uniform Quantization



For $M=2^n$ levels, step size Δ:

$\Delta = 2V/2^n = V(2^{-n+1})$

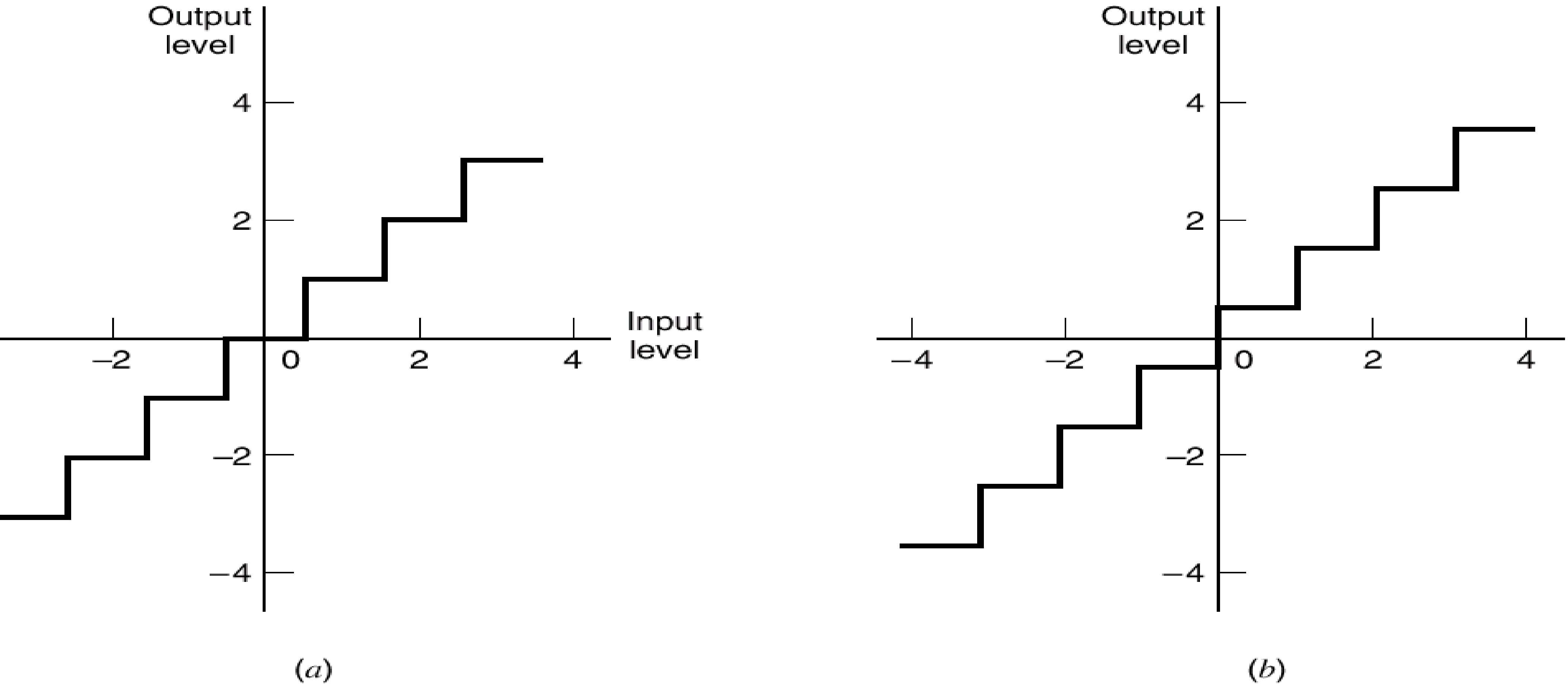
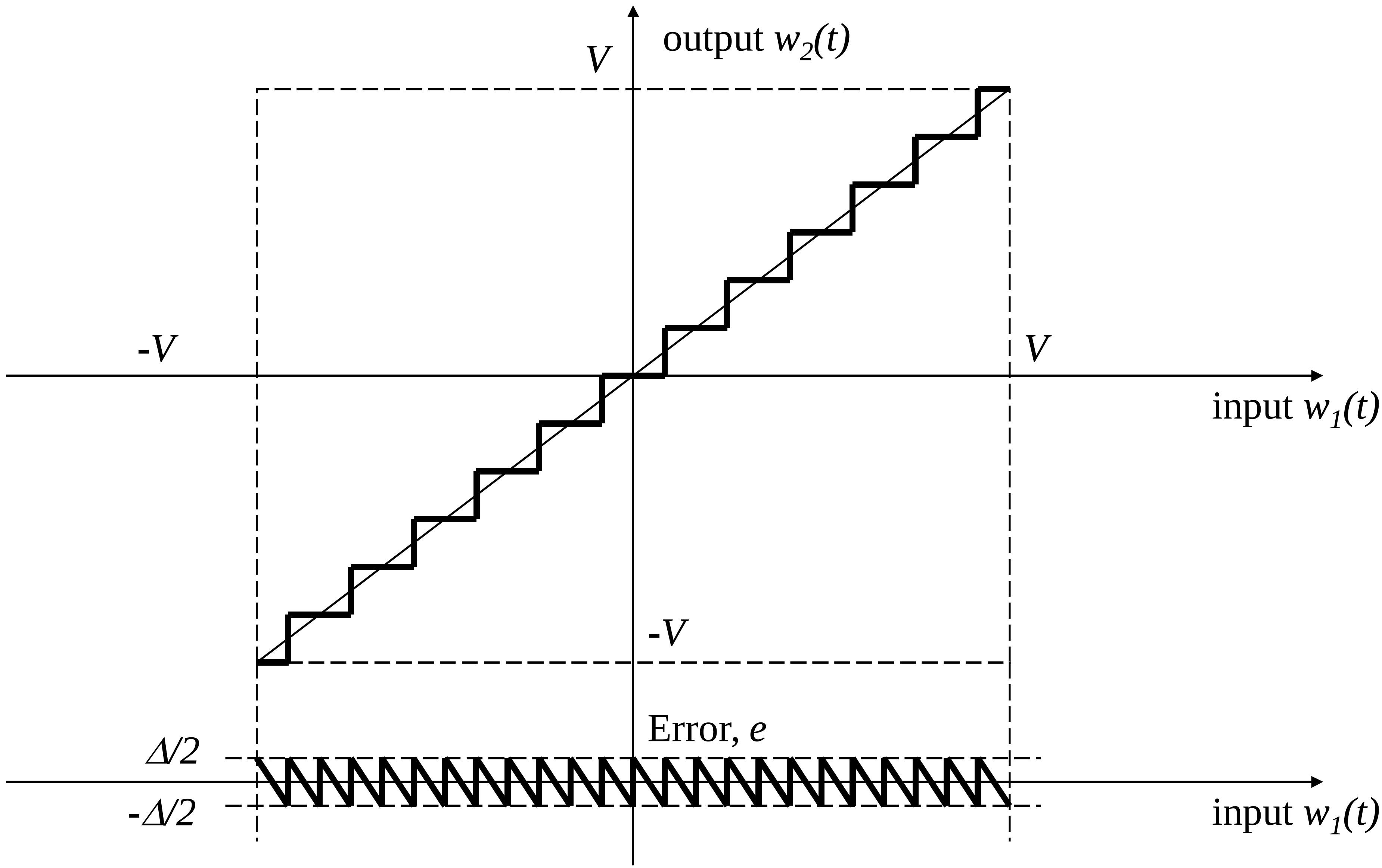
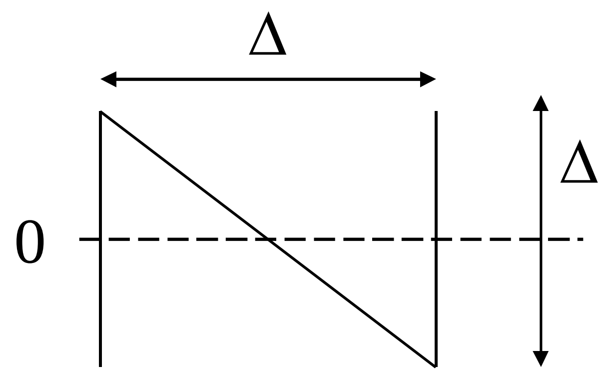


Figure 3.10 Two types of quantization: (*a*) midtread and (*b*) midrise.

Quantization Error, *e*



Error is symmetric around zero.



Average error power :

$$\frac{1}{2V}\int_{-V}^{V} e^2(s)\,ds = \frac{2}{\Delta}\int_{0}^{\frac{\Delta}{2}} x^2\,dx = \frac{2}{\Delta}\left(\frac{\left(\frac{\Delta}{2}\right)^3}{3}\right) = \frac{\Delta^2}{12} = \frac{\left(V \bullet 2^{-n+1}\right)^2}{12} = \frac{V^2}{3}2^{-2n}$$

Suppose the input signal is a triangular wave between $-V$ and $+V$.

Then the average signal power is $\frac{V^2}{3}$.

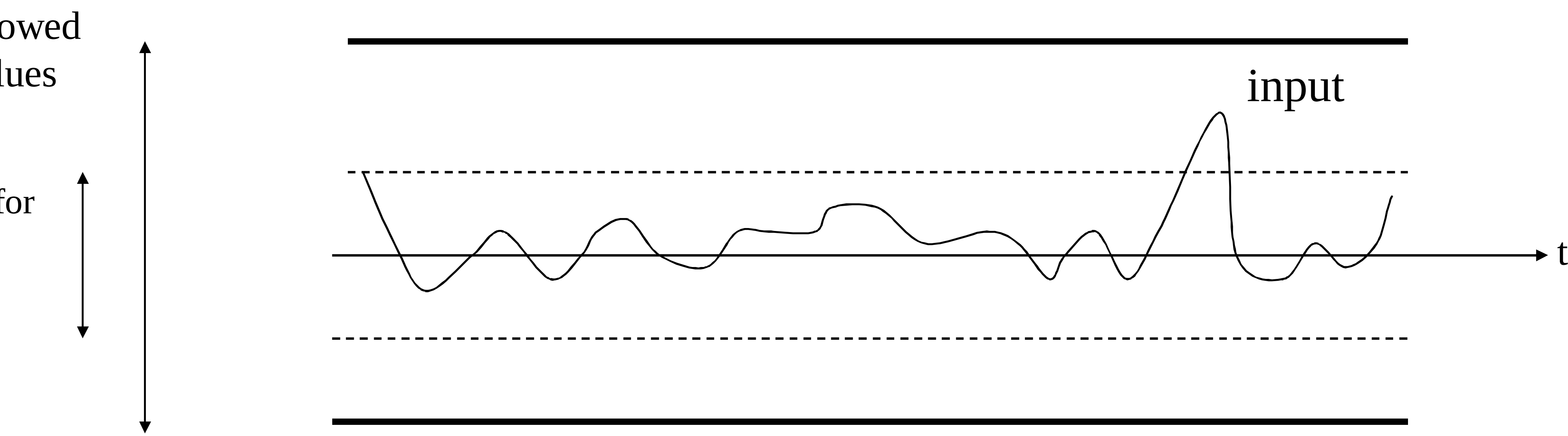
$$\Rightarrow \left(\frac{S}{N}\right)_{\text{out}} = 2^{2n}$$

Definition. The dynamic range of an input signal is the ratio of the largest to the smallest power levels which the input signal can take on and be reproduced with the acceptable signal distortion.

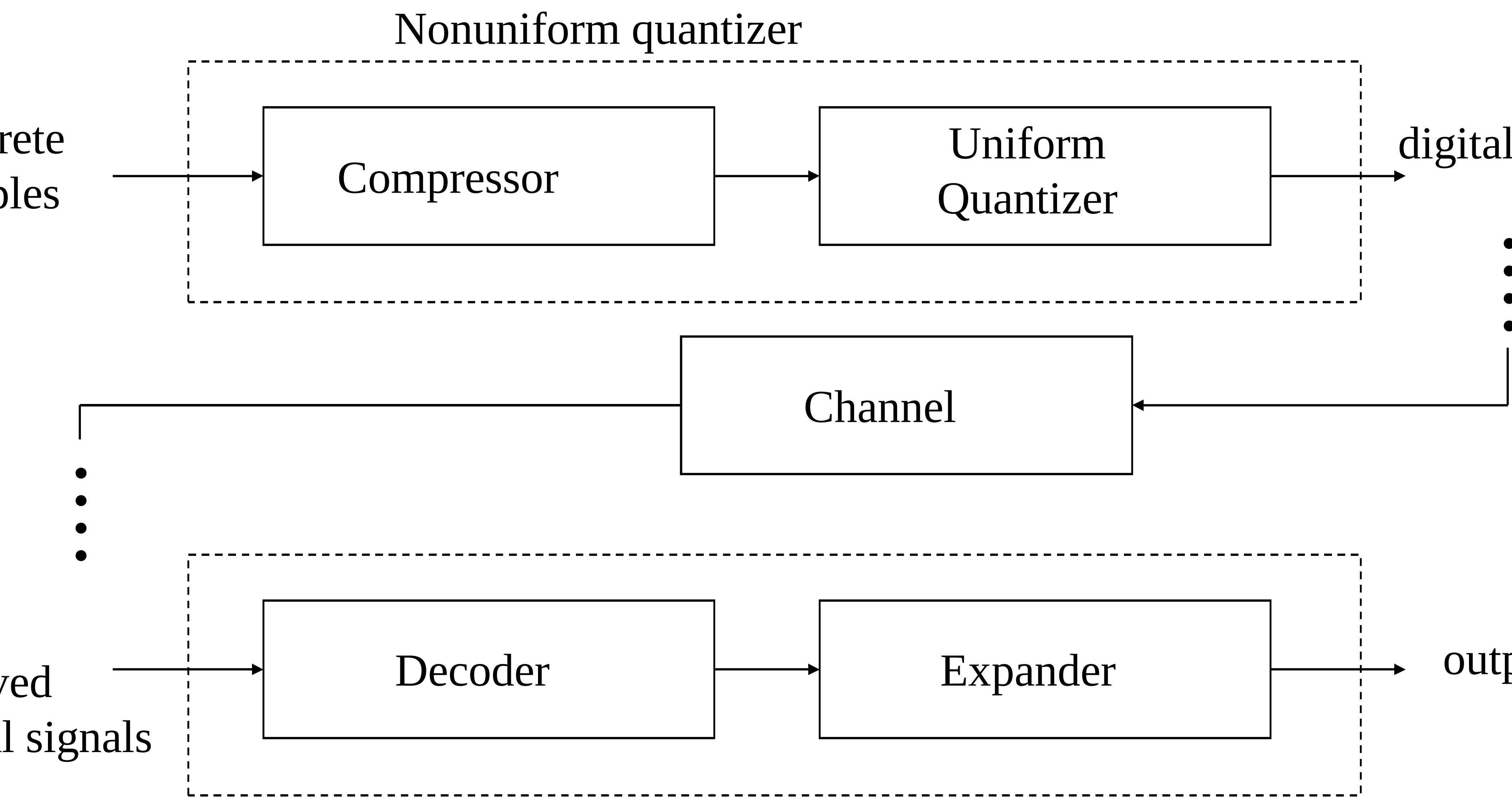
The dynamic range of the quantizer input in the PCM system is $6n$ dB.

Nonuniform Quantizer

Used to reduce quantization error and increase the dynamic range when input signa
not uniformly distributed over its allowed range of values.



ressing-and-expanding” is called “companding.”



Line codes:

1. Unipolar nonreturn-to-zero (NRZ) Signaling
2. Polar nonreturn-to-zero(NRZ) Signaling
3. Unipor nonreturn-to-zero (RZ) Signaling
4. Bipolar nonreturn-to-zero (BRZ) Signaling
5. Split-phase (Manchester code)

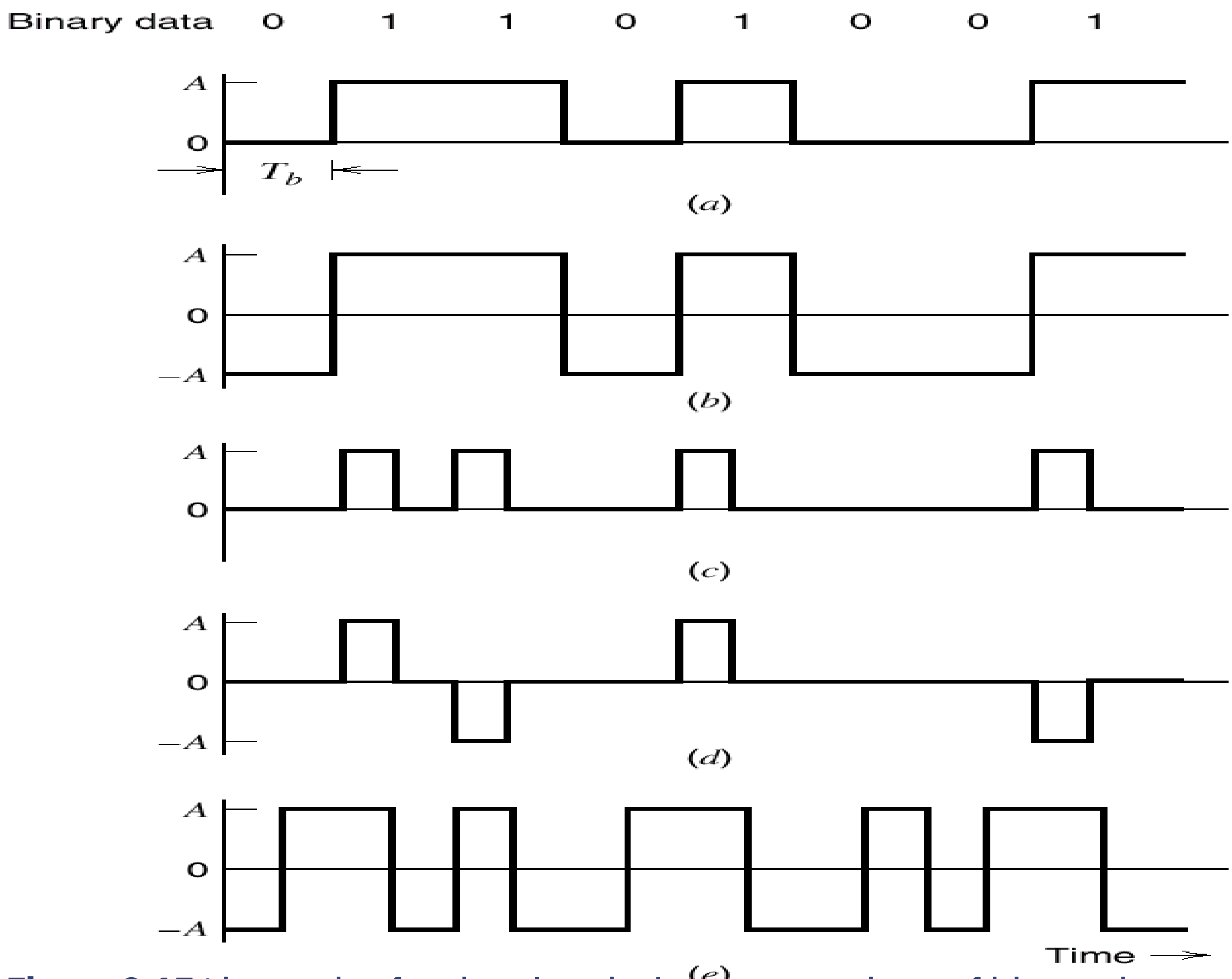
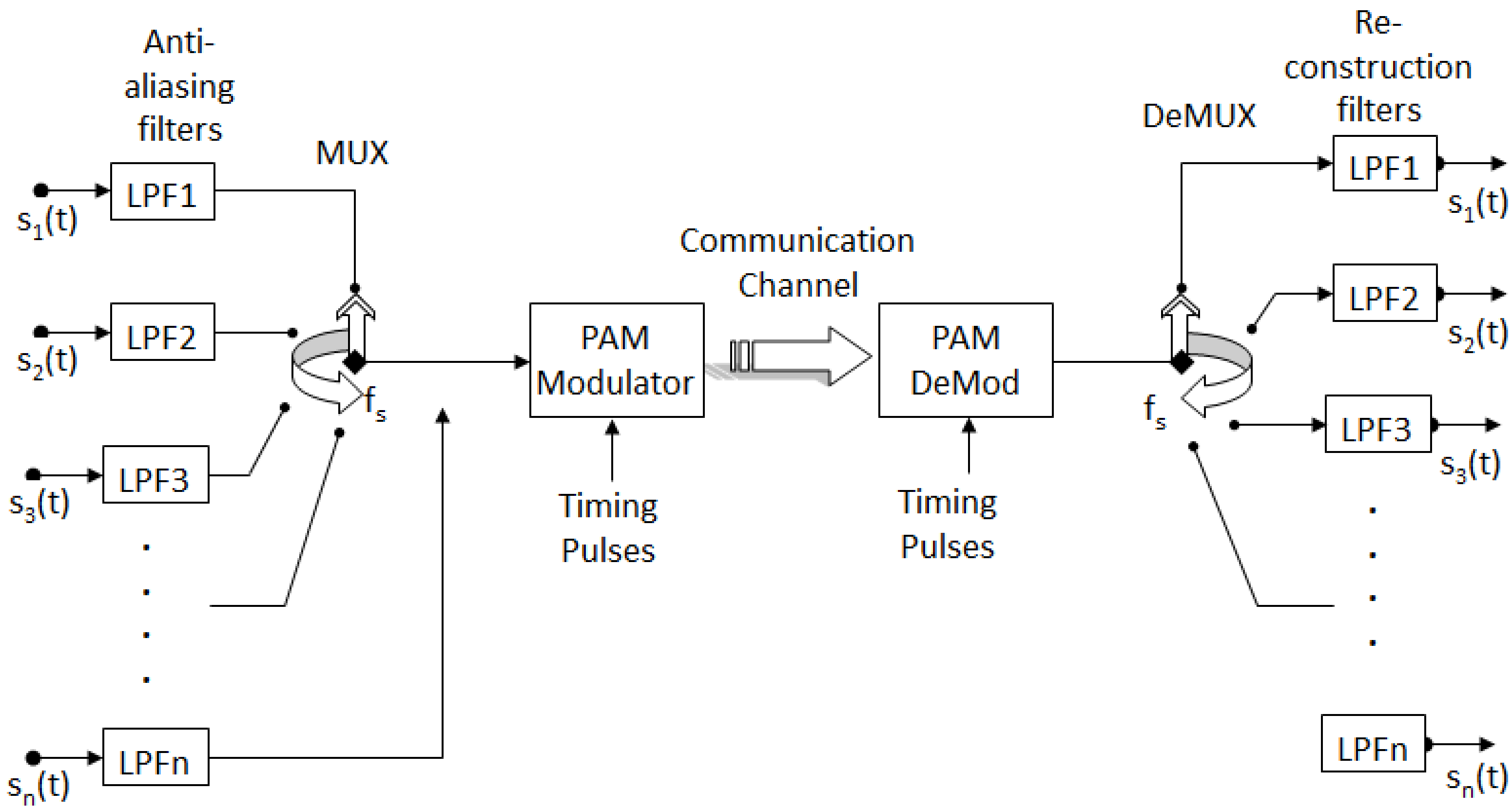


Figure 3.15 Line codes for the electrical representations of binary data.
(*a*) Unipolar NRZ signaling. (*b*) Polar NRZ signaling.
(c) Unipolar RZ signaling. (*d*) Bipolar RZ signaling.
(*e*) Split-phase or Manchester code.

Application of Sampling Theorem – PAM/TDM

Design of PAM/TDM System



UNIT-II DIGITAL MODULATION

Pulse Code Modulation (PCM)

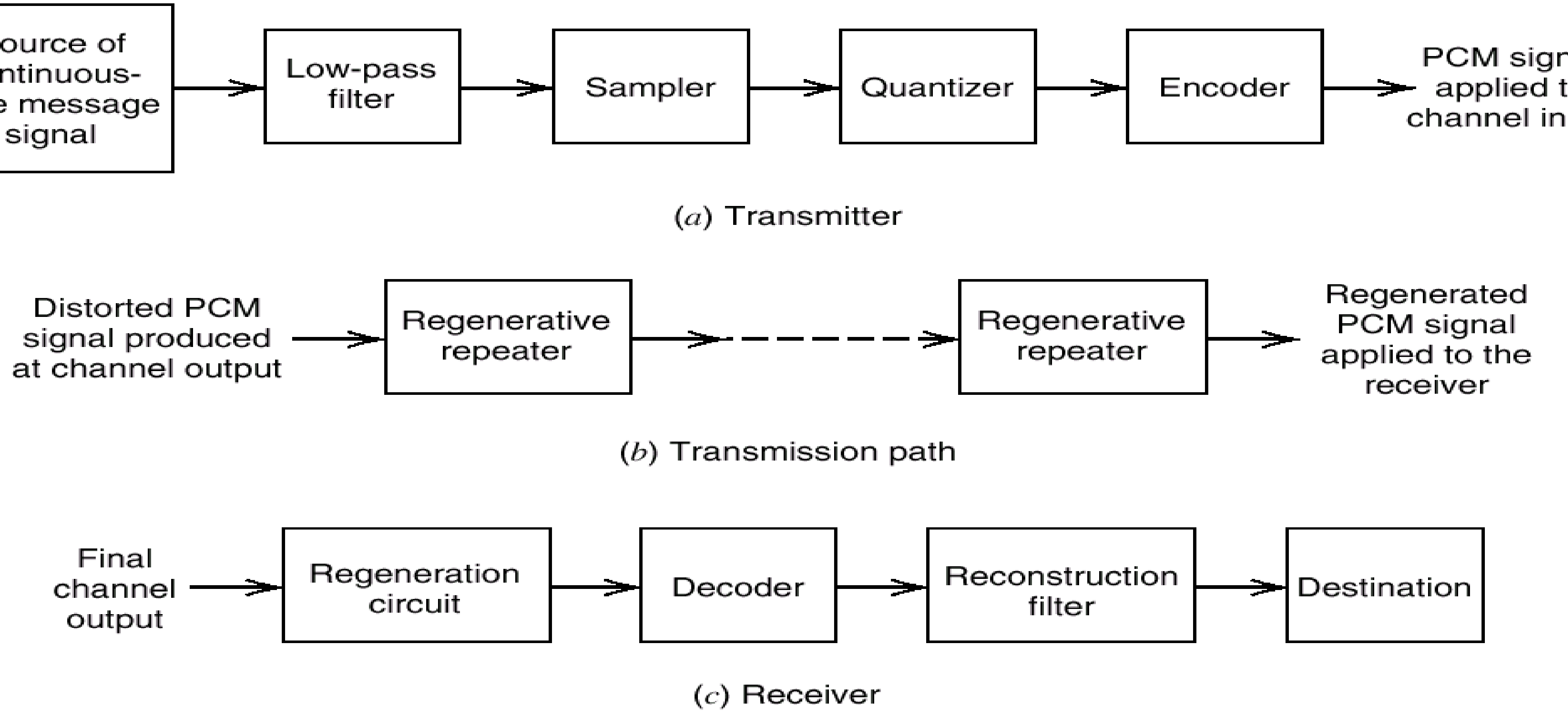
1. Block Diagram of PCM

2 PCM Sampling

3 Quantization of Sampled Signal

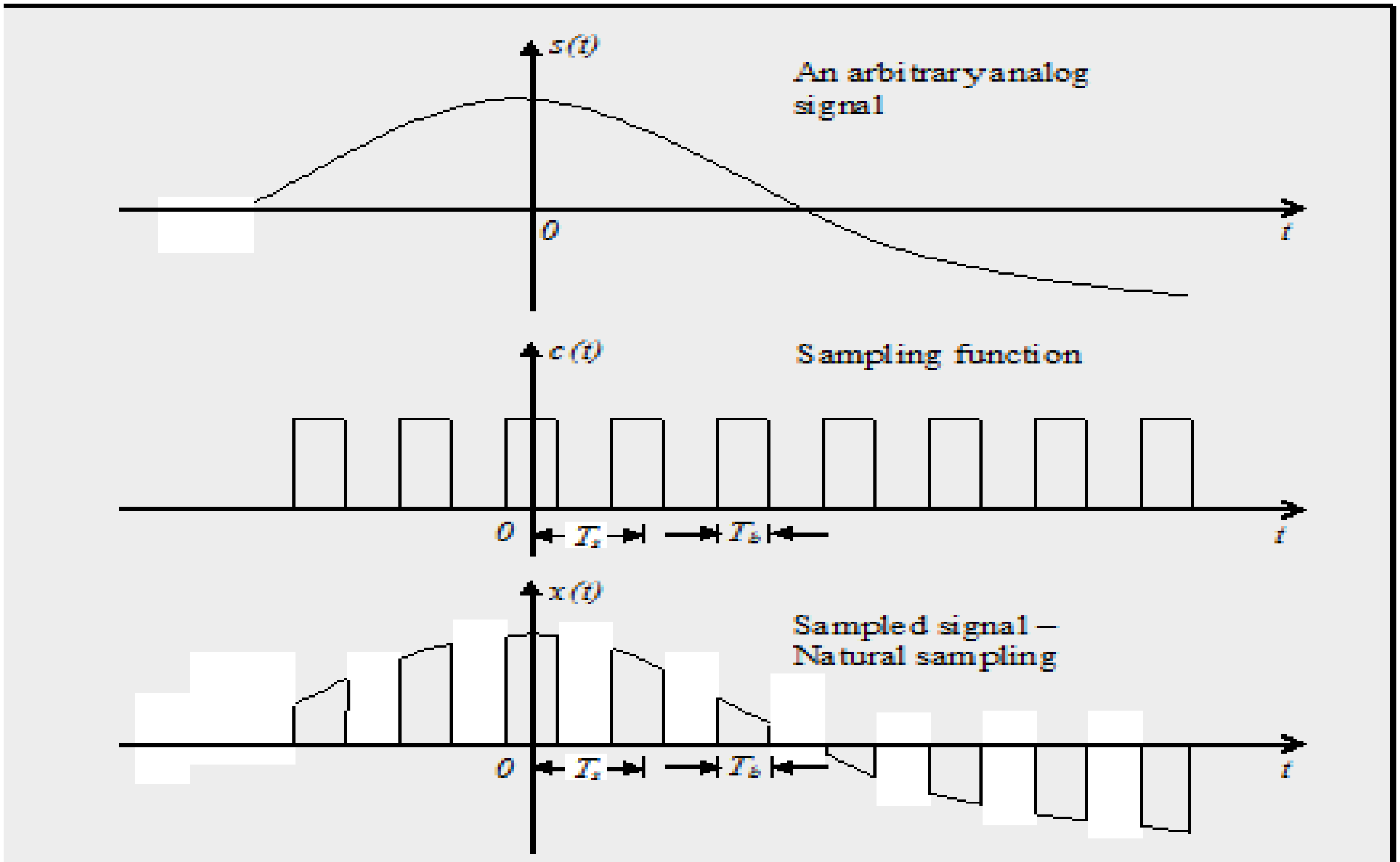
4 Encoding of Quantized Sampled Signal

Pulse Code Modulation



The basic elements of a PCM system.

PCM Sampling



he Process of Natural Sampling

Quantization of Sampled Signal

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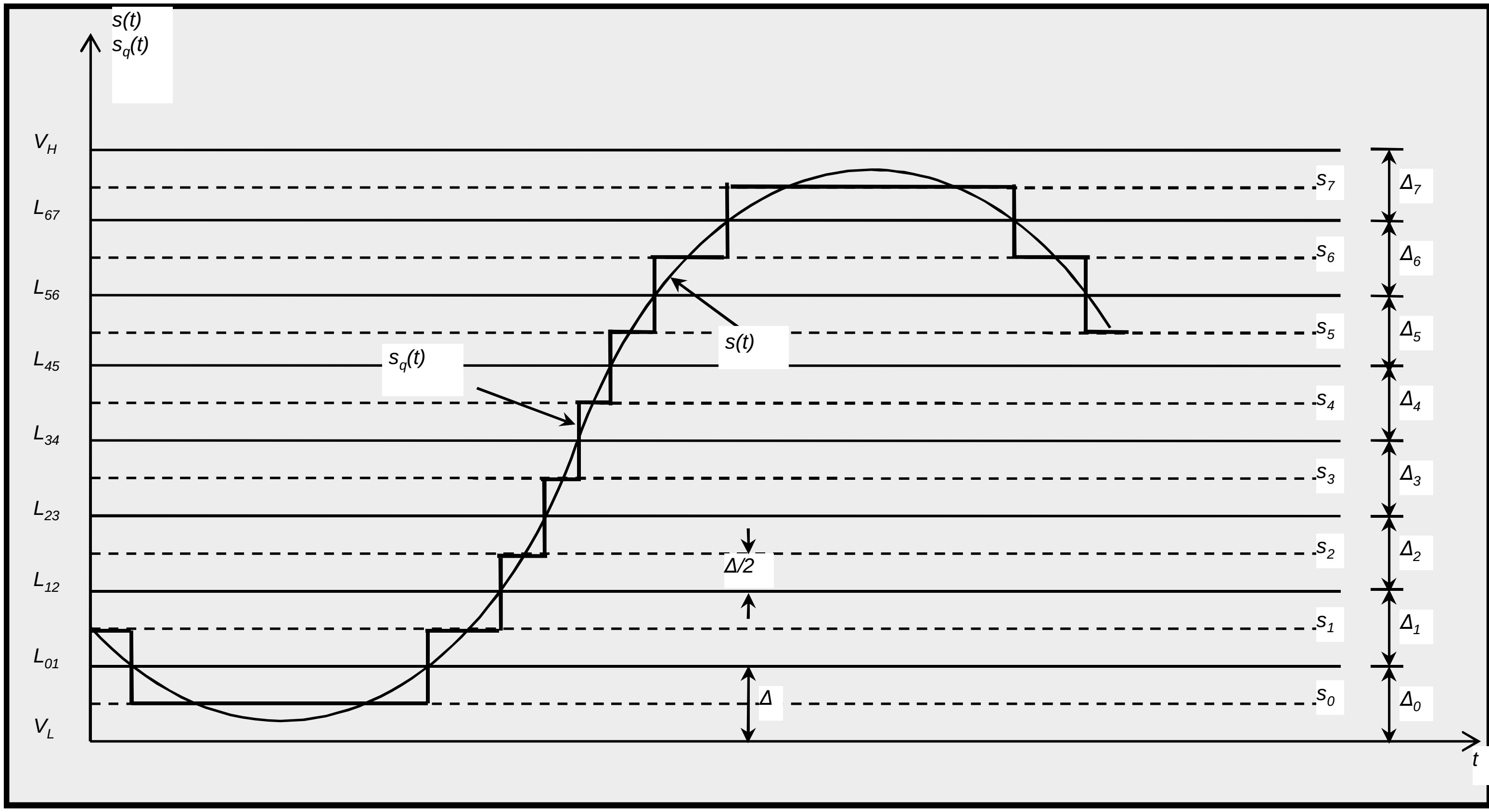
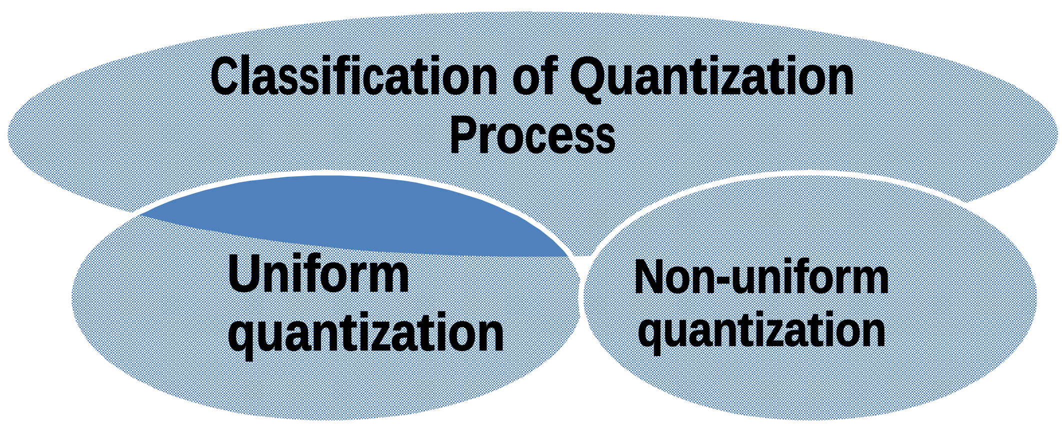
antization



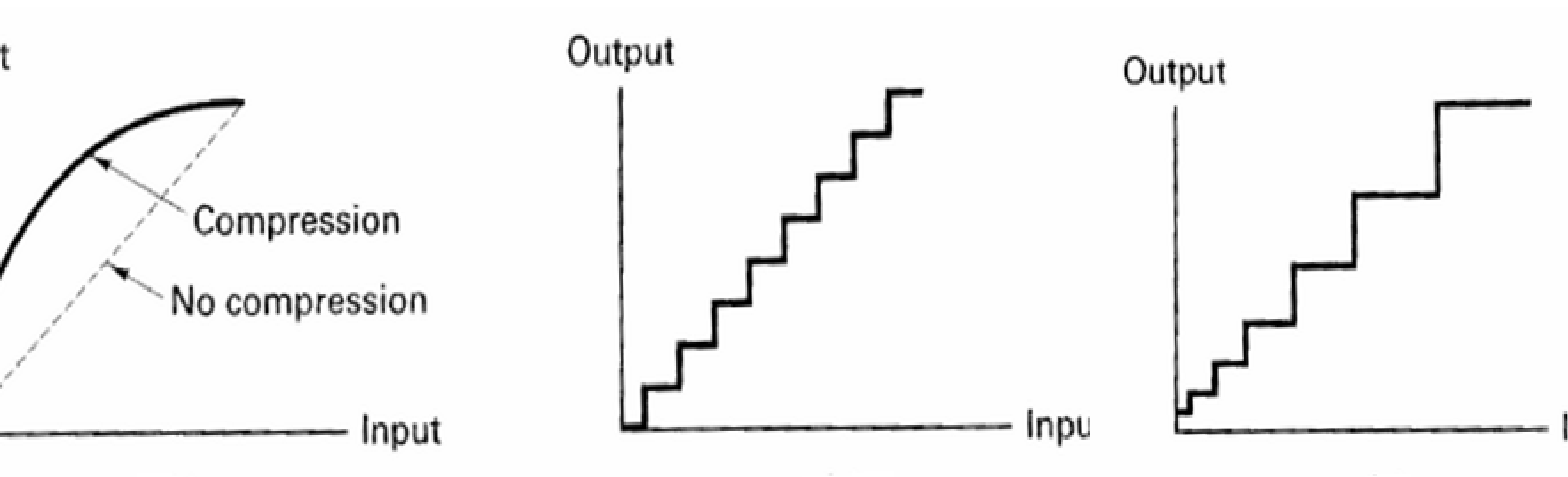
Quantization Error and Classification

Quantization is the conversion of an analog sample of the information signal into discrete form. Thus, an infinite number of possible levels are converted to a finite number of conditions.

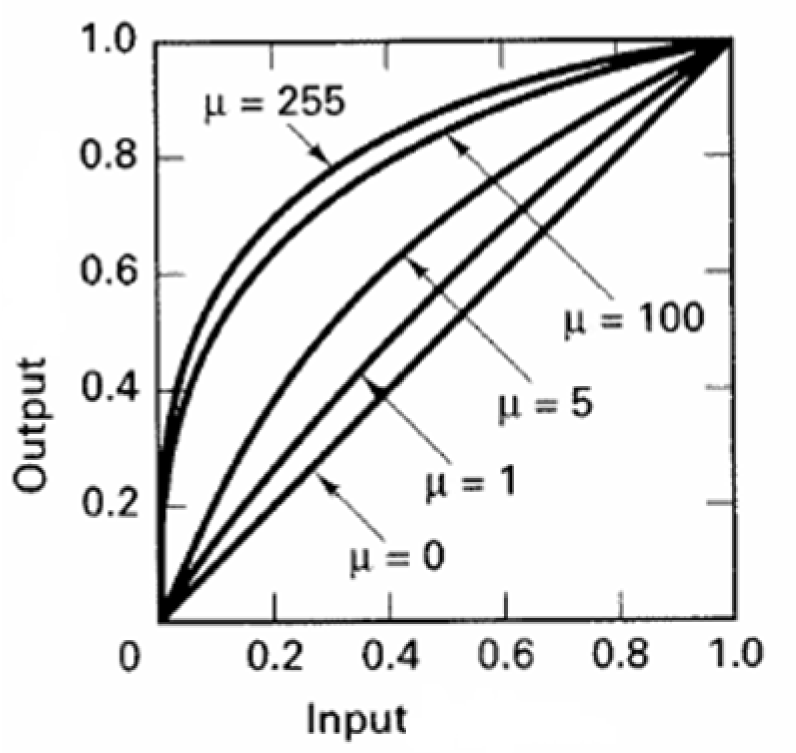
Quantization error is defined as the difference between rounding off sample values of an analog signal to the nearest permissible level of the quantizer during the process of quantization.

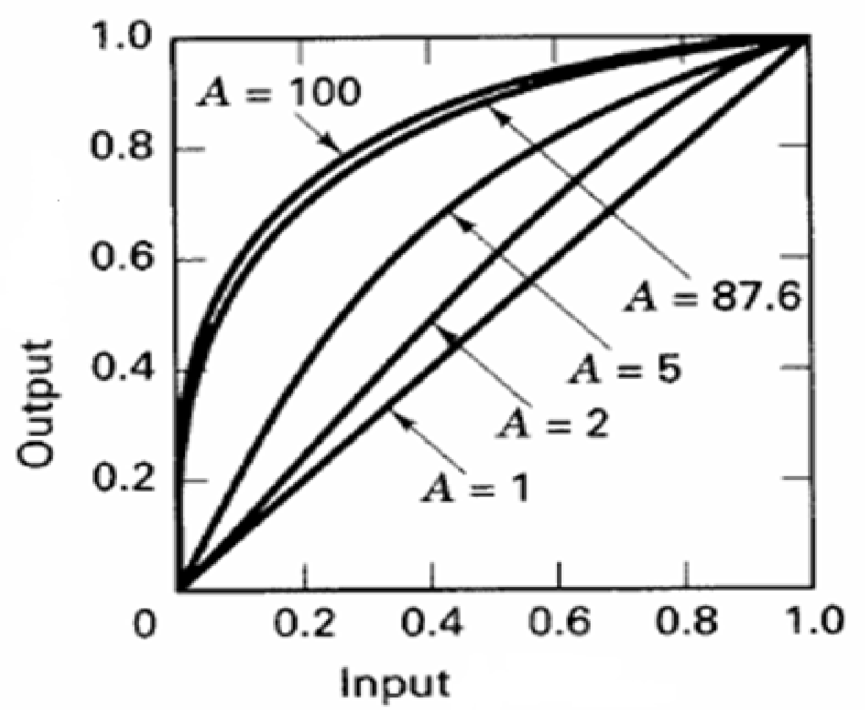


Characteristics of Compressor, Uniform and Non-uniform Quantizer

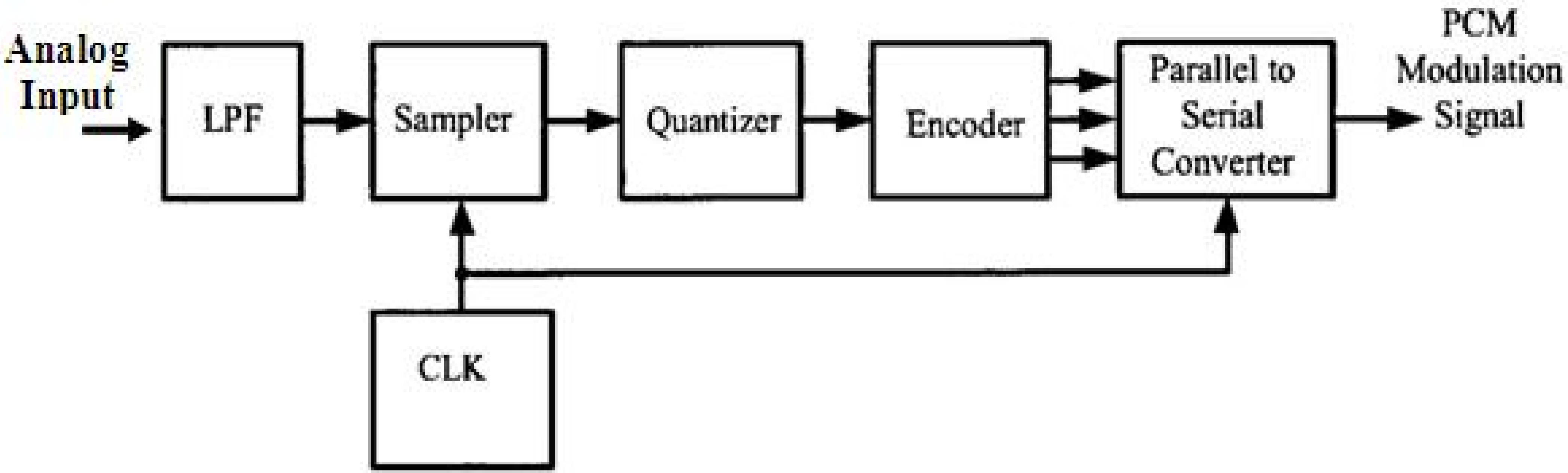


μ-law and A-law Compression Characteristics





Encoding of Quantized Sampled Signal



PCM – Functional Blocks

PCM System Parameters

PCM Data Rate (bps) = $2nf_m$

PCM Bandwidth (Hz) = (1/2) PCM Data Rate = nf_m

Dynamic Range (dB) = $20 \log (2^n - 1)$

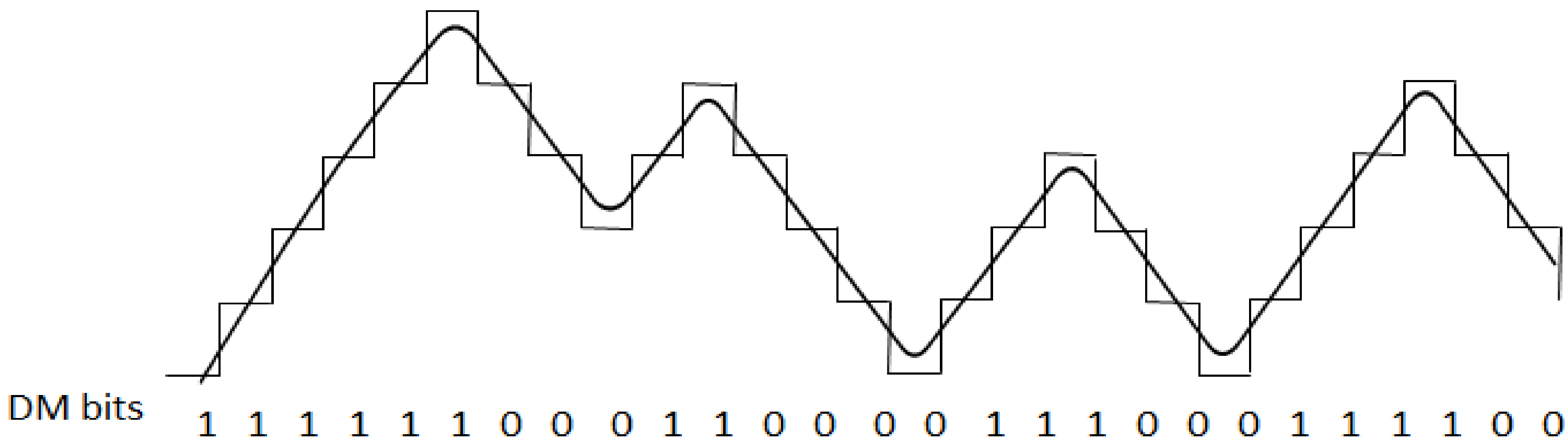
Coding Efficiency (%) = [(minimum bits)/(actual bits)] x 100

Where n is number of PCM encoding bits and f_m is the highest frequency component of information signal

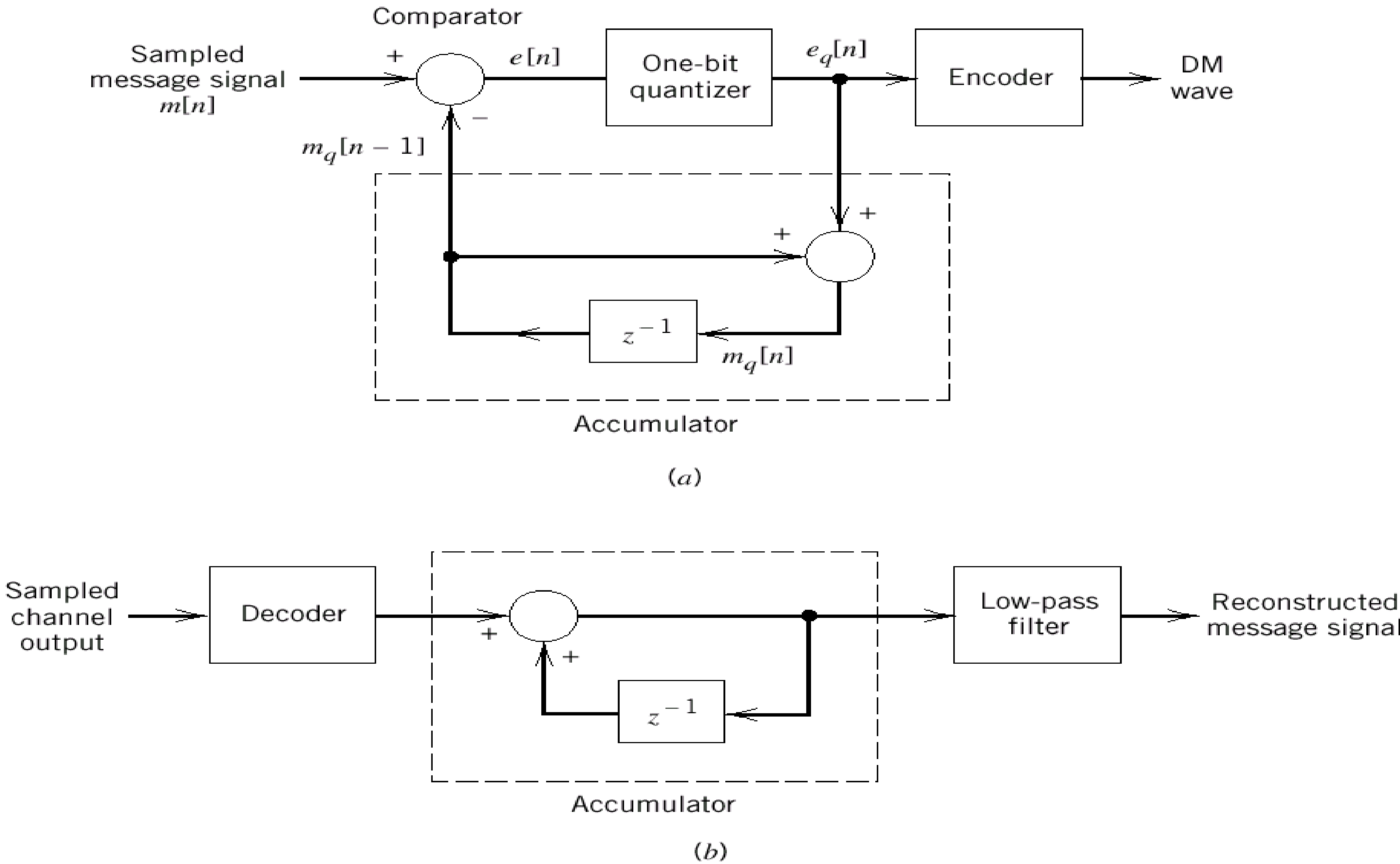
DELTA MODULATION

Essence of Delta Modulation (DM)

Delta modulation (DM) uses a single-bit DPCM code to achieve digital transmission of analog signals



An Ideal Delta Modulation Waveform



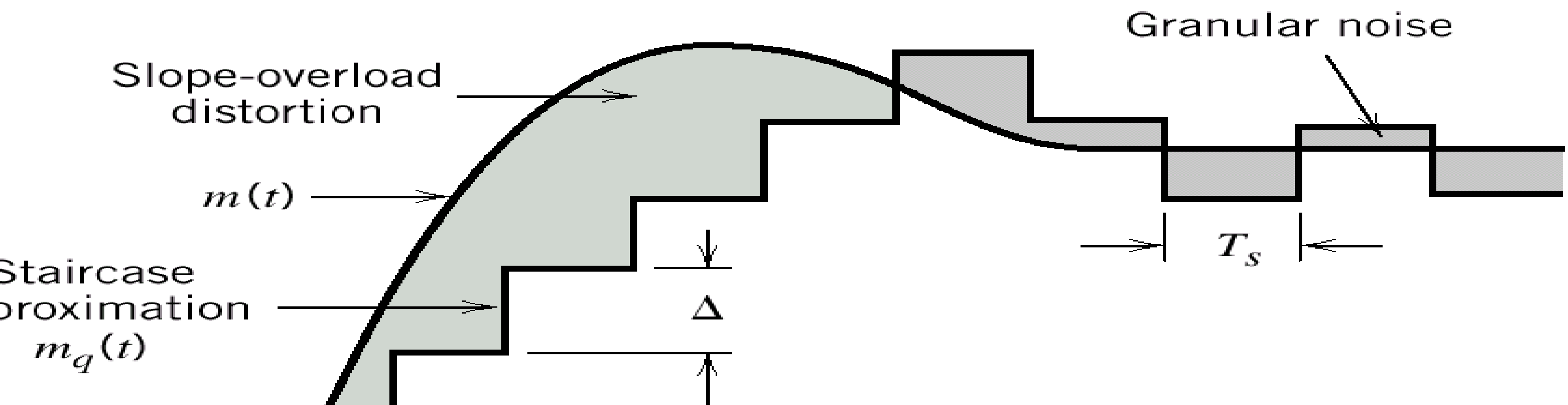
DM system. (*a*) Transmitter. (*b*) Receiver.

ator consists of a comparator, a quantizer, and an accumulator

utput of the accumulator is

$$m_q[n] = \Delta \sum_{i=1}^{n} \operatorname{sgn}(e[i])$$

$$= \sum_{i=1}^{n} e_q[i] \qquad (3.55)$$



o types of quantization errors :

pe overload distortion and **granular noise**

a-Sigma modulation (sigma-delta modulation)

$\Delta - \Sigma$ modulation which has an **integrator** can

eve the draw back of delta modulation (**differentiator**)

eficial effects of using integrator:

Pre-emphasize the low-frequency content

ncrease correlation between adjacent samples

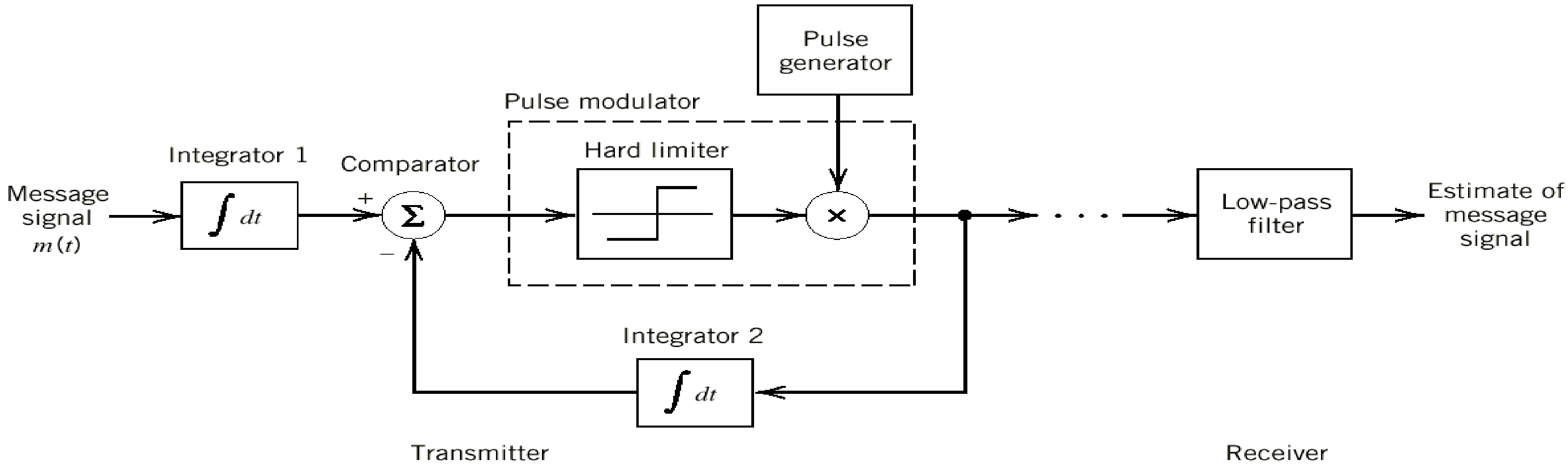
educe the variance of the error signal at the quantizer input)

Simplify receiver design

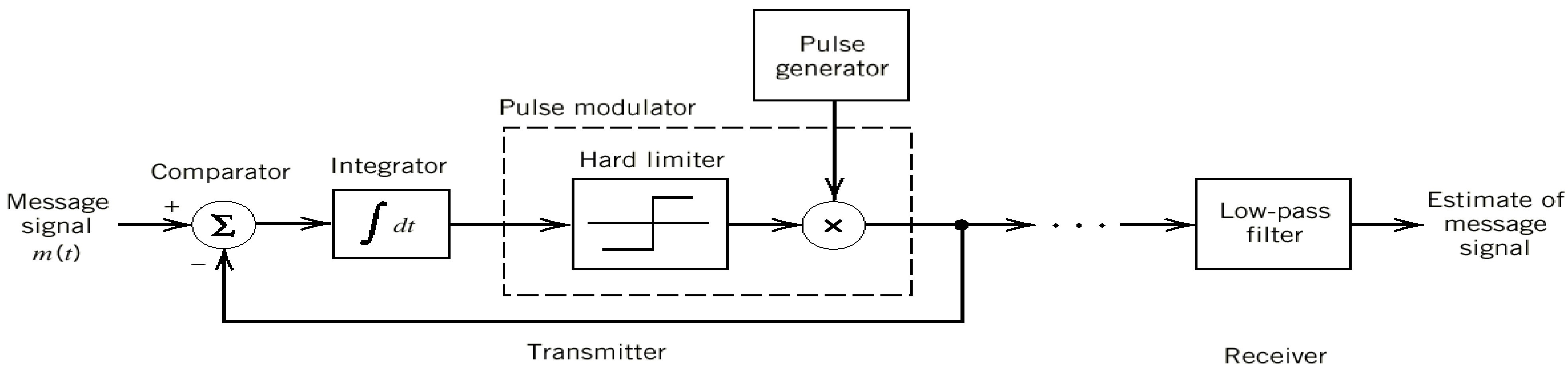
use the transmitter has an integrator , the receiver

ists simply of a low-pass filter.

differentiator in the conventional DM receiver is cancelled by the integrator)



(a)



(b)

ivalent versions of delta-sigma modulation system.

Differential Pulse-Code Modulation (DPCM)

PCM has the sampling rate higher than the **Nyquist rate** .The encode signal contains redundant information.
ciently remove this redundancy.

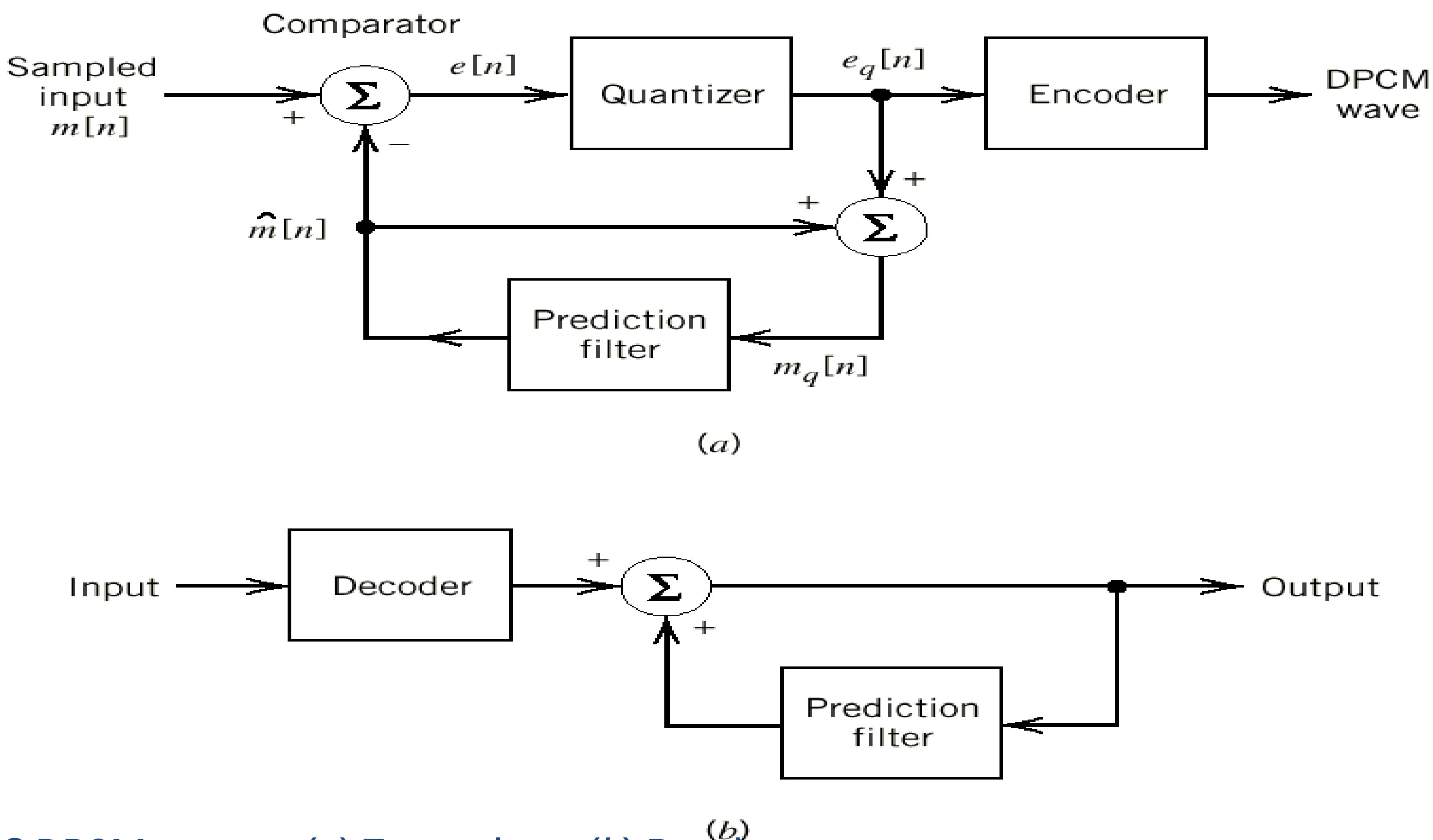


Figure 3.28 DPCM system. (*a*) Transmitter. (*b*) Receiver.

Adaptive Differential Pulse-Code Modulation (ADPCM)

ed for coding speech at low bit rates , we have two aims in mind:

Remove redundancies from the speech signal as far as possible.

Assign the available bits in a perceptually efficient manner.

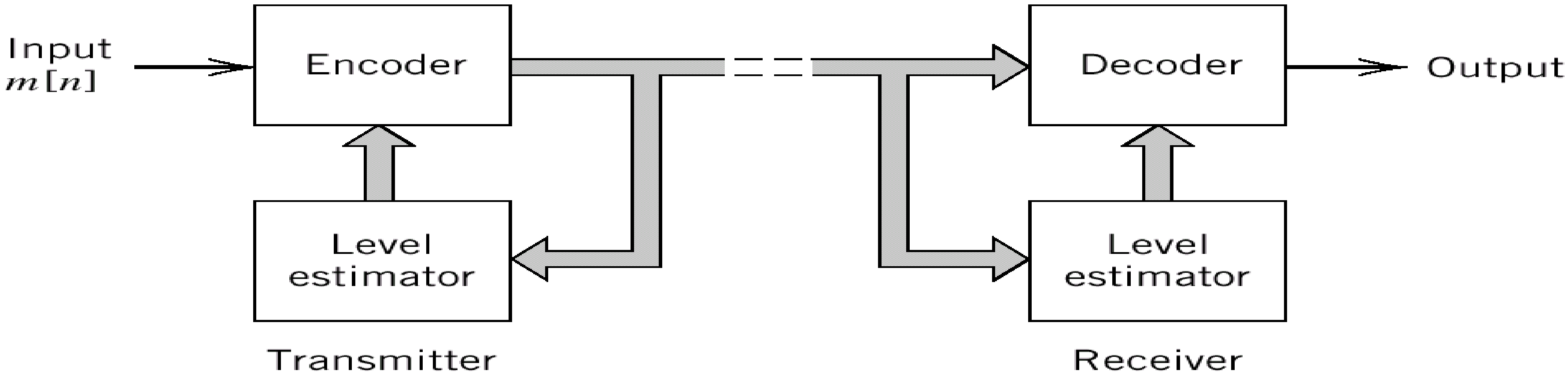
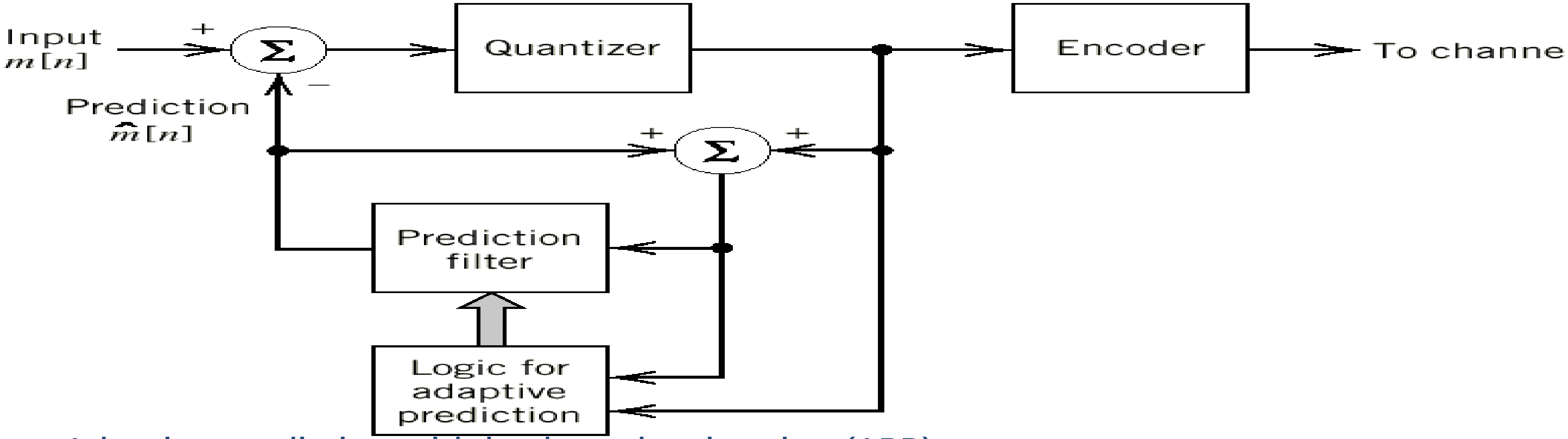


Figure 3.29 Adaptive quantization with backward estimation (AQB).



Adaptive prediction with backward estimation (APB).

Comparison of PCM and DM Techniques

S. No.	Parameter	PCM	DPCM	DM	ADM
1.	Number of bits per sample	4/8/16 bits	More than one bit but less than PCM	One bit	One bit
2.	Number of levels	Depends on number of bits	Fixed number of levels	Two levels	Two levels
3.	Step size	Fixed or variable	Fixed or variable	Fixed	Variable
4.	Transmission bandwidth	More bandwidth needed	Lesser than PCM	Lowest	Lowest
5.	Feedback	Does not exist	Exists	Exists	Exists
6.	Quantization noise/distortion	Quantization noise depends on number of bits	Quantization noise & slope overload	slope overload & granular noise	Quantization noise only
7.	Complexity of implementation	Complex	Simple	Simple	Simple

UNIT-III
Basband Pulse Transmission

Transmit and Receive Formatting

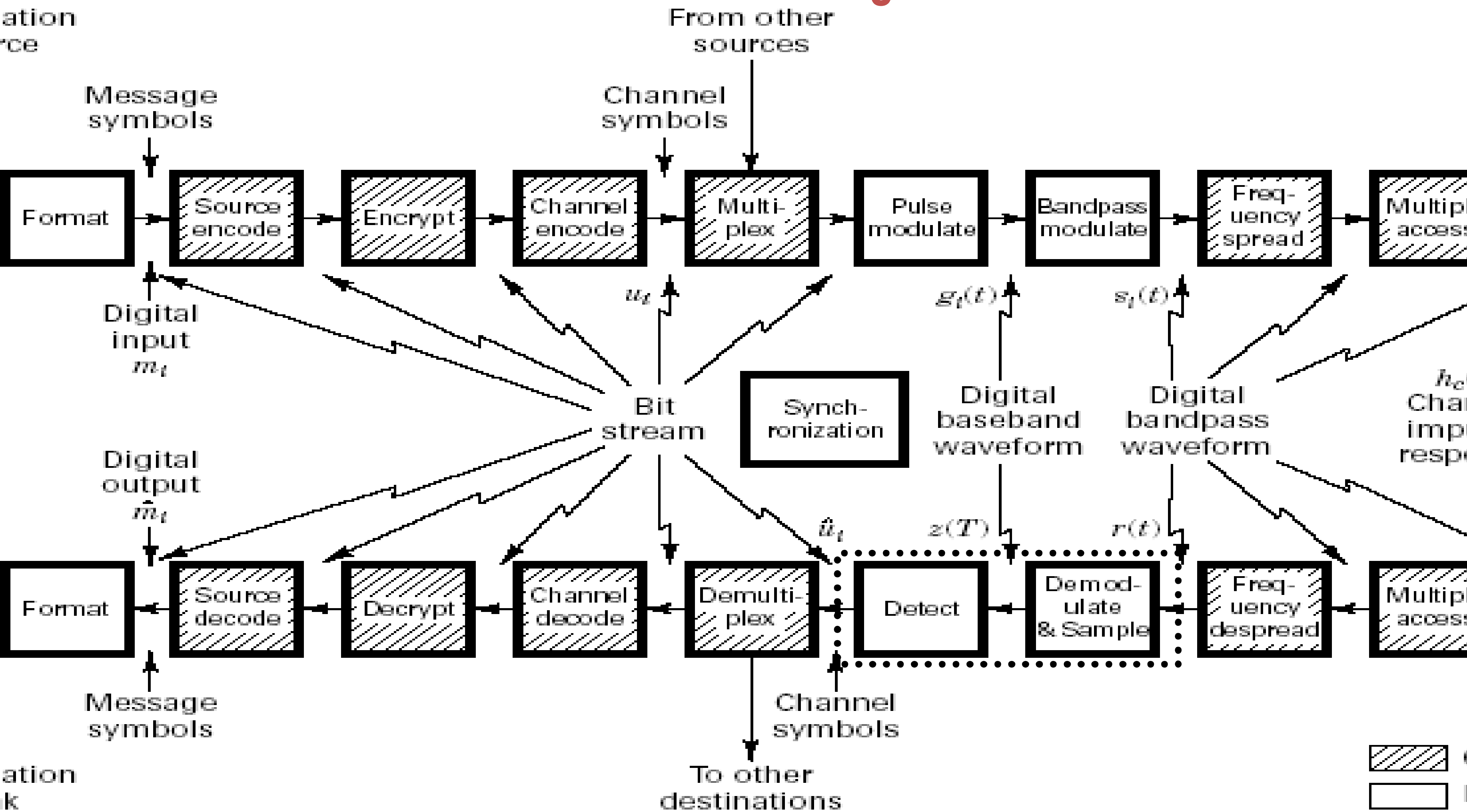


Figure 1.2 Block diagram of a typical digital communication system.

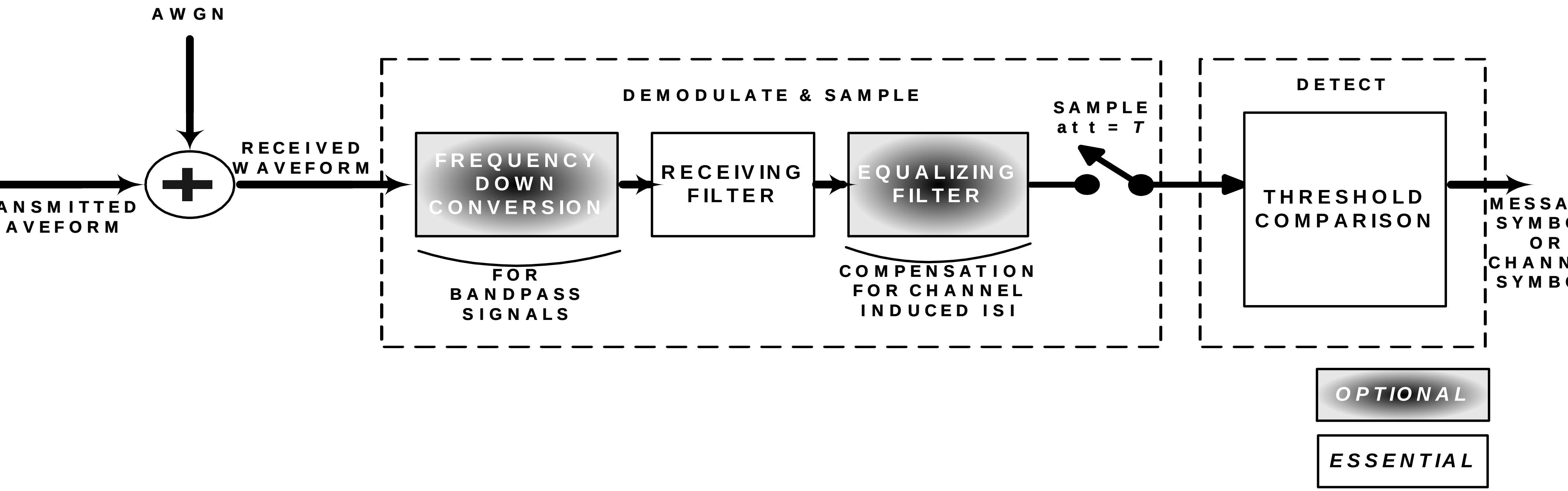
Sources of Error in received Signal

Major sources of errors:

– Thermal noise (AWGN)
  - disturbs the signal in an additive fashion (Additive)
  - has flat spectral density for all frequencies of interest (White)
  - is modeled by Gaussian random process (Gaussian Noise)

– Inter-Symbol Interference (ISI)
  - Due to the filtering effect of transmitter, channel and receiver, symbols are "smeared".

Receiver Structure



Demodulation/Detection of digital signals

Receiver Structure contd

The digital receiver performs **two basic functions**:

– **Demodulation**

– **Detection**

Why demodulate a baseband signal???

– Channel and the transmitter’s filter causes ISI which “smears” the transmitted pulses

– Required to recover a waveform to be sampled at t = nT.

Detection

– decision-making process of selecting possible digital symbol

Steps in designing the receiver

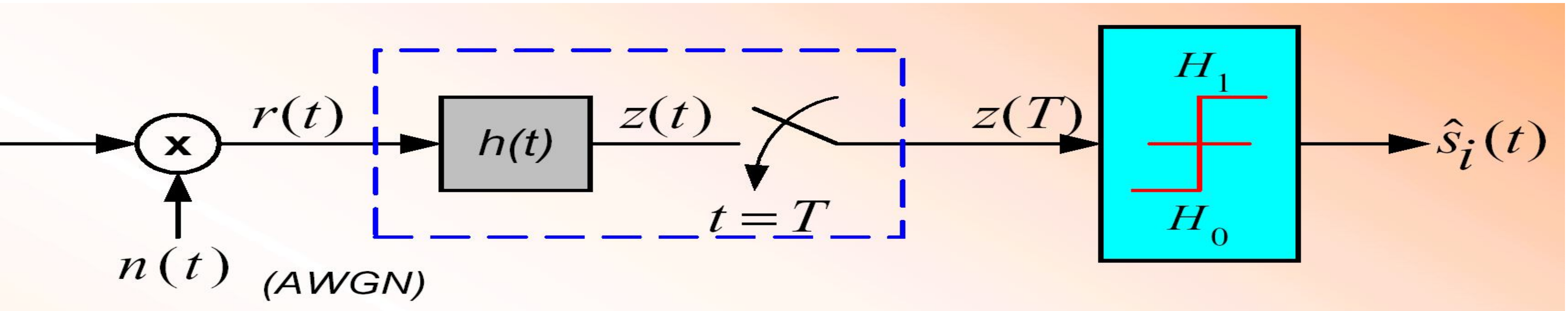
Find optimum solution for receiver design with the following goals:

1. Maximize SNR
2. Minimize ISI

Steps in design:

- Model the received signal
- Find separate solutions for each of the goals.

Detection of Binary Signal in Gaussian Noise



covery of signal at the receiver consist of two parts

ilter

- Reduces the received signal to a single variable *z(T)*
- *z(T)* is called the ***test statistics***

etector (or decision circuit)

- Compares the *z(T)* to some threshold level γ_0 , i.e.,

$$z(T) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_0$$

where H_1 and H_0 are the two possible binary hypothesis

Finding optimized filter for AWGN channel

Assuming Channel with response equal to impulse function

Detection of Binary Signal in Gaussian Noise

- For any binary channel, the transmitted signal over a symbol interval (0,T) is:

$$s_i(t) = \begin{cases} s_0(t) & 0 \le t \le T \qquad \text{for a binary } 0 \\ s_1(t) & 0 \le t \le T \qquad \text{for a binary } 1 \end{cases}$$

- The received signal r(t) degraded by noise n(t) and possibly degraded by the impulse res the channel $h_c(t)$, is

$$r(t) = s_i(t) * h_c(t) + n(t) \qquad i = 1,2$$

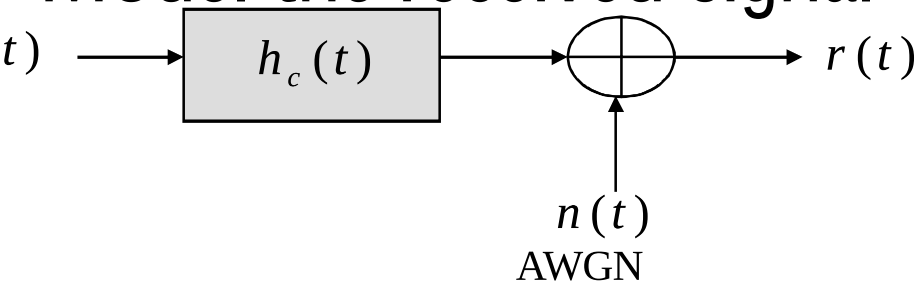
Where n(t) is assumed to be zero mean AWGN process

- For ideal distortionless channel where $h_c(t)$ is an impulse function and convolution with produces no degradation, r(t) can be represented as:

$$r(t) = s_i(t) + n(t) \qquad i = 1,2 \qquad 0 \le t \le T$$

Design the receiver filter to maximize the SNR

Model the received signal

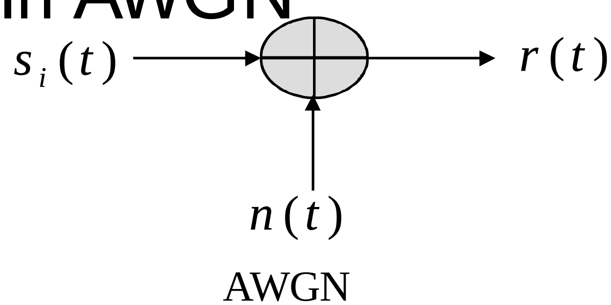


$$r(t) = s_i(t) * h_c(t) + n(t)$$

Simplify the model:

Received signal in AWGN

Ideal channels
$h_c(t) = \delta(t)$



$$r(t) = s_i(t) + n(t)$$

Find Filter Transfer Function $H_0(f)$

ctive: To maximizes $(S/N)_T$ and find h(t)

ing signal $a_i(t)$ at filter output in terms of filter transfer function H(f)

$$a_i(t) = \int_{-\infty}^{\infty} H(f)\,S(f)\,e^{j2\pi ft}\,df$$

the filter transfer funtion and S(f) is the Fourier transform of input signal s(t)

ded PSD of i/p noise is $N_0/2$

noise power can be expressed as:

$$\sigma_0^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

$(S/N)_T$:

$$\left(\frac{S}{N}\right)_T = \frac{\left|\int_{-\infty}^{\infty} H(f)\,S(f)\,e^{j2\pi fT}\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df}$$

- For H(f) = H_{opt} (f) to maximize $(S/N)_T$ use Schwarz's Inequality:

$$\left| \int_{-\infty}^{\infty} f_1(x)\; f_2(x)dx \right|^2 \le \int_{-\infty}^{\infty} |f_1(x)|^2\, dx \int_{-\infty}^{\infty} |f_2(x)|^2\, dx$$

- Equality holds if $f_1(x) = k\, f^*_2(x)$ where k is arbitrary constant and * indicates complex conjugate
- Associate H(f) with $f_1(x)$ and $S(f)\, e^{j2\pi fT}$ with $f_2(x)$ to get:

$$\left| \int_{-\infty}^{\infty} H(f)\, S(f)\, e^{j2\pi fT}\, df \right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |S(f)|^2\, df$$

- Substitute yields to:

$$\left(\frac{S}{N} \right)_T \le \frac{2}{N_0} \int_{-\infty}^{\infty} |S(f)|^2\, df$$

$$\max\left(\frac{S}{N}\right)_T \le \frac{2E}{N_0}$$ and energy E of the input signal s(t):

$$E = \int_{-\infty}^{\infty} \left| S(f) \right|^2 df$$

s $(S/N)_T$ depends on input signal energy

power spectral density of noise and

on the particular shape of the waveform

lity for $\max\left(\frac{S}{N}\right)_T \le \frac{2E}{N_0}$ holds for optimum filter transfer function $H_0(f)$

that:

$$H(f) = H_0(f) = kS^*(f)\, e^{-j2\pi fT}$$

$$h(t) = \Im^{-1}\left\{ kS^*(f)\, e^{-j2\pi fT} \right\} \quad (3.55)$$

eal valued s(t):

$$h(t) = \begin{cases} kS(T-t) & 0 \le t \le T \\ 0 & else\ where \end{cases}$$

mpulse response of a filter producing maximum output signal-to-noise ratio is the
or image of message signal s(t), delayed by symbol time duration T.

filter designed is called a ***MATCHED FILTER***

$$\boxed{h(t) = \begin{cases} kS(T-t) & 0 \le t \le T \\ 0 & else \quad where \end{cases}}$$

ned as:

a linear filter designed to provide the maximum
signal-to-noise power ratio at its output for a given
transmitted symbol waveform

Matched Filter Output of a rectangular Pulse

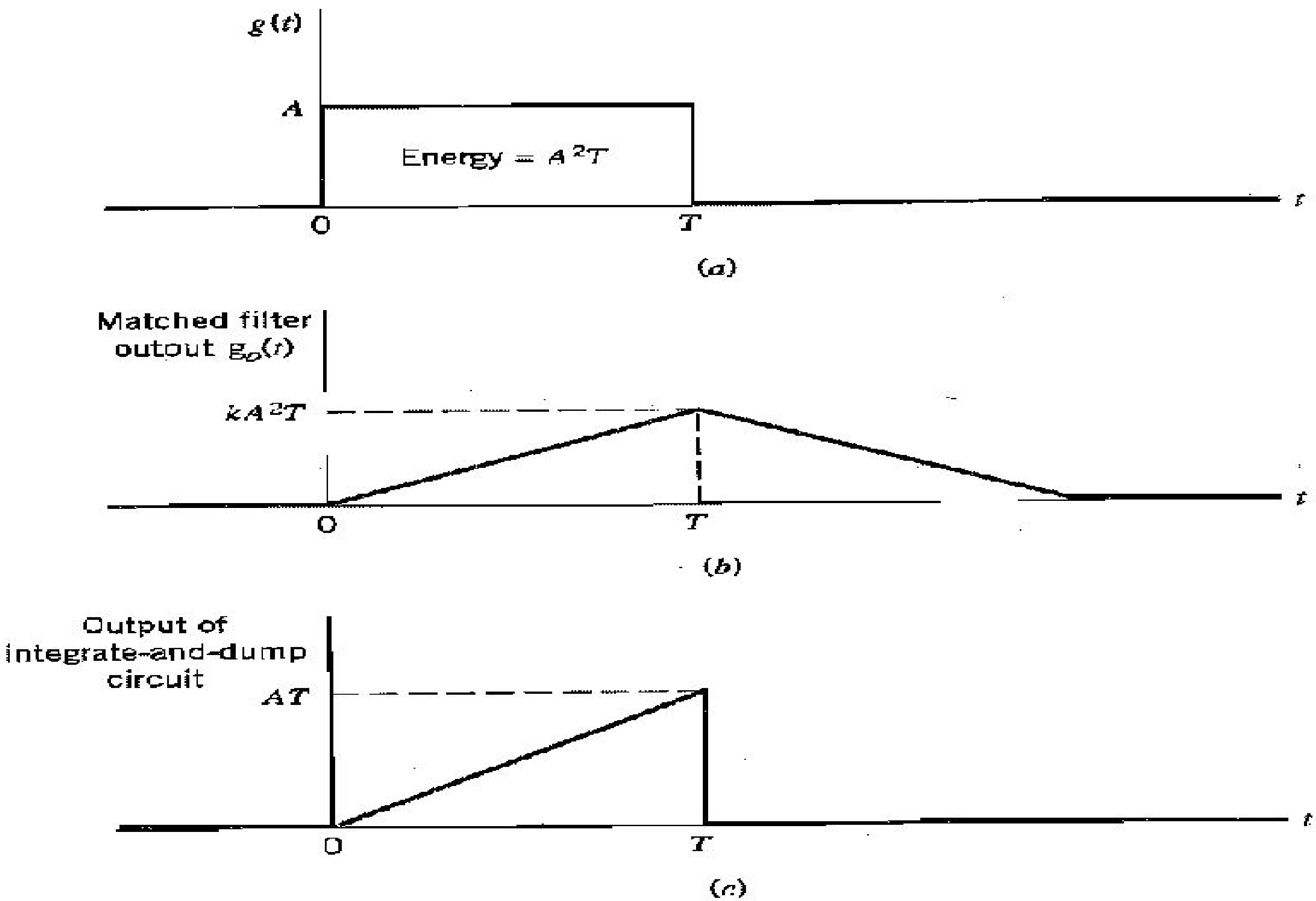


FIGURE 4.2 (*a*) Rectangular pulse. (*b*) Matched filter output. (*c*) Integrator output.

Replacing Matched filter with Integrator

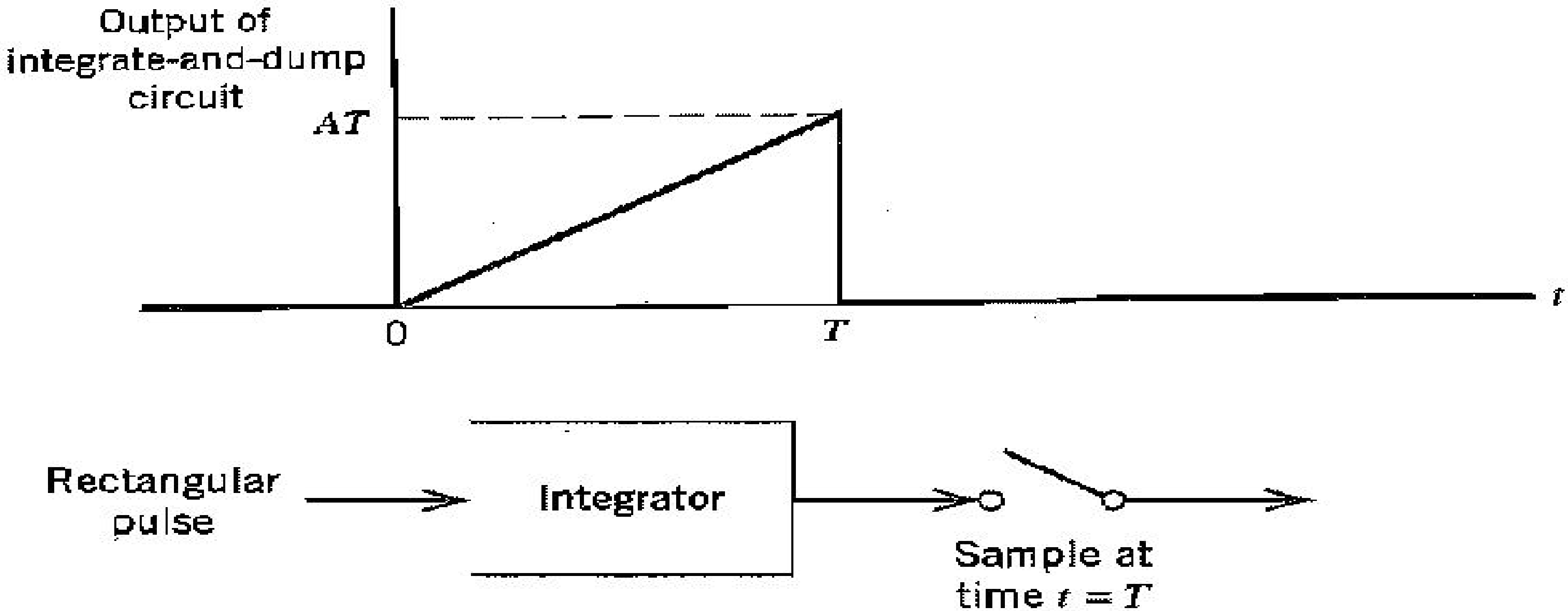
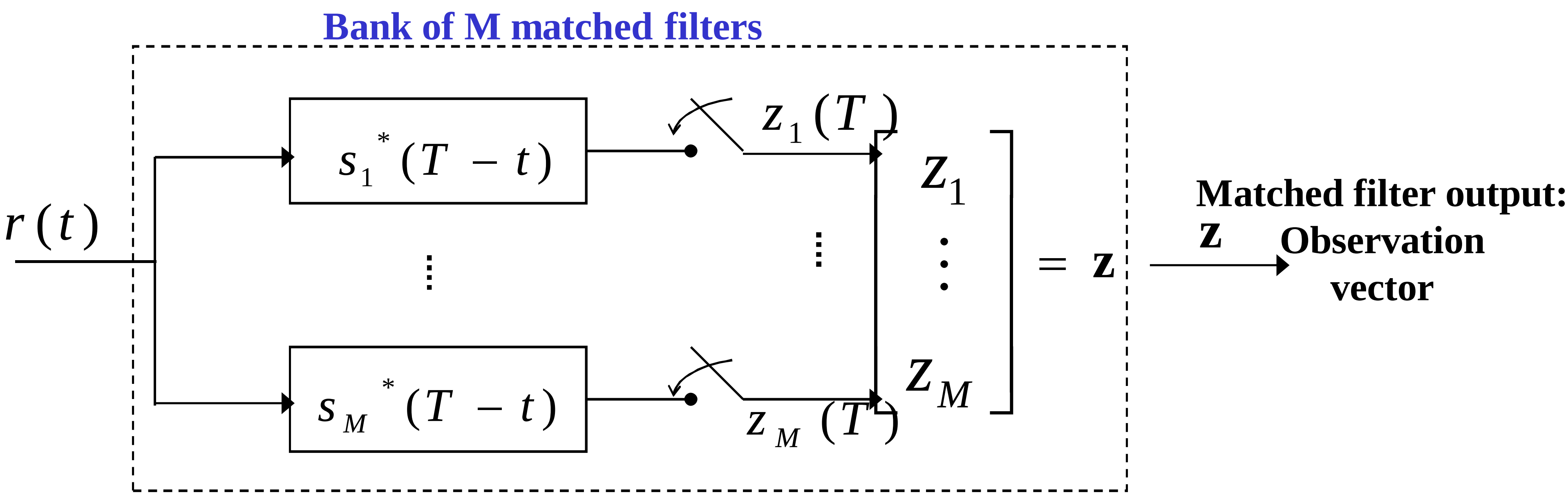


FIGURE 4.3 Integrate-and-dump circuit.

Implementation of matched filter receiver



$$z_i = r(t) * s^*_i(T-t) \quad i = 1, \ldots, M$$

$$\mathbf{z} = (z_1(T), z_2(T), \ldots, z_M(T)) = (z_1, z_2, \ldots, z_M)$$

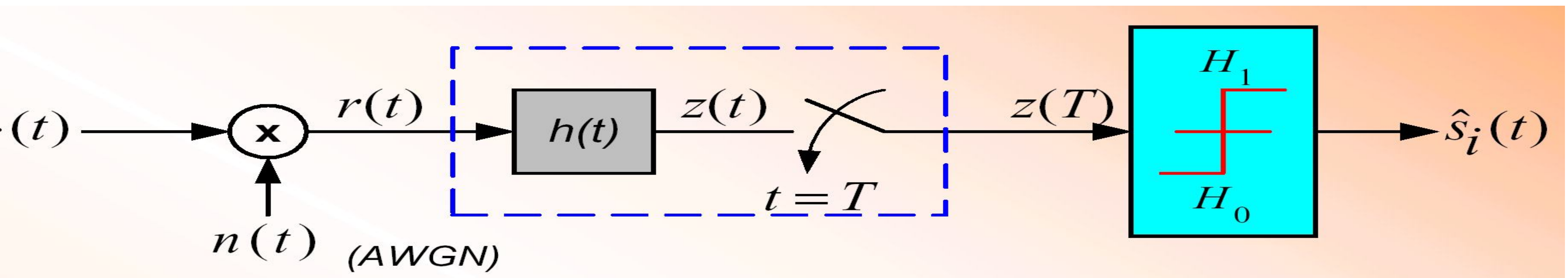
Detection

Max. Likelihood Detector

Probability of Error

Detection

- ed filter reduces the received signal to a single variable *z(T), after which the detection of symbol is carried out*
- ncept of ***maximum likelihood detector*** is based on Statistical Decision Theory
- s us to
  - mulate the decision rule that operates on the data
  - imize the detection criterion



$$z(T) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_0$$

Probabilities Review

P[s_1] $\rightarrow$ *a priori probabilities*

hese probabilities are known before transmission

obability of the received sample

), p(z|s_1)

nditional pdf of received signal z, conditioned on the class s_i

], P[s_1|z] $\rightarrow$ *a posteriori probabilities*

fter examining the sample, we make a refinement of our previous knowledge

$_0$], P[s_0|s_1]

ong decision (error)

$_1$], P[s_0|s_0]

rrect decision

How to Choose the threshold?

num Likelihood Ratio test and Maximum a posteriori (MAP) criterion:

$$p(s_0 | z) > p(s_1 | z) -> H_0$$

$$p(s_1 | z) > p(s_0 | z) -> H_1$$

em is that a posteriori probabilities are not known.

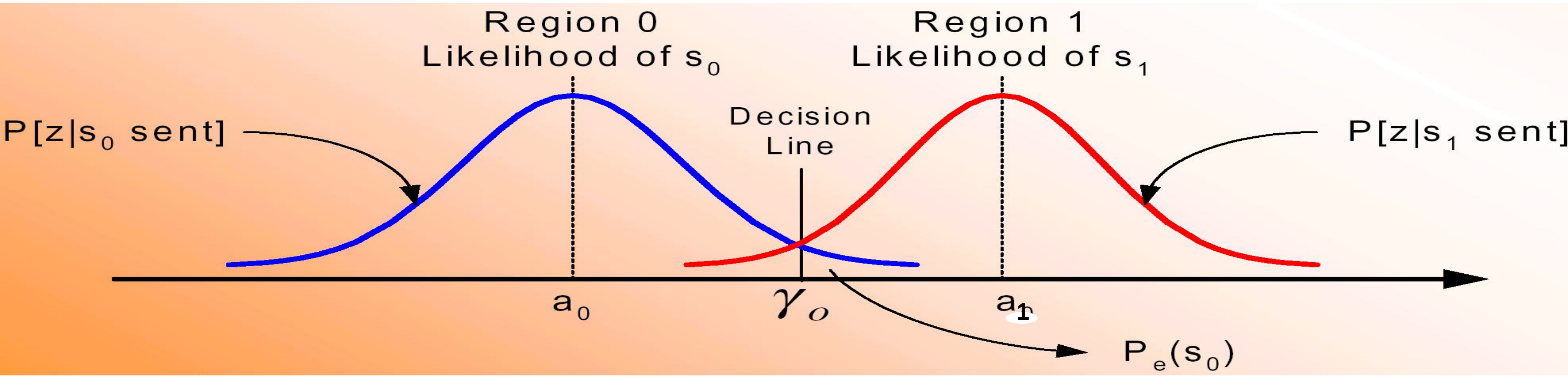
on: Use Bay's theorem:

$$p(s_i | z) = \frac{p(z | s_i) p(s_i)}{p(z)}$$

$$\Rightarrow \frac{p(z|s_1)P(s_1)}{P(z)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{p(z|s_0)P(s_0)}{P(z)} \Rightarrow p(z|s_1)P(s_1) \underset{H_0}{\overset{H_1}{\gtrless}} p(z|s_0)P(s_0$$

s means that if received signal is positive, s_1 (t) was sent, else s_0 (t) was sent

Likelihood of S_o and S_1



MAP criterion:

$$L(z) \triangleq \frac{p(z|s_1)}{p(z|s_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(s_0)}{P(s_1)} \Leftarrow \textit{likelihood ratio test (LRT)}$$

When the two signals, $s_0(t)$ and $s_1(t)$, are equally likely, i.e., $P(s_0) = P(s_1) = 0.5$, then the decision rul
becomes

$$L(z) = \frac{p(z|s_1)}{p(z|s_0)} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \Leftarrow \max \textit{ likelihood ratio test}$$

s is known as ***maximum likelihood ratio test*** because we are selecting
hypothesis that corresponds to the signal with the maximum likelihood.

erms of the Bayes criterion, it implies that the cost of both types of error is the same tuting the pdfs

$$H_0 : \qquad p(z \mid s_0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_0}{\sigma_0} \right)^2 \right]$$

$$H_1 : \qquad p(z \mid s_1) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_1}{\sigma_0} \right)^2 \right]$$

$$L(z) = \frac{p(z \mid s_1)}{p(z \mid s_0)} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \Rightarrow \frac{\frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_o^2}(z - a_1)^2 \right]}{\frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_0^2}(z - a_0)^2 \right]} \underset{H_0}{\overset{H_1}{\gtrless}} 1$$

Hence:

$$\exp\left[\frac{z(a_1-a_0)}{\sigma_0^2}-\frac{(a_1^2-a_0^2)}{2\sigma_0^2}\right]\begin{matrix}>\\<\end{matrix} 1$$

Taking the log, both sides will give

$$\ln\{L(z)\}=\frac{z(a_1-a_0)}{\sigma_0^2}-\frac{(a_1^2-a_0^2)}{2\sigma_0^2}\ \begin{matrix}H_1\\>\\<\\H_0\end{matrix}\ 0$$

$$\Rightarrow\quad \frac{z(a_1-a_0)}{\sigma_0^2}\ \begin{matrix}H_1\\>\\<\\H_0\end{matrix}\ \frac{(a_1^2-a_0^2)}{2\sigma_0^2}=\frac{(a_1+a_0)(a_1-a_0)}{2\sigma_0^2}$$

- Hence

$$z \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\sigma_0^2 (a_1 + a_0)(a_1 - a_0)}{2\sigma_0^2 (a_1 - a_0)}$$

$$z \underset{H_0}{\overset{H_1}{\gtrless}} \frac{(a_1 + a_0)}{2} \underline{\Delta}\, \gamma_0$$

where z is the *minimum error criterion* and γ_0 is *optimum threshold*

- For antipodal signal, $s_1(t) = -s_0(t) \Rightarrow a_1 = -a_0$

$$z \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

Probability of Error

vill occur if

s sent $\rightarrow s_0$ is received

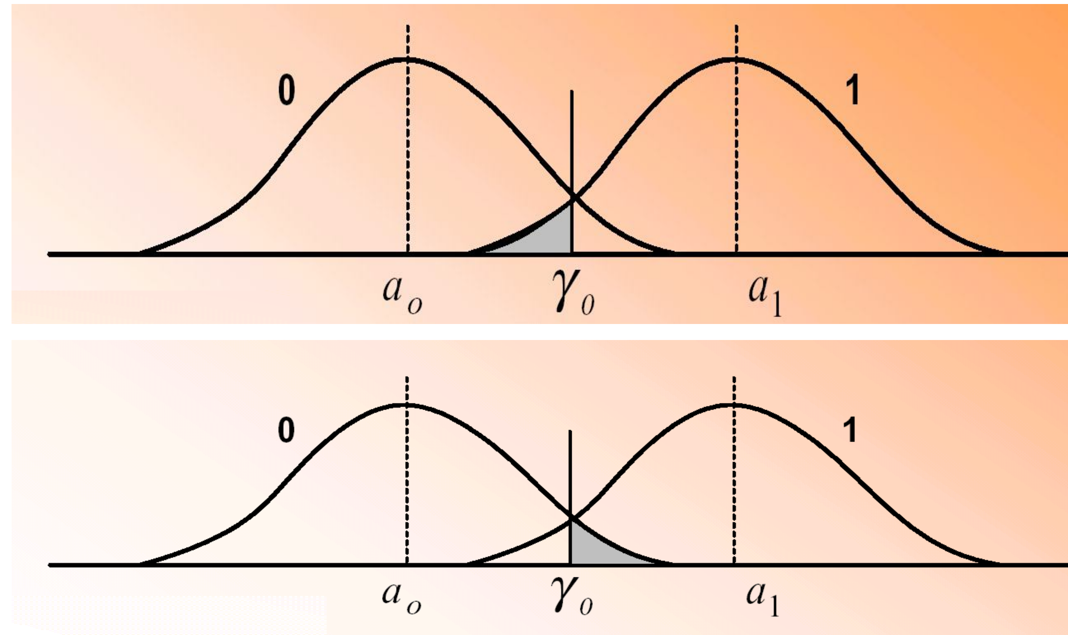
$$P(H_0 \mid s_1) = P(e \mid s_1)$$

$$P(e \mid s_1) = \int_{-\infty}^{\gamma_0} p(z \mid s_1)\, dz$$

s sent $\rightarrow s_1$ is received

$$P(H_1 \mid s_0) = P(e \mid s_0)$$

$$P(e \mid s_0) = \int_{\gamma_0}^{\infty} p(z \mid s_0)\, dz$$



The total probability of error is sum of the errors

$$P_B = \sum_{i=1}^{2} P(e, s_i) = P(e \mid s_1) P(s_1) + P(e \mid s_0) P(s_0)$$

$$= P(H_0 \mid s_1) P(s_1) + P(H_1 \mid s_0) P(s_0)$$

nals are equally probable

$$P_B = P(H_0 | s_1)P(s_1) + P(H_1 | s_0)P(s_0)$$
$$= \frac{1}{2}\left[P(H_0 | s_1) + P(H_1 | s_0)\right]$$

, the probability of bit error P_B, is the probability that an incorrect hypothesis is made

rically, P_B is the area under the tail of either of the conditional distributions $p(z|s_1)$ or $p(z|s_0)$

$$P_B = \int_{\gamma_0}^{\infty} P(H_1 | s_0)\,dz = \int_{\gamma_0}^{\infty} p(z | s_0)\,dz$$
$$= \int_{\gamma_0}^{\infty} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_0}{\sigma_0}\right)^2\right]dz$$

Inter-Symbol Interference (ISI)

- SI in the detection process due to the filtering effects of the ystem
- Overall equivalent system transfer function

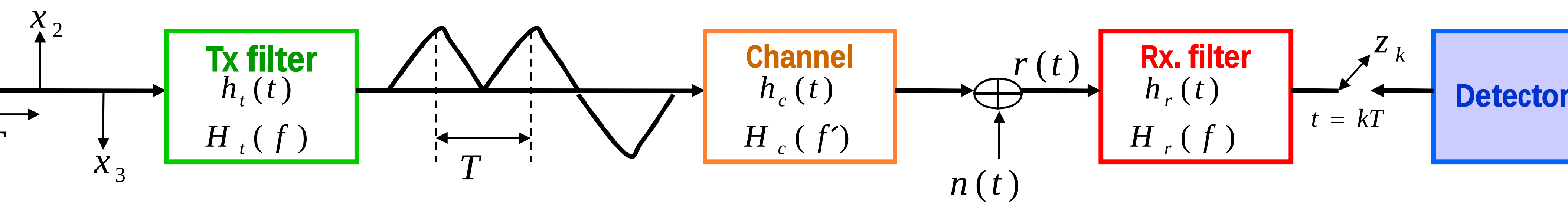
$$H(f) = H_t(f)H_c(f)H_r(f)$$

- creates echoes and hence time dispersion
- causes ISI at sampling time

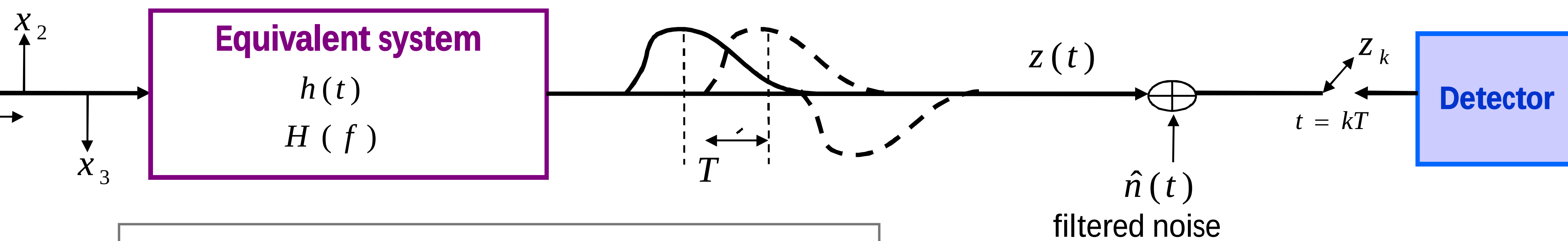
$$z_k = s_k + n_k + \sum_{i \neq k} \alpha_i s_i$$

Inter-symbol interference

Baseband system model



Equivalent model



$$H(f) = H_t(f)H_c(f)H_r(f)$$

Nyquist bandwidth constraint

Nyquist bandwidth constraint:

- The theoretical minimum required system bandwidth to detect *Rs* [symbols/s] without ISI is *Rs/2* [Hz].
- Equivalently, a system with bandwidth *W=1/2T=Rs/2* [Hz] can support a maxim transmission rate of *2W=1/T=Rs* [symbols/s] without ISI.

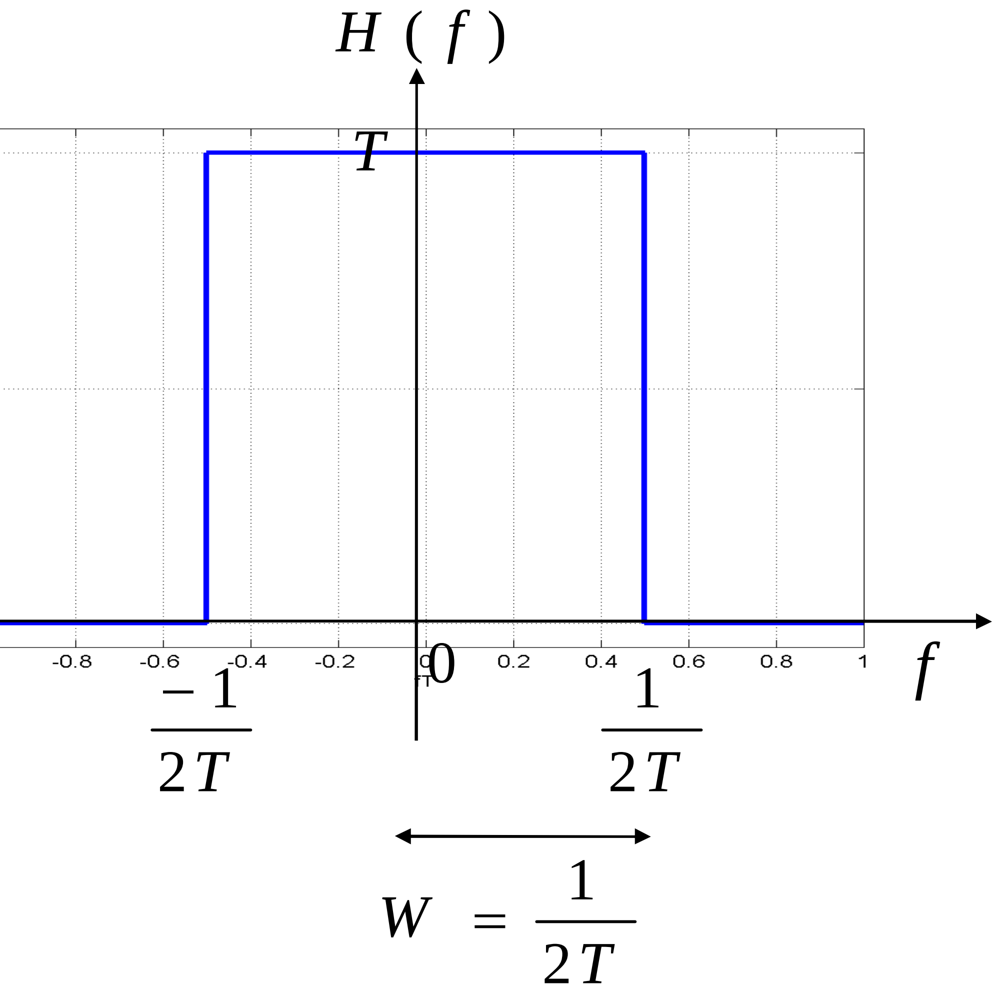
$$\boxed{\frac{1}{2T} = \frac{R_s}{2} \leq W \Rightarrow \frac{R_s}{W} \geq 2 \quad \text{[symbol/s/} \quad \text{Hz]}}$$

Bandwidth efficiency, *R/W* [bits/s/Hz] :

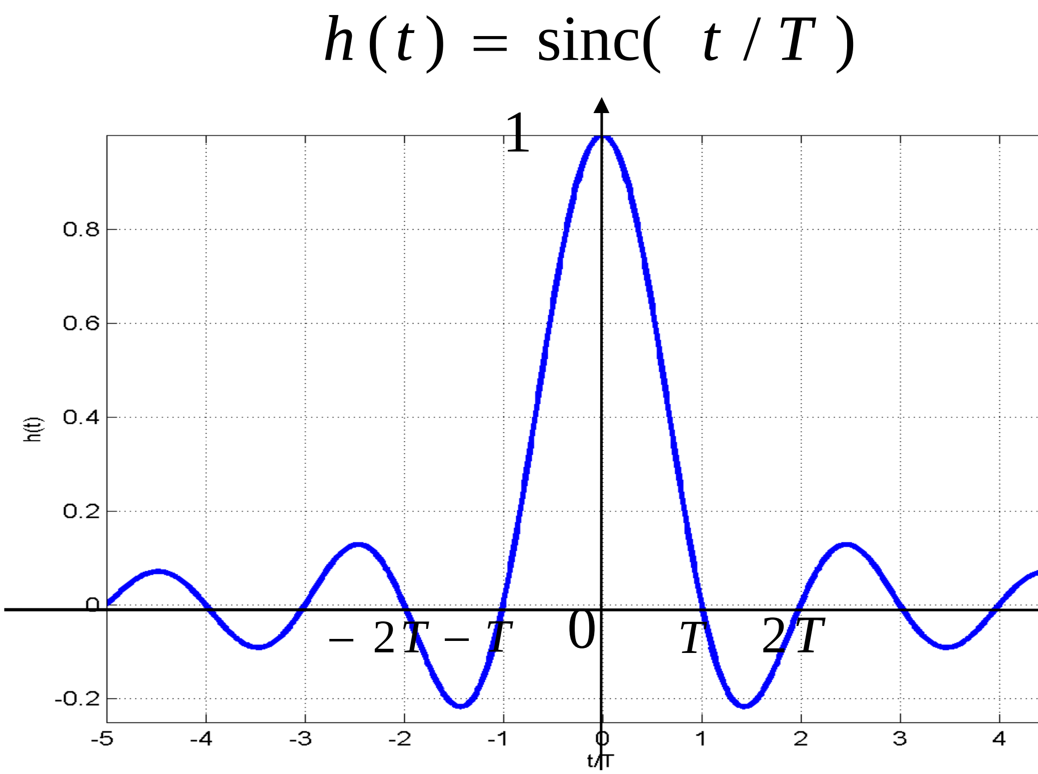
- An important measure in DCs representing data throughput per hertz of bandv
- Showing how efficiently the bandwidth resources are used by signaling technic

Ideal Nyquist pulse (filter)

Ideal Nyquist filter



Ideal Nyquist pulse



Nyquist pulses (filters)

Nyquist pulses (filters):

– Pulses (filters) which results in no ISI at the sampling time.

Nyquist filter:

– Its transfer function in frequency domain is obtained by convolving a rectangular function with any real even-symmetric frequency function

Nyquist pulse:

– Its shape can be represented by a sinc(t/T) function multiply by another time function.

Example of Nyquist filters: Raised-Cosine filter

Pulse shaping to reduce ISI

Goals and trade-off in pulse-shaping

- Reduce ISI
- Efficient bandwidth utilization
- Robustness to timing error (small side lobes)

The raised cosine filter

aised-Cosine Filter

- A Nyquist pulse (No ISI at the sampling time)

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ \cos^2\left[\dfrac{\pi}{4} \dfrac{|f| + W - 2W_0}{W - W_0}\right] & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| > W \end{cases}$$

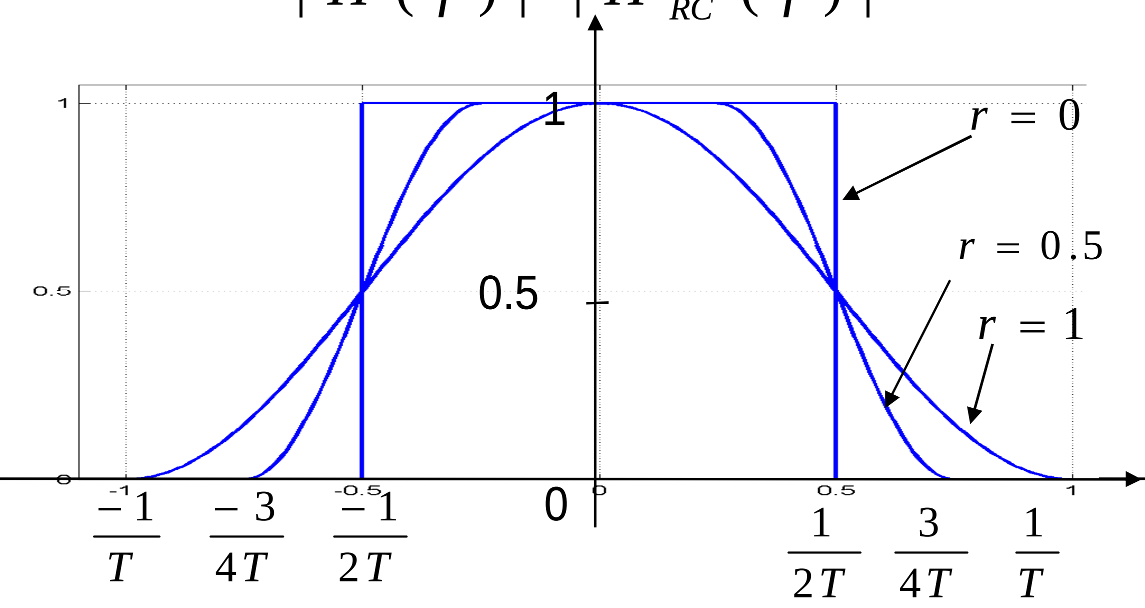
$$h(t) = 2W_0(\mathrm{sinc}(2W_0 t)) \frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}$$

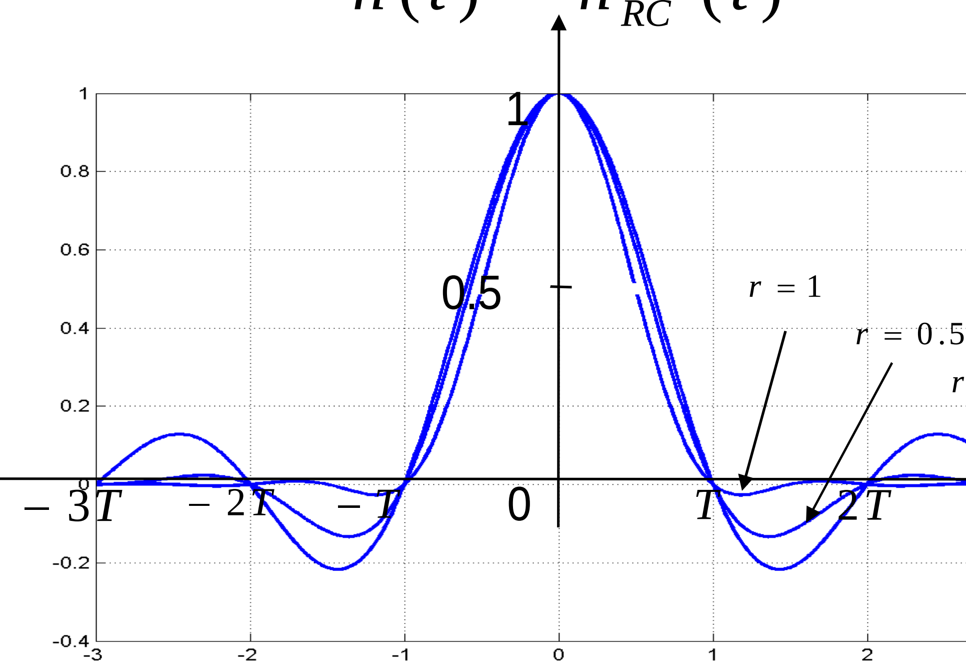
Excess bandwidth: $W - W_0$

Roll-off factor $r = \dfrac{W - W_0}{W_0}$

$0 \le r \le 1$

The Raised cosine filter – cont'd





Baseband $\quad W_{\text{sSB}} = (1 + r)\frac{R_s}{2}$

Passband $\quad W_{\text{DSB}} = (1 + r)R_s$

Pulse shaping and equalization to remove ISI

No ISI at the sampling time

$$H_{\mathrm{RC}}(f) = H_t(f)H_c(f)H_r(f)H_e(f)$$

quare-Root Raised Cosine (SRRC) filter and Equalizer

$$H_{\mathrm{RC}}(f) = H_t(f)H_r(f)$$

$$H_r(f) = H_t(f) = \sqrt{H_{\mathrm{RC}}(f)} = H_{\mathrm{SRRC}}(f)$$

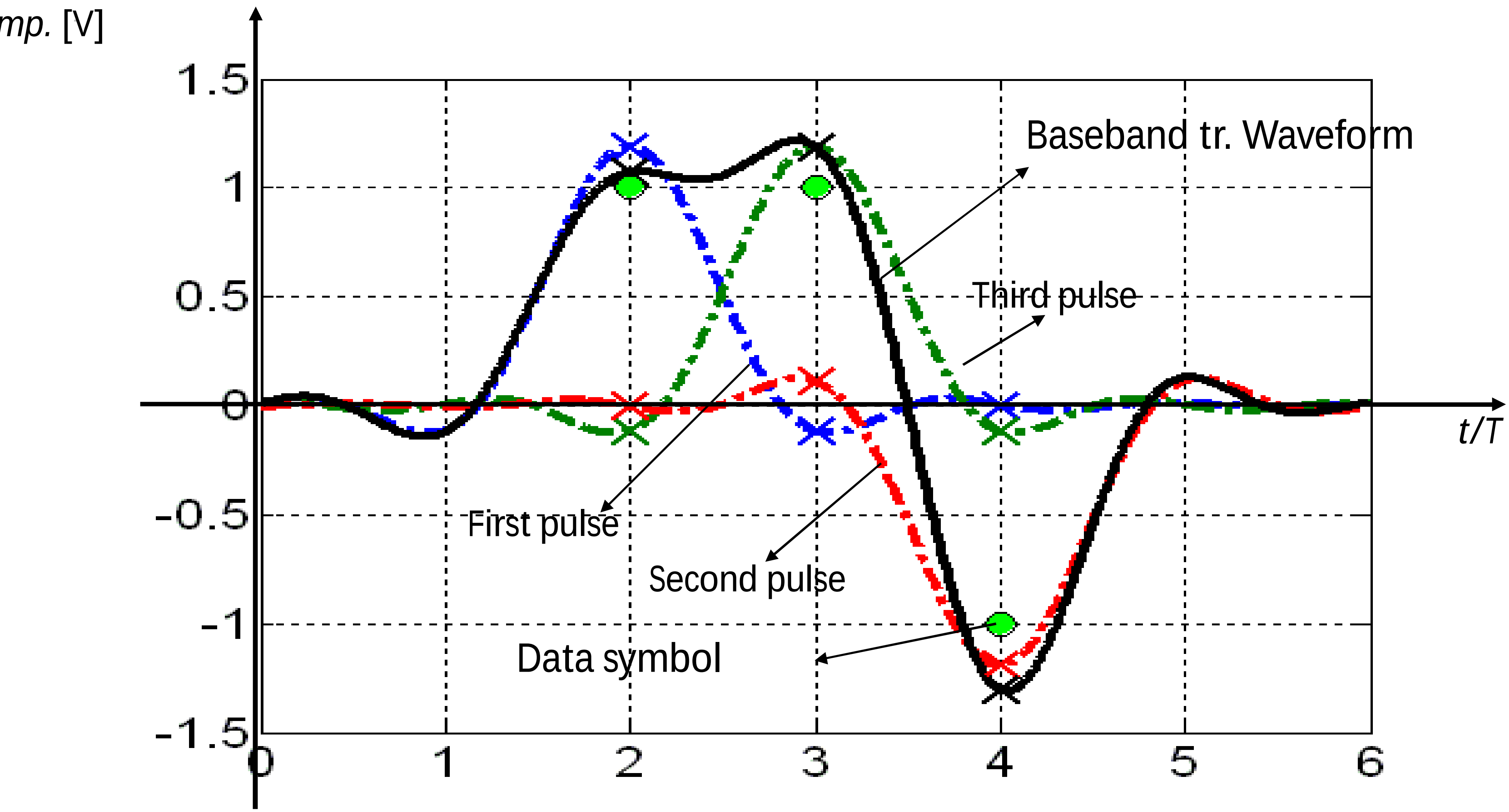
Taking care of ISI caused by tr. filter

$$H_e(f) = \frac{1}{H_c(f)}$$

Taking care of ISI caused by channel

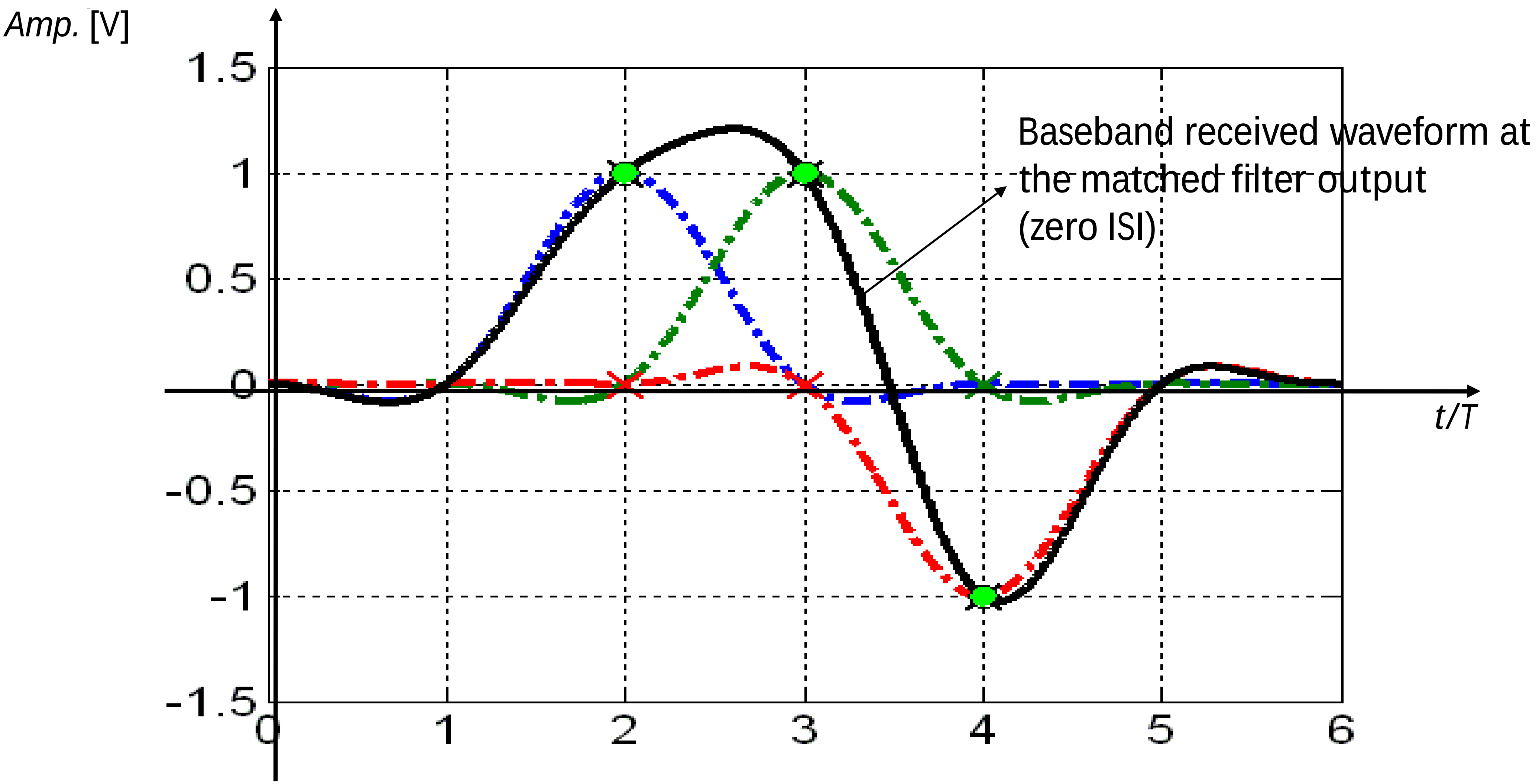
Example of pulse shaping

Square-root Raised-Cosine (SRRC) pulse shaping



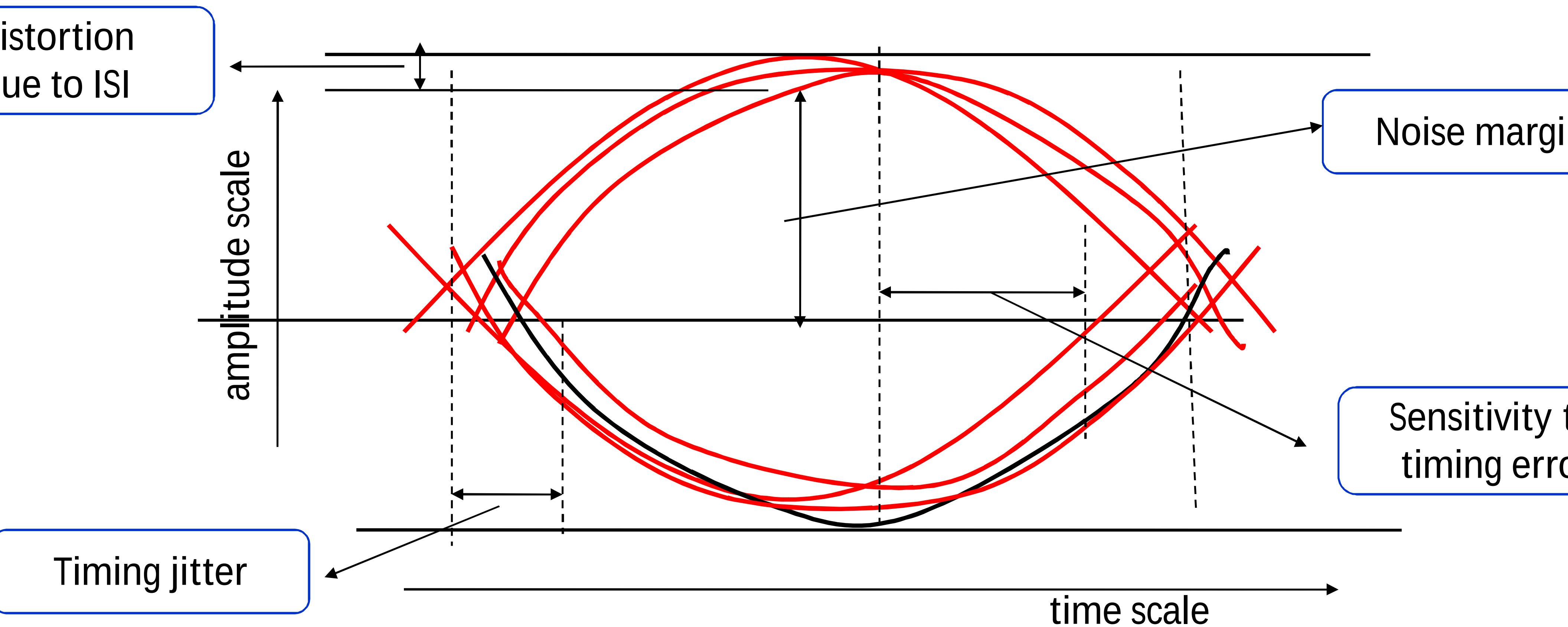
Example of pulse shaping ...

Raised Cosine pulse at the output of matched filter



Eye pattern

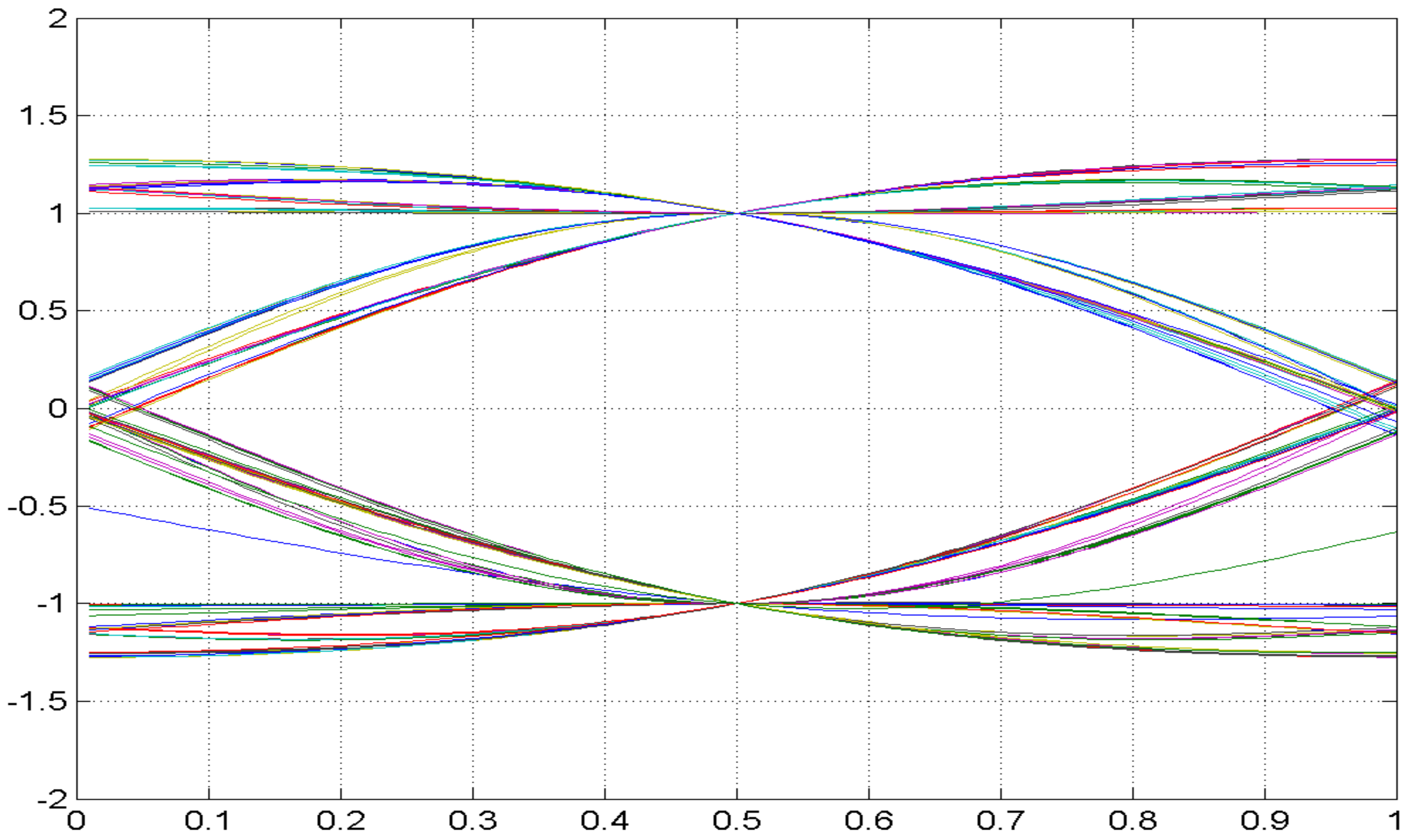
Eye pattern: Display on an oscilloscope which sweeps the system response to a baseband signal at the rate *1/T* (*T* symbol duration)



Example of eye pattern:

Binary-PAM, SRRQ pulse

Perfect channel (no noise and no ISI)

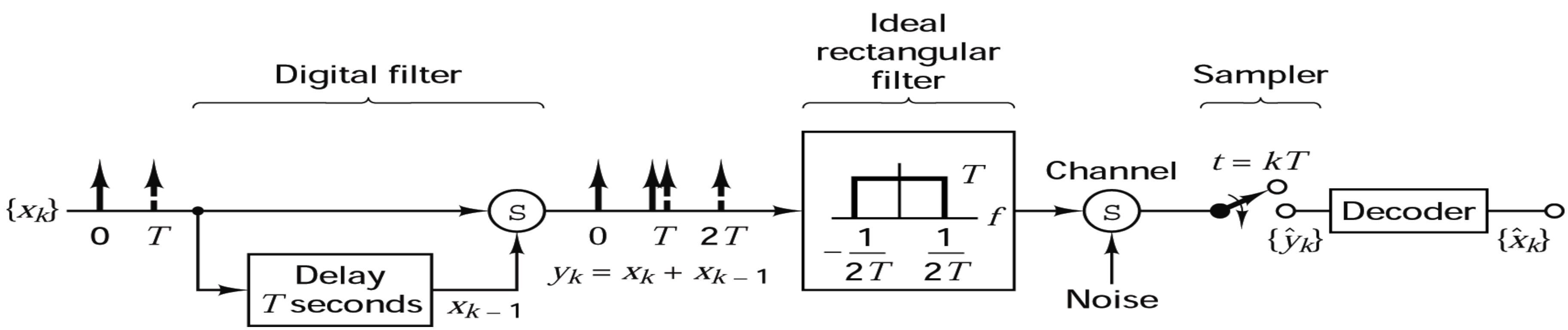


Correlative Coding

Transmit 2W symbols/s with zero ISI, using the theoretical minimum bandwidth of W Hz, without infinitely sharp filters.

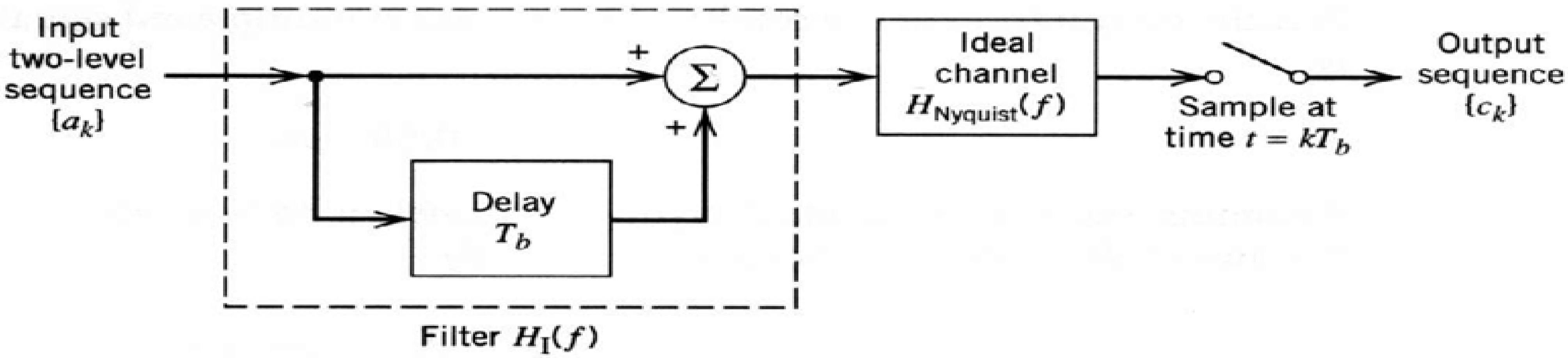
Correlative coding (or duobinary signaling or partial response signaling) introduces some controlled amount of ISI into the data stream rather than trying to eliminate ISI completely

Doubinary signaling



Duobinary signaling

Du



$$\begin{aligned} H_{\text{I}}(f) &= H_{\text{Nyquist}}(f)[1 + \exp(-j2\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j\pi f T_b) + \exp(-j\pi f T_b)] \exp(-j\pi f T_b) \\ &= 2H_{\text{Nyquist}}(f) \cos(\pi f T_b) \exp(-j\pi f T_b) \end{aligned}$$

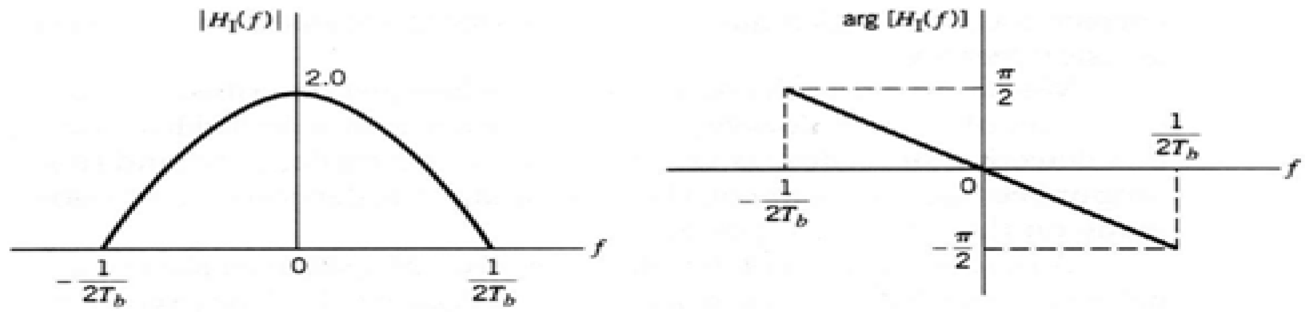
$$H_{\text{Nyquist}}(f) = \begin{cases} 1, & |f| \leq 1/2T_b \\ 0, & \text{otherwise} \end{cases}$$

Duobinary signal and Nyguist Criteria

Nygu

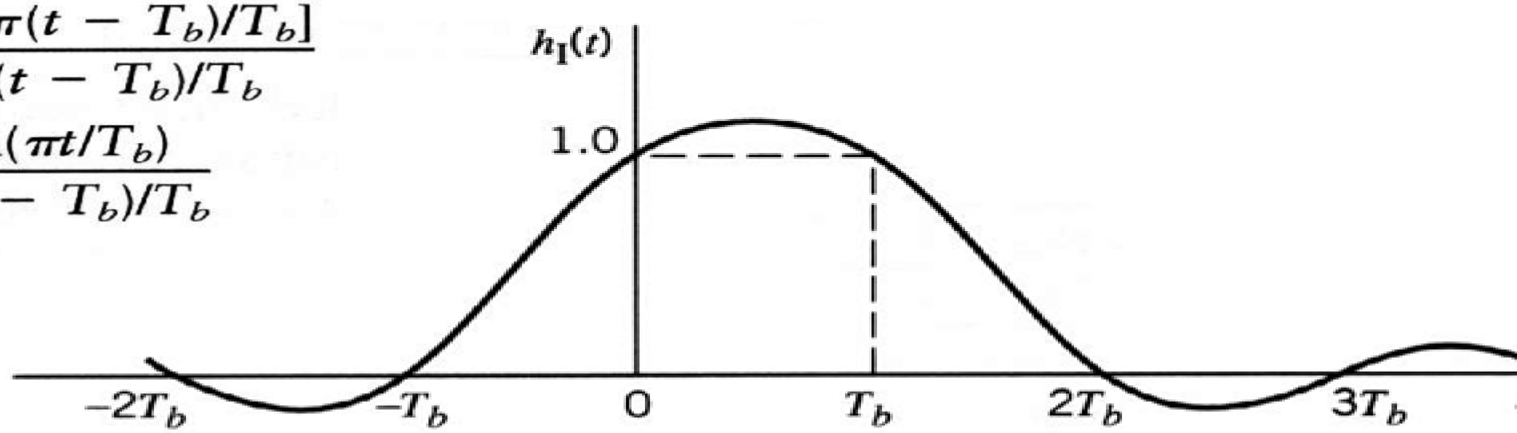
$$H_I(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases}$$

th



(a) (b)

$$\begin{aligned} h_I(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t - T_b)/T_b]}{\pi(t - T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t - T_b)/T_b} \\ &= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)} \end{aligned}$$



Differential Coding

The response of a pulse is spread over more than one signalin
interval.

$$\hat{a}_k = c_k - \hat{a}_{k-1}$$

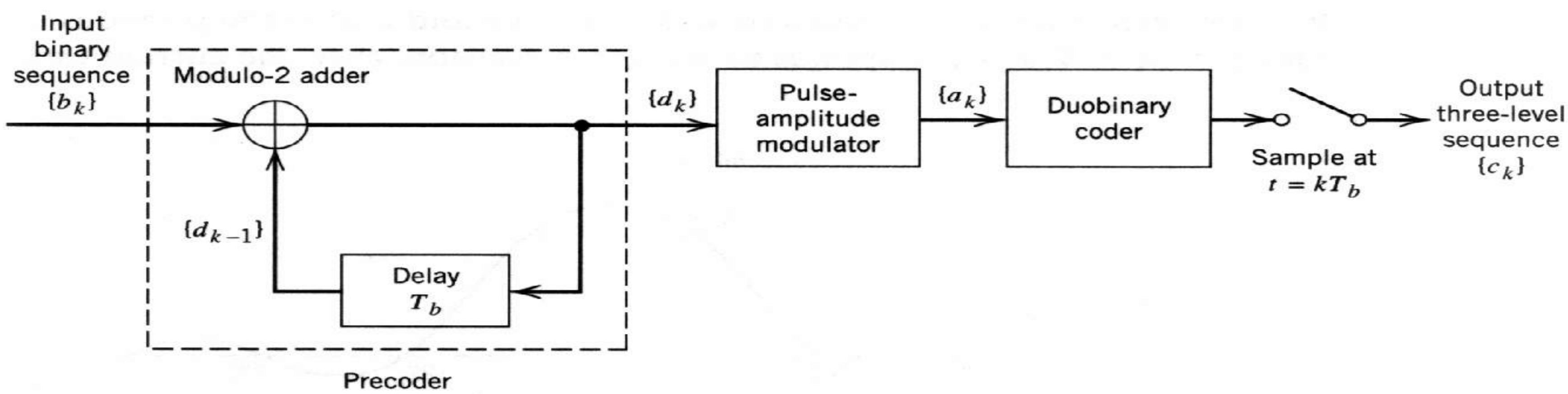
The response is partial in any signaling interval.

Detection :

– M

To a

(pre



Modified duobinary signaling

- Modified duobinary signaling
  - – In duobinary signaling, H(f) is nonzero at the origin.
  - – We can correct this deficiency by using the class IV partial response

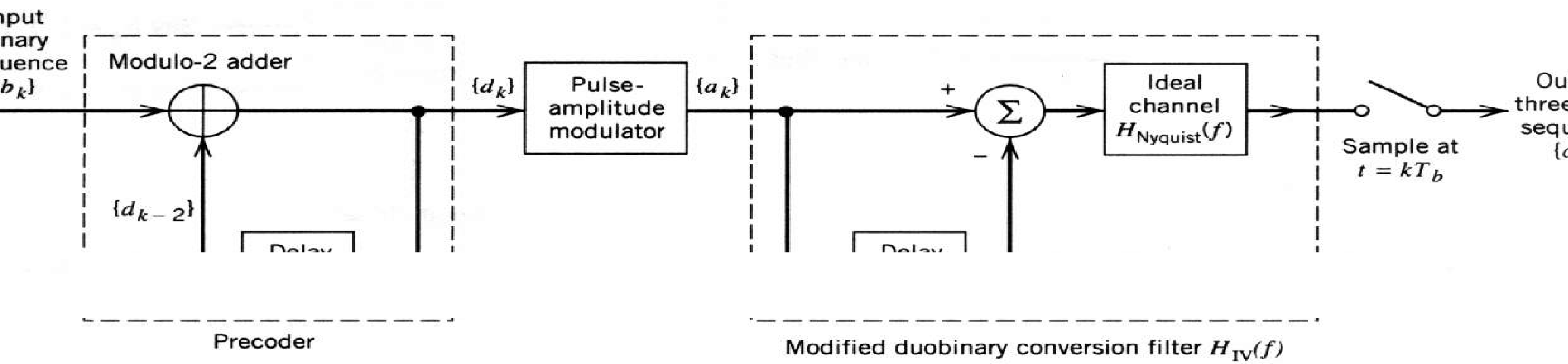


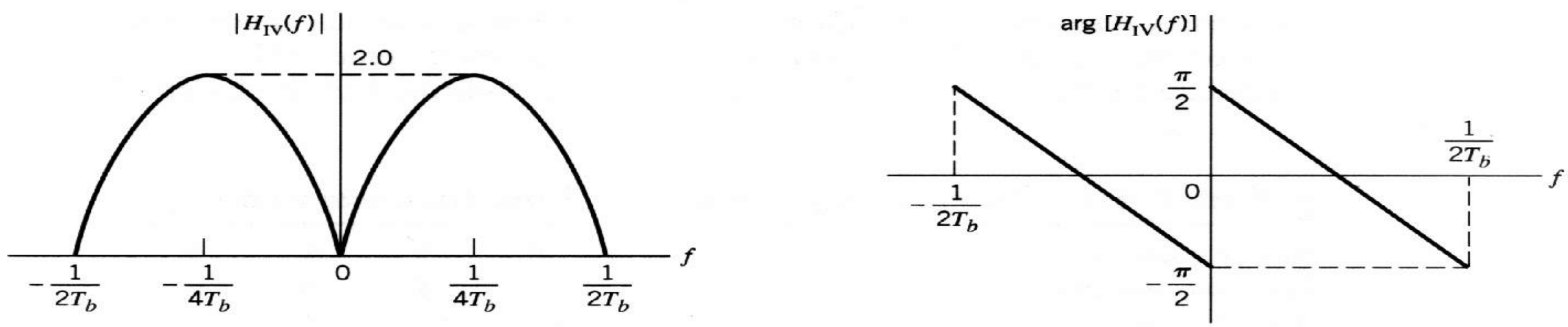
FIGURE 4.16 Modified duobinary signaling scheme.

Modified duobinary signaling

Spectrur

$$H_{IV}(f) = H_{Nyquist}(f)[1 - \exp(-j4\pi f T_b)]$$
$$= 2jH_{Nyquist}(f)\sin(2\pi f T_b)\exp(-j2\pi f T_b)$$

$$H_{IV}(f) = \begin{cases} 2j\sin(2\pi f T_b)\exp(-j2\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{elsewhere} \end{cases}$$



(a) (b)

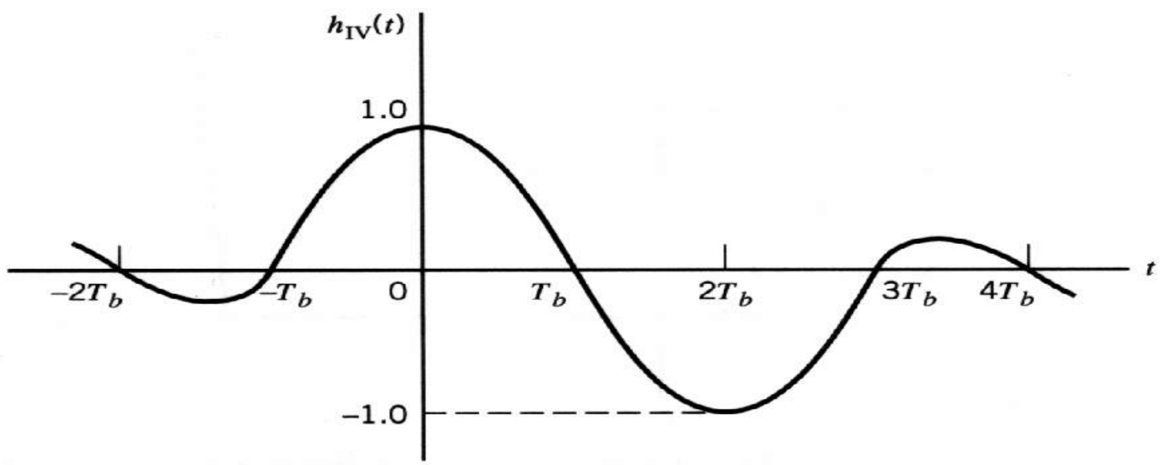
Modified duobinary signaling

Time S ving 2, 0, and -2?

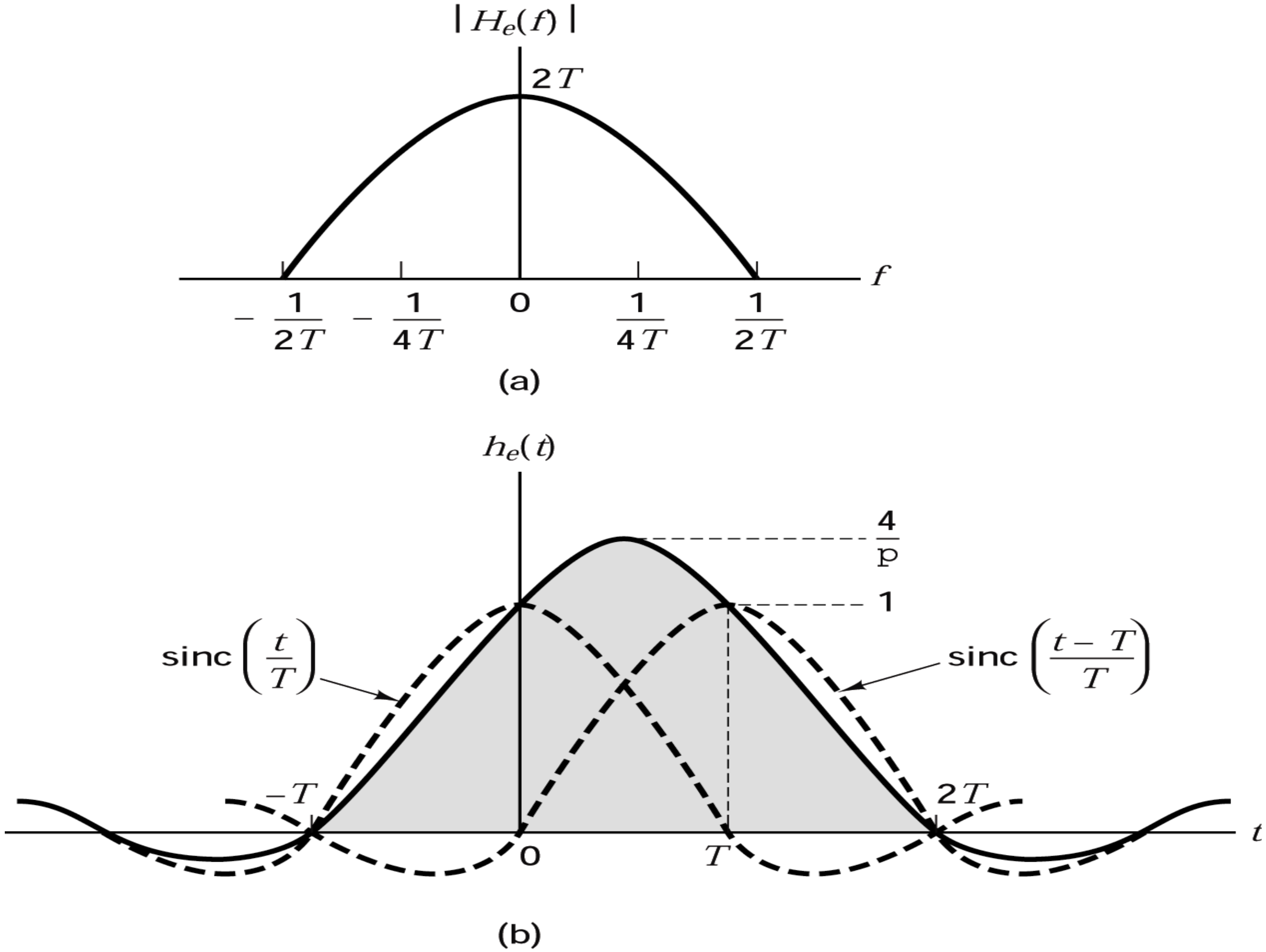
$$h_{IV}(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t - 2T_b)/T_b)]}{\pi(t - 2T_b)/T_b}$$

$$= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t - 2T_b)/T_b}$$

$$= \frac{2T_b^2 \sin(\pi t/T_b)}{\pi t(2T_b - t)}$$



Duobinary Transfer Function



Comparison of Binary with Duobinary Signaling

Binary signaling assumes the transmitted pulse amplitude are independent of one another

Duobinary signaling introduces correlation between pulse amplitudes

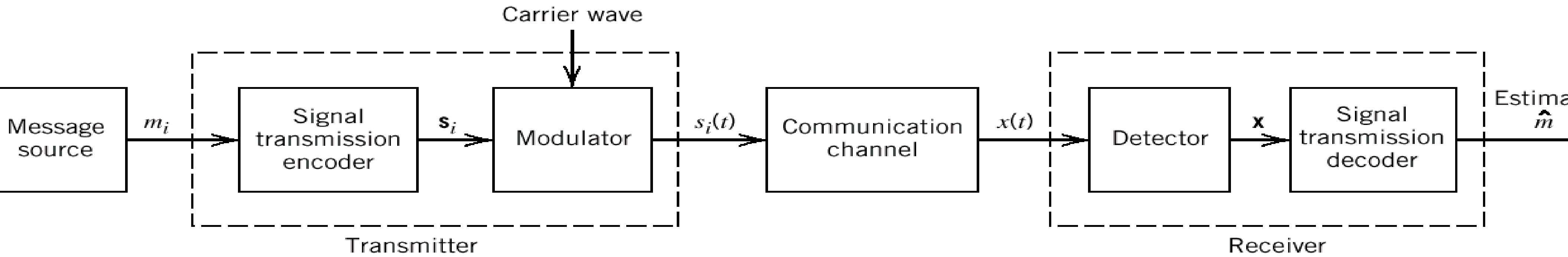
Duobinary technique achieve zero ISI signal transmission using a smaller system bandwidth

Duobinary coding requires three levels, compared with the usual two levels for binary coding

Duobinary signaling requires more power than binary signaling (~2.5 dB greater SNR than binary signaling)

Pass-band Data Transmission

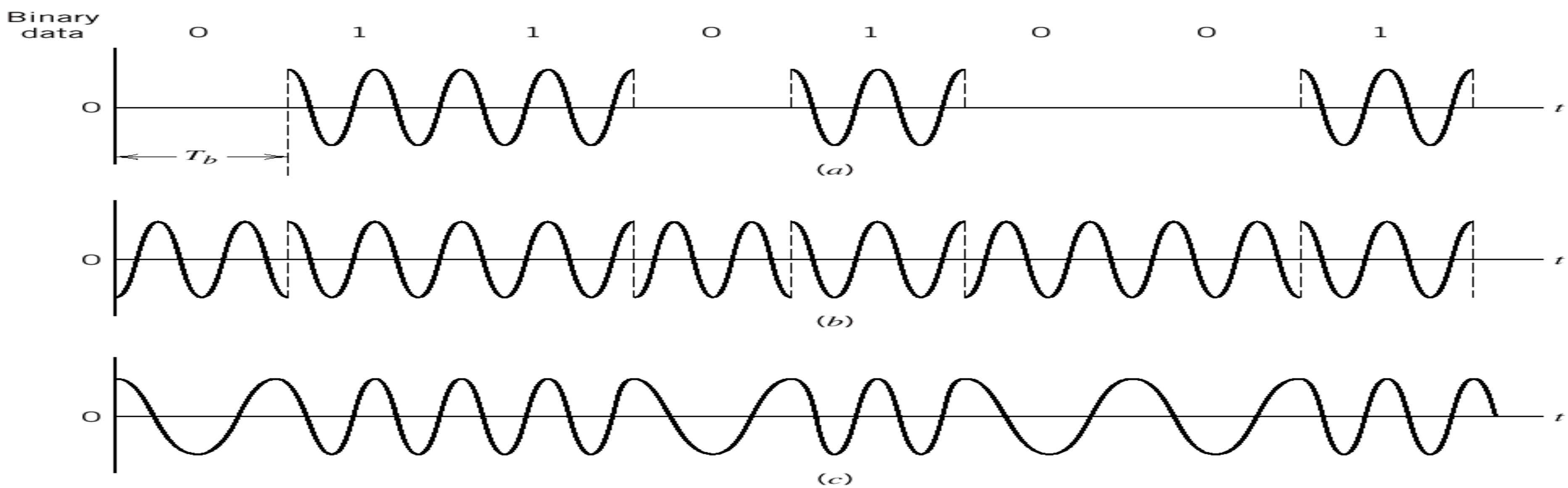
Block Diagram



Functional model of pass-band data transmission system.

Signaling

Illustrative waveforms for the three basic forms of signaling binary information. (*a*) Amplitude-shift keying. (*b*) Phase-shift keying. (*c*) Frequency-shift keying with continuous phase.



What do we want to study?

We are going to study and compare different modulation techniques in terms of

- Probability of errors
- Power Spectrum
- Bandwidth efficiency

$$\rho = \frac{R_b}{B} \qquad \text{Bits/s/Hz}$$

Coherent PSK

Binary Phase Shift Keying (BPSK)

– Consider the system with 2 basis functions

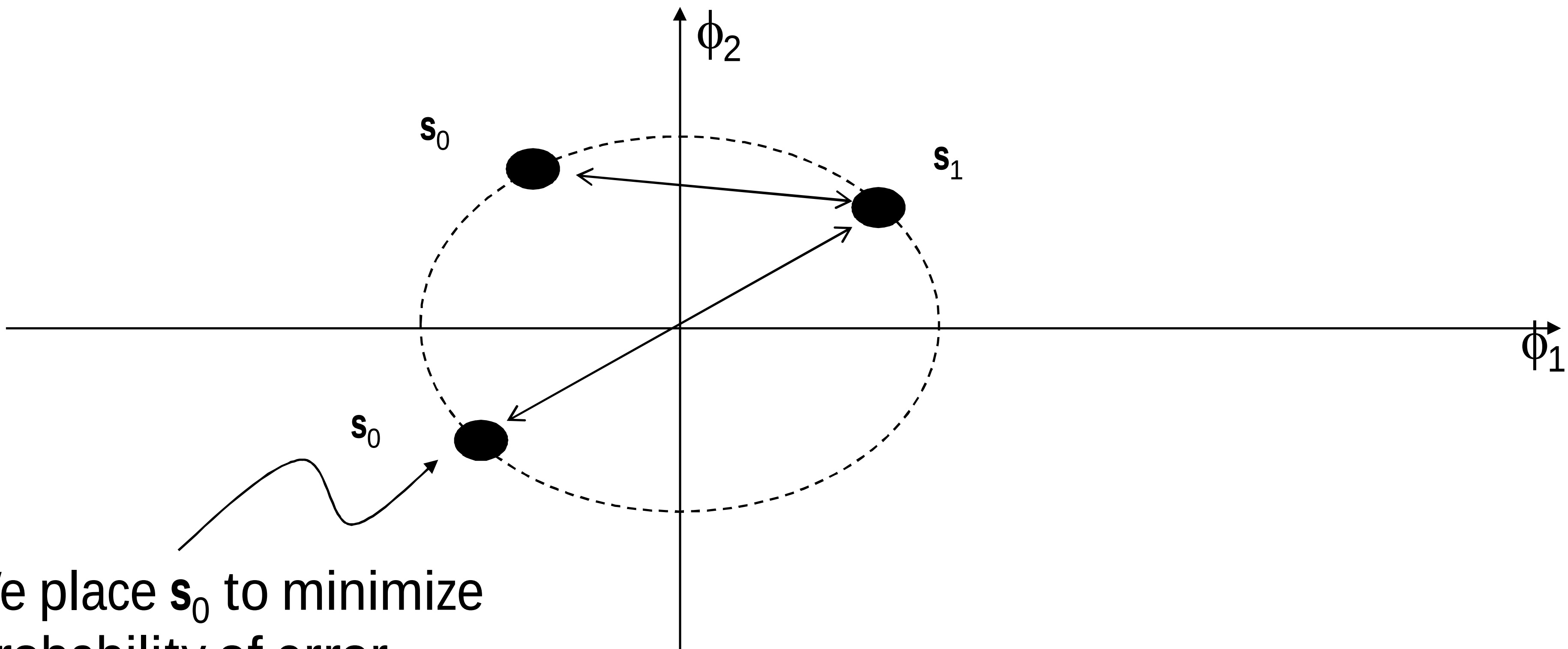
$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos \; 2\pi f_c t$$

– and

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \sin \; 2\pi f_c t$$

BPSK

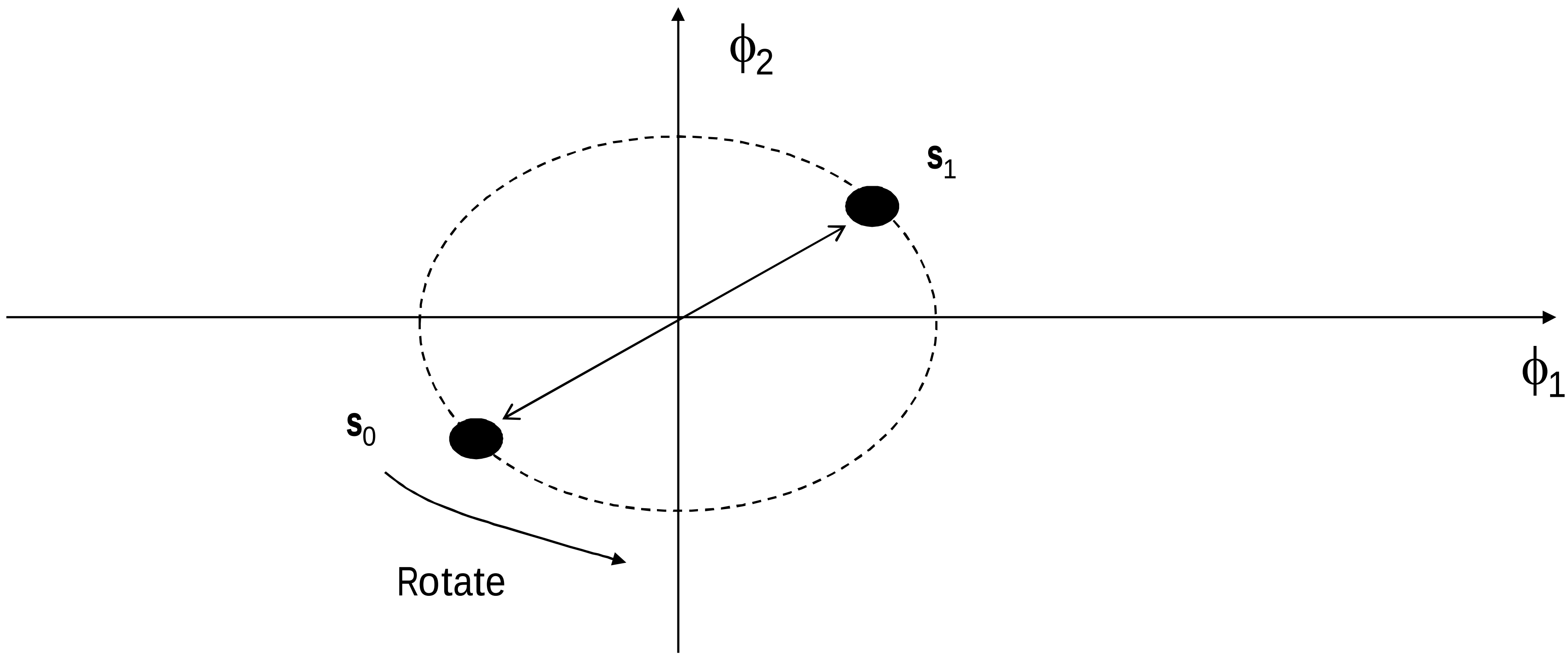
If we want to fix that for both symbols (0 and 1) the transmitted energies are equal, we have



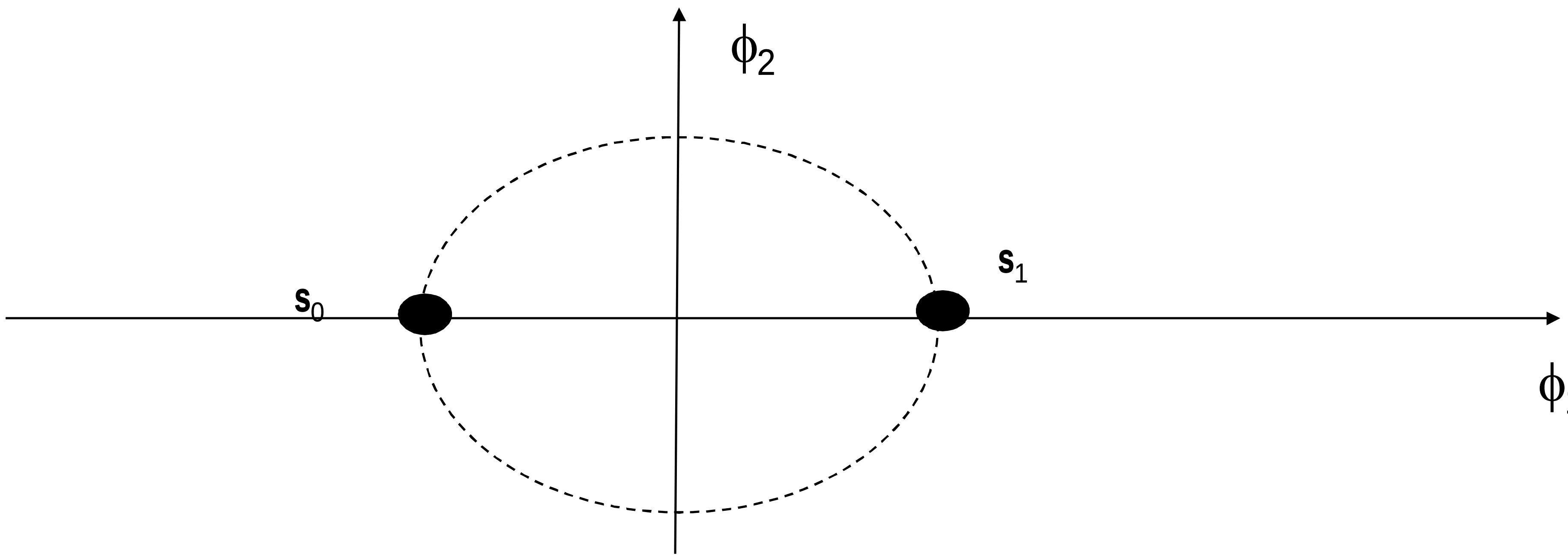
We place $\mathbf{s}_0$ to minimize probability of error

BPSK

We found that phase of $\mathbf{s}_1$ and $\mathbf{s}_0$ are 180 degree difference.
We can rotate $\mathbf{s}_1$ and $\mathbf{s}_0$



BPSK



We observe that ϕ_2 has nothing to do with signals. Hence, only one basis function is sufficient to represent the signals

BPSK

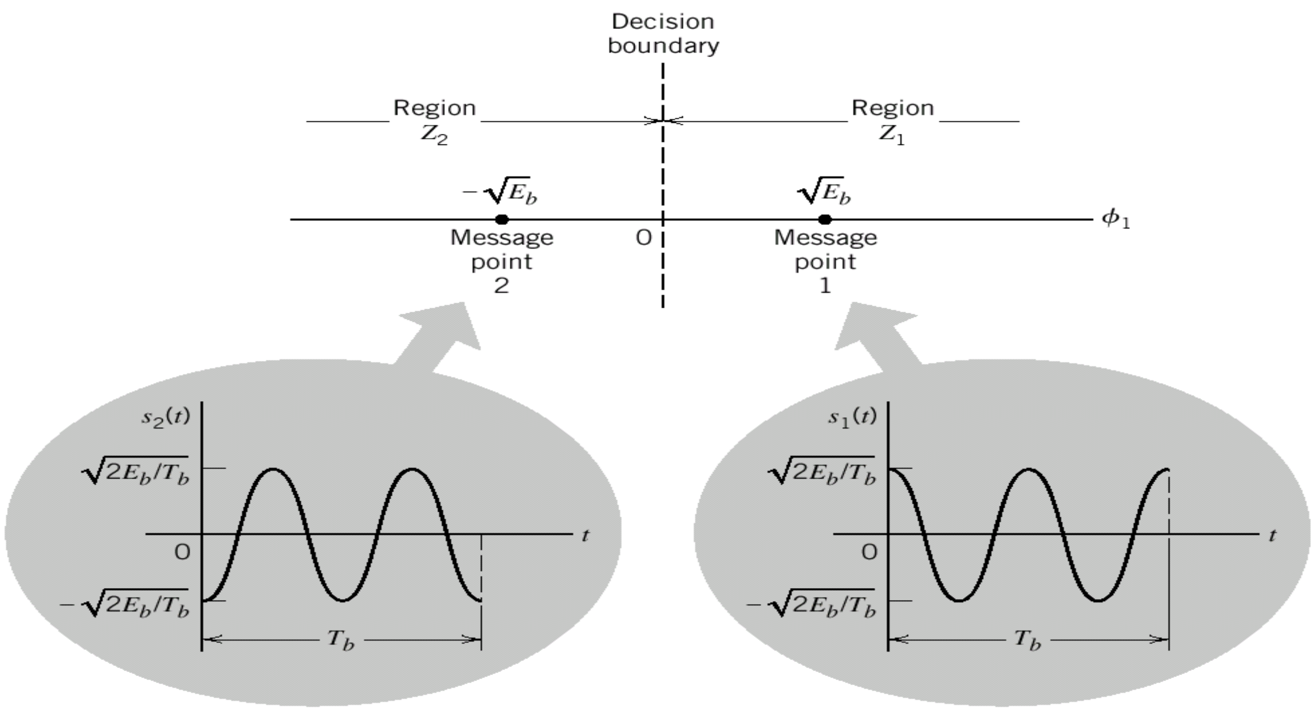
Finally, we have

$$s_1(t) = \sqrt{E_b}\,\phi_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos\ 2\pi f_c t$$

$$s_0(t) = -\sqrt{E_b}\,\phi_1(t) = -\sqrt{\frac{2E_b}{T_b}}\cos\ 2\pi f_c t$$

BPSK

Signal-space diagram for coherent binary PSK system. The waveforms depicting the transmitted signals $s_1(t)$ and $s_2(t)$, displayed in the inserts, assume $n_c = 2$.



BPSK

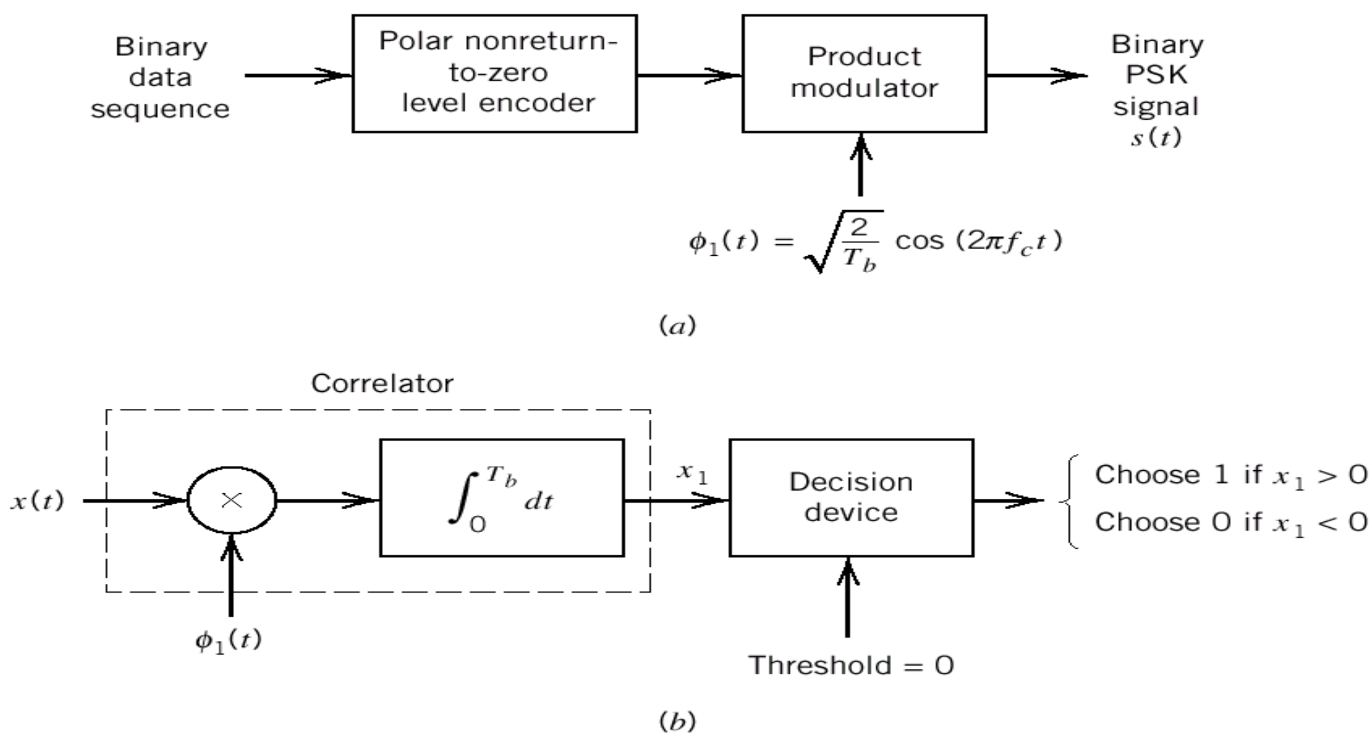
- Probability of error calculation. In the case of equally likely ($\Pr(m_0)=\Pr(m_1)$), we have

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BPSK

Block diagrams for (*a*) binary PSK transmitter and (*b*) coherent binary PSK receiver.



Quadriphase-Shift Keying (QPSK)

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right]; \quad 0 \le t < T$$

T is symbol duration

E is signal energy per symbol

There are 4 symbols for i = 1, 2, 3, and 4

QPSK

$$s_i(t) = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\sqrt{\frac{2}{T}}\cos(2\pi f_c t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

$$= \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t); \quad 0 \le t < T$$

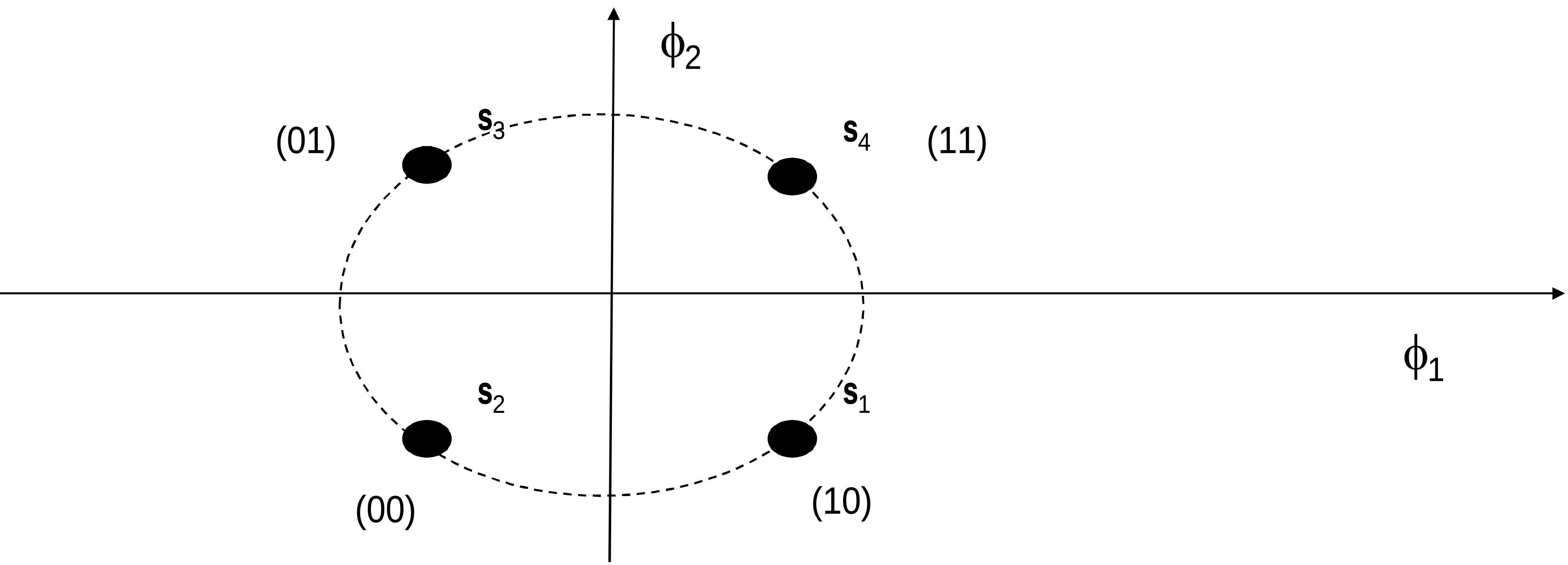
Which we can write in vector format as

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E}\cos\left((2i-1)\frac{\pi}{4}\right) \\ -\sqrt{E}\sin\left((2i-1)\frac{\pi}{4}\right) \end{bmatrix}$$

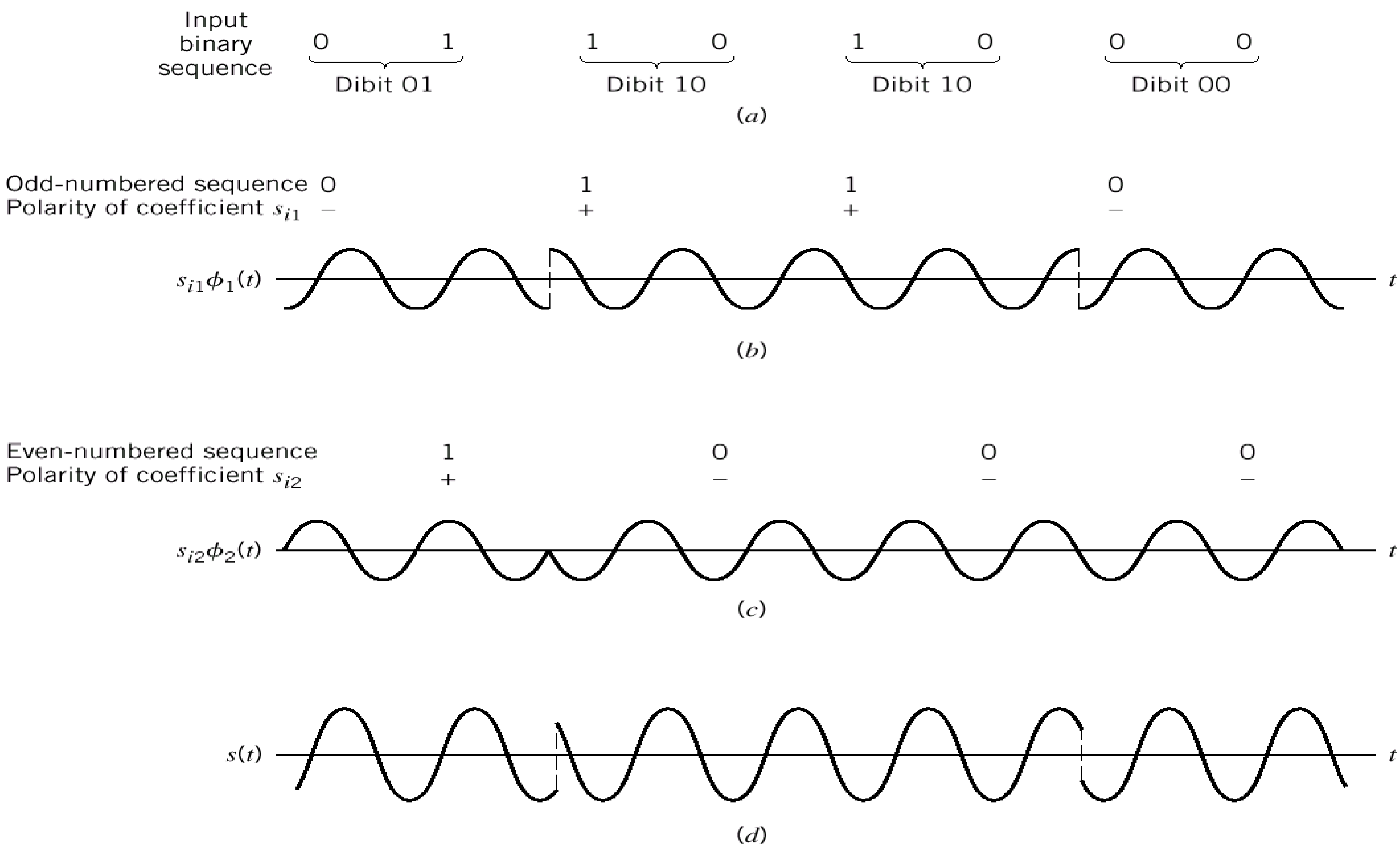
QPSK

i	Input Dibit	Phase of QPSK signaling	Coordinate of Message point	
			s_{i1}	s_{i2}
1	10	$\pi / 4$	$\sqrt{E / 2}$	$-\sqrt{E / 2}$
2	00	$3\pi / 4$	$-\sqrt{E / 2}$	$-\sqrt{E / 2}$
3	01	$5\pi / 4$	$-\sqrt{E / 2}$	$\sqrt{E / 2}$
4	11	$7\pi / 4$	$\sqrt{E / 2}$	$\sqrt{E / 2}$

QPSK

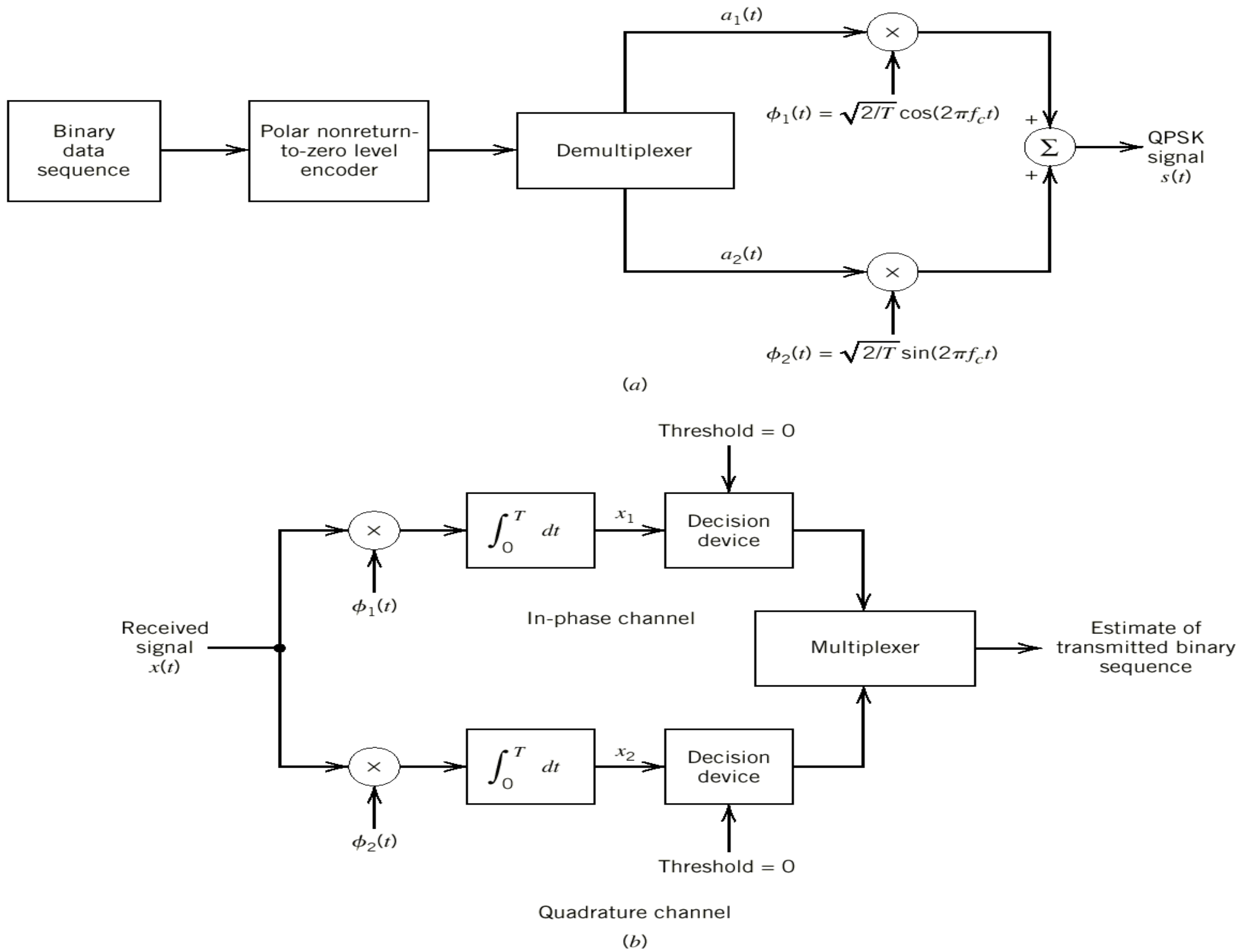


QPSK signals



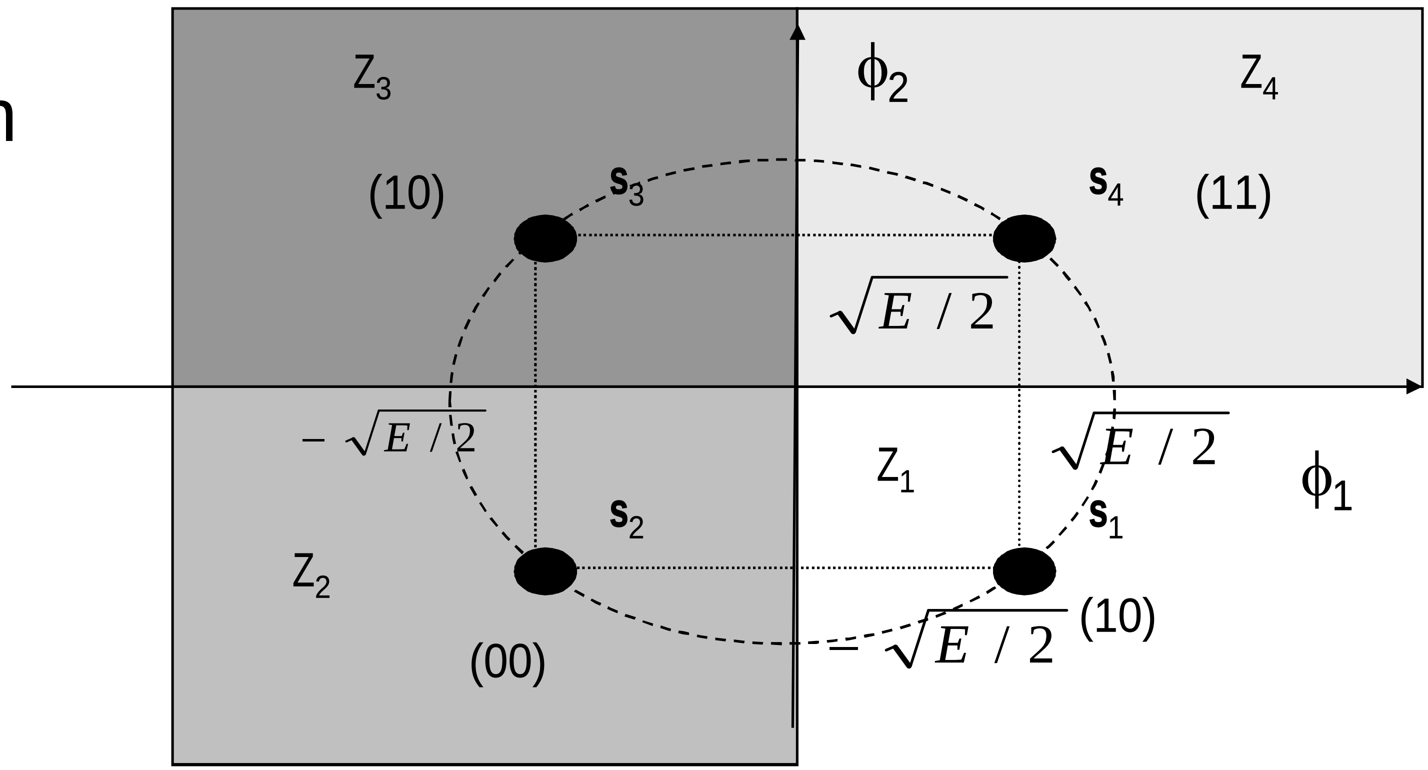
QPSK

Block diagrams of (*a*) QPSK transmitter and (*b*) coherent QPSK receiver.



QPSK: Error Probability QPSK

Consider signal constellation given in the figure



QPSK

can treat QPSK as the combination of 2 independent
K over the interval $T=2T_b$

e the first bit is transmitted by ϕ_1 and the second bit is
nsmitted by ϕ_2.

bability of error for each channel is given by

$$P' = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{12}}{2\sqrt{N_0}}\right) = \frac{1}{2}\operatorname{erf}c\left(\sqrt{\frac{E}{2N_0}}\right)$$

QPSK

mbol is to be received correctly both bits must be received ectly.

ce, the average probability of correct decision is given by

ch gives the probability of errors equal to $P_c = (1 - P')^2$

$$- P_C = \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$\mathrm{fc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

QPSK

e one symbol of QPSK consists of two bits, we have $E = 2E_b$.

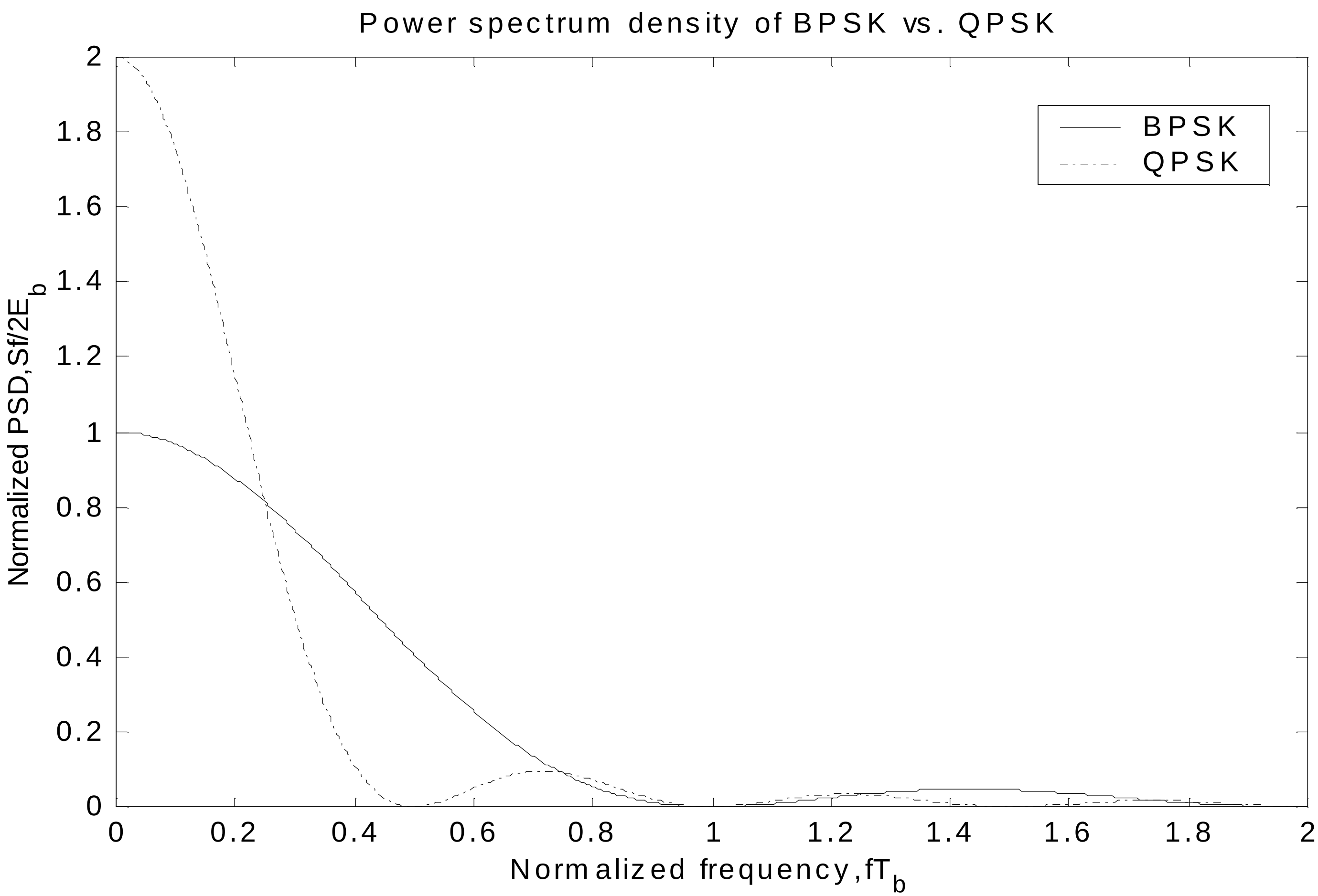
$$Pe\,(\text{per symbol}) \approx \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

above probability is the error probability per symbol. The avg. ability of error per bit

$$\text{it}) = \frac{1}{2} Pe\,(\text{per symbol}) \approx \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

ch is exactly the same as BPSK .

BPSK vs QPSK

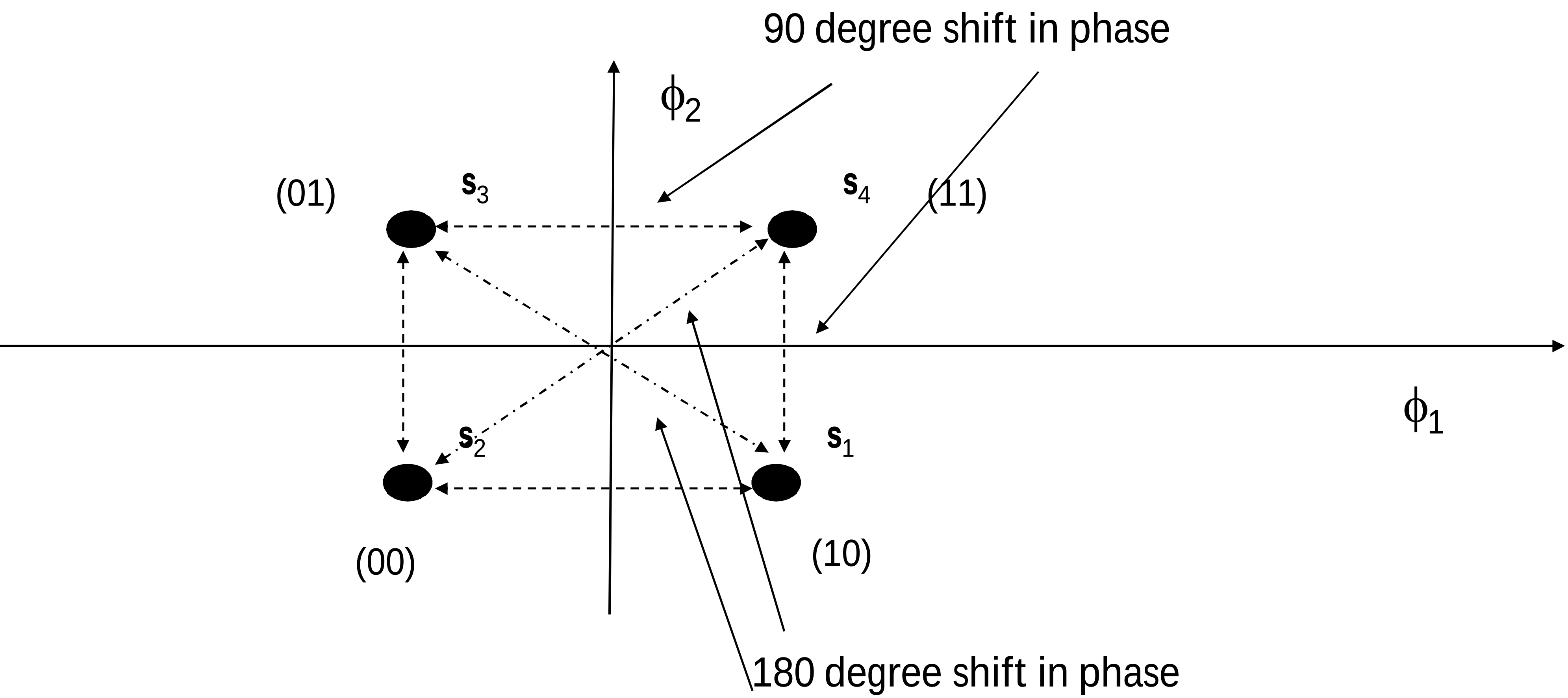


Power spectrum density of BPSK vs. QPSK
BPSK
QPSK
Normalized PSD,Sf/2E$_b$
Normalized frequency,fT$_b$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

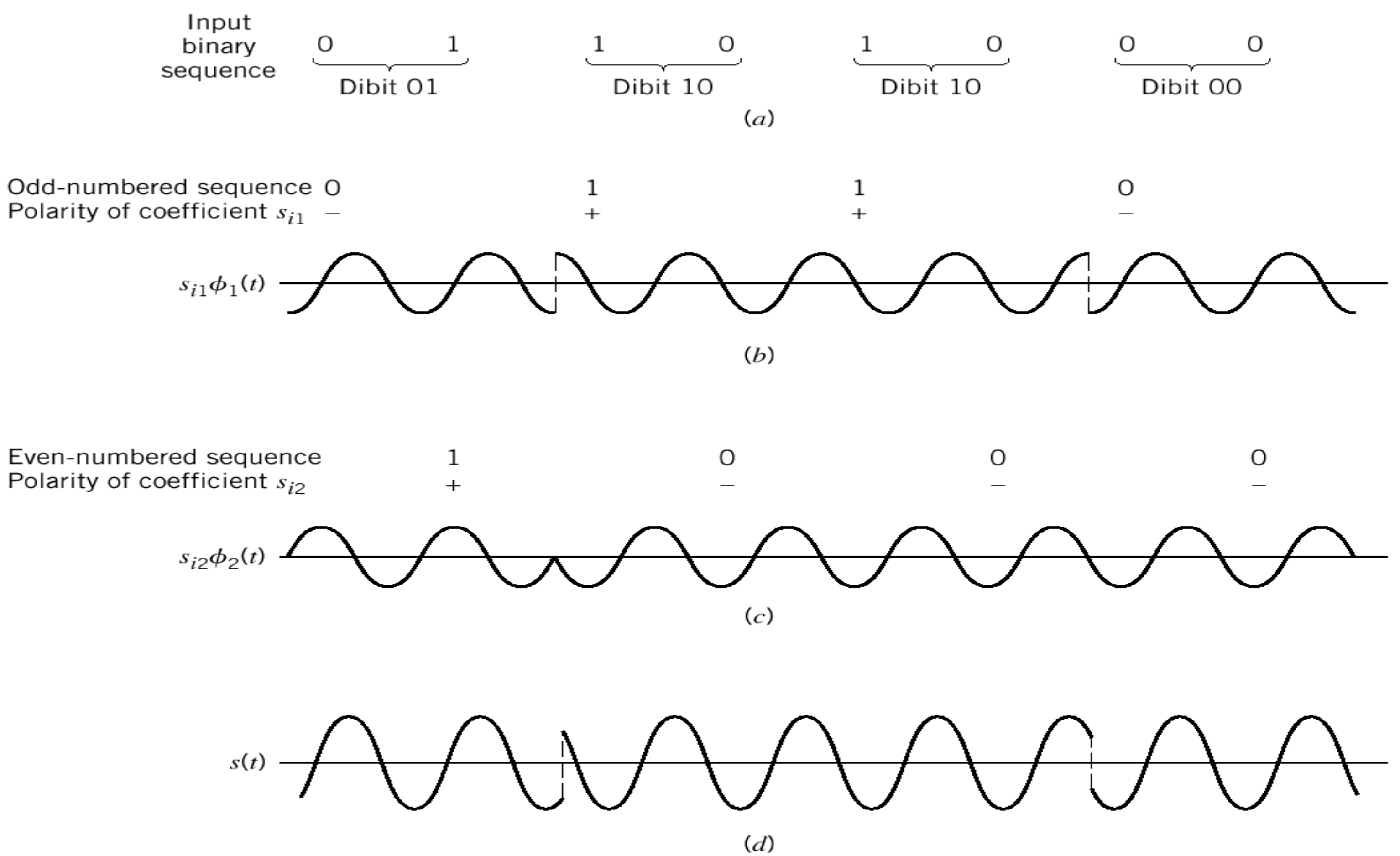
QPSK

- Conclusion
  - QPSK is capable of transmitting data twice as faster as BPSK with th same energy per bit.
  - We will also learn in the future that QPSK has half of the bandwidth of BPSK.

OFFSET QPSK


90 degree shift in phase
ϕ_2
(01)
s_3
s_4
(11)
ϕ_1
s_2
s_1
(00)
(10)
180 degree shift in phase

OFFSET QPSK



Input binary sequence
0 1
1 0
1 0
0 0
Dibit 01
Dibit 10
Dibit 10
Dibit 00
(a)
Odd-numbered sequence 0 1 1 0
Polarity of coefficient s_{i1} − + + −
$s_{i1}\phi_1(t)$
t
(b)
Even-numbered sequence 1 0 0 0
Polarity of coefficient s_{i2} + − − −
$s_{i2}\phi_2(t)$
t
(c)
$s(t)$
t
(d)

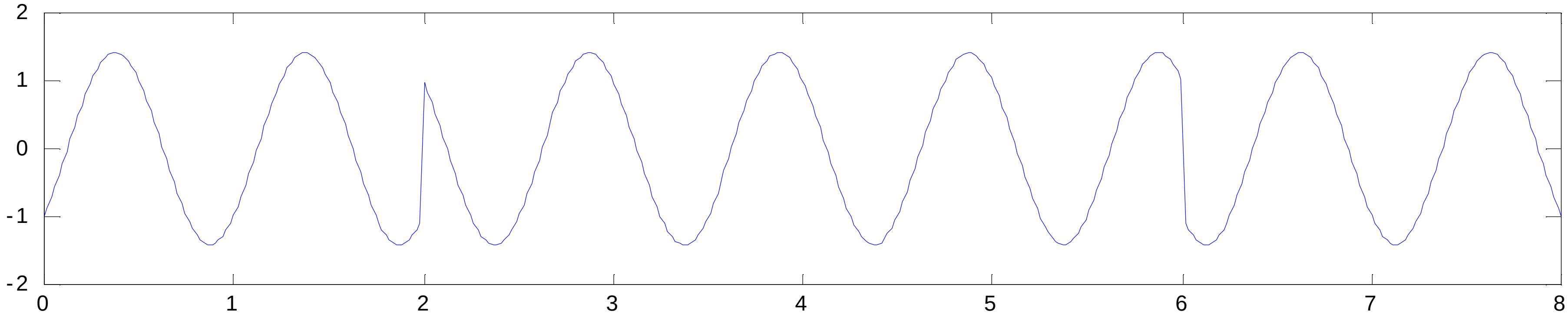
OFFSET QPSK

Whenever both bits are changed simultaneously, 180 degree phase-shift occurs.

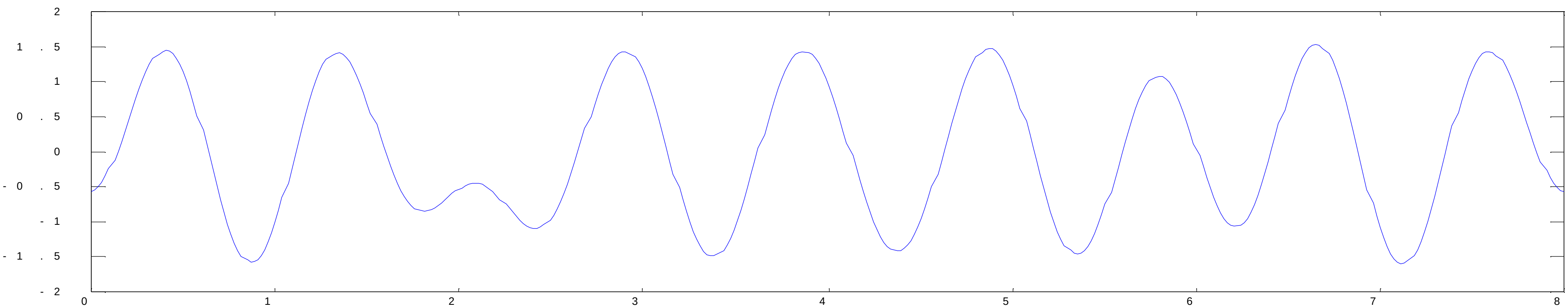
At 180 phase-shift, the amplitude of the transmitted signal changes very rapidly costing amplitude fluctuation.

This signal may be distorted when is passed through the filter or nonlinear amplifier.

OFFSET QPSK



Original Signal



Filtered signal

OFFSET QPSK

To solve the amplitude fluctuation problem, we propose the offset QPSK.

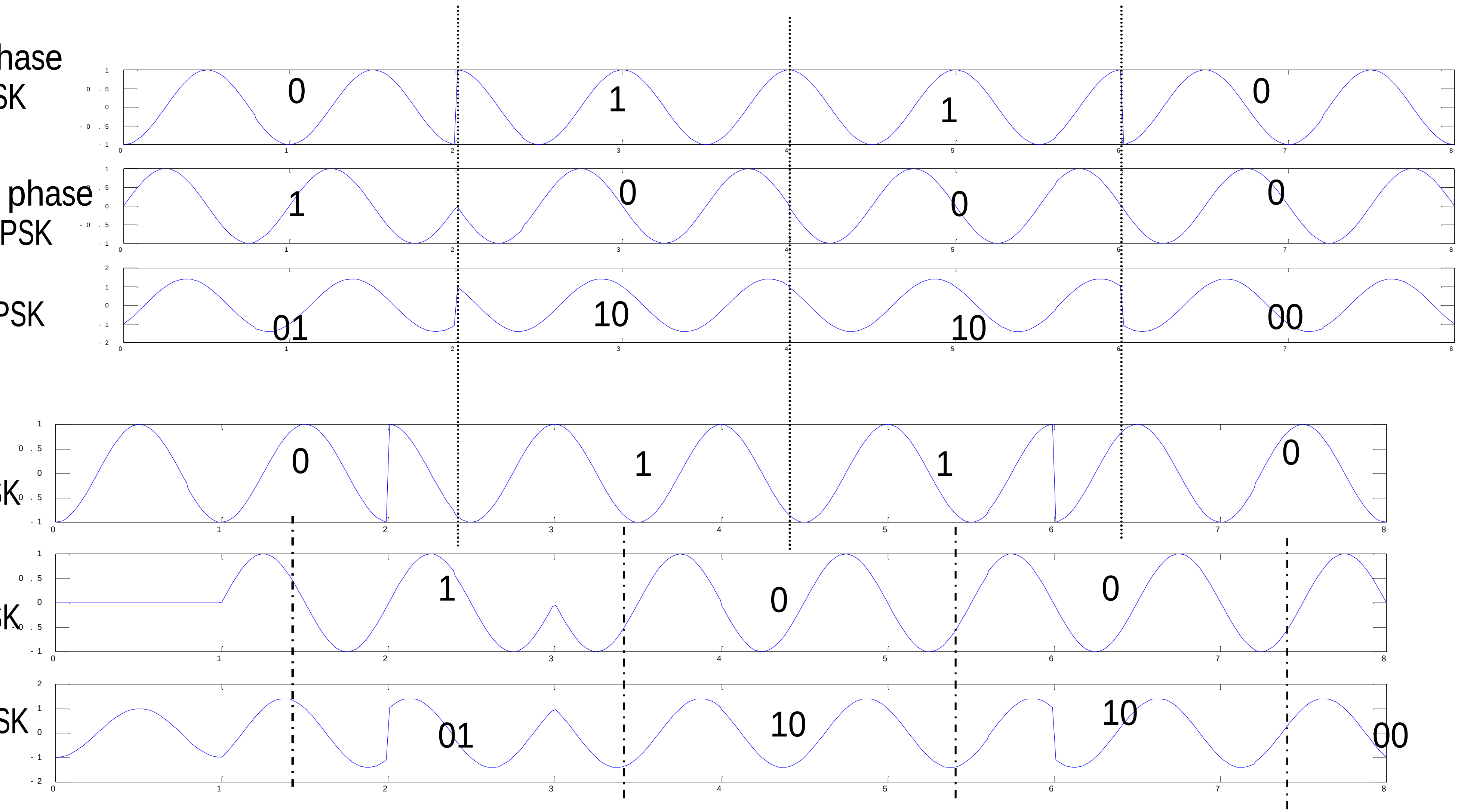
Offset QPSK delay the data in quadrature component by T/2 seconds (half of symbol).

Now, no way that both bits can change at the same time.

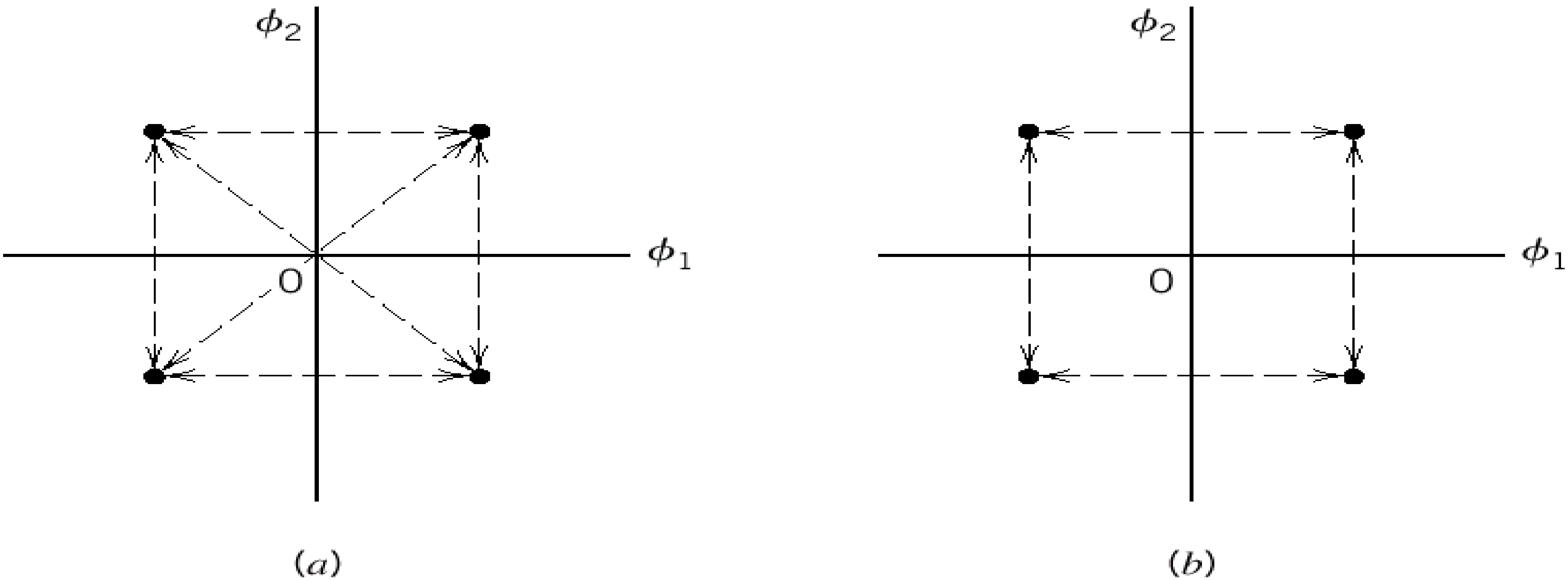
OFFSET QPSK

In the offset QPSK, the phase of the signal can change by ±90 or 0 degree only while in the QPSK the phase of the signal car change by ±180 ±90 or 0 degree.

OFFSET QPSK


hase
SK
phase
PSK
PSK
SK
SK
SK
0 1 1 0
1 0 0 0
01 10 10 00
0 1 1 0
1 0 0
01 10 10 00

Offset QPSK



Possible paths for switching between the message points in (*a*) QPSK and (*b*) offset QPSK.

OFFSET QPSK

dwidths of the offset QPSK and the regular QPSK is the
ne.

m signal constellation we have that

$$P_e \approx \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

ich is exactly the same as the regular QPSK.

M-array PSK

At a moment, there are M possible symbol values being sent for M different phase values,

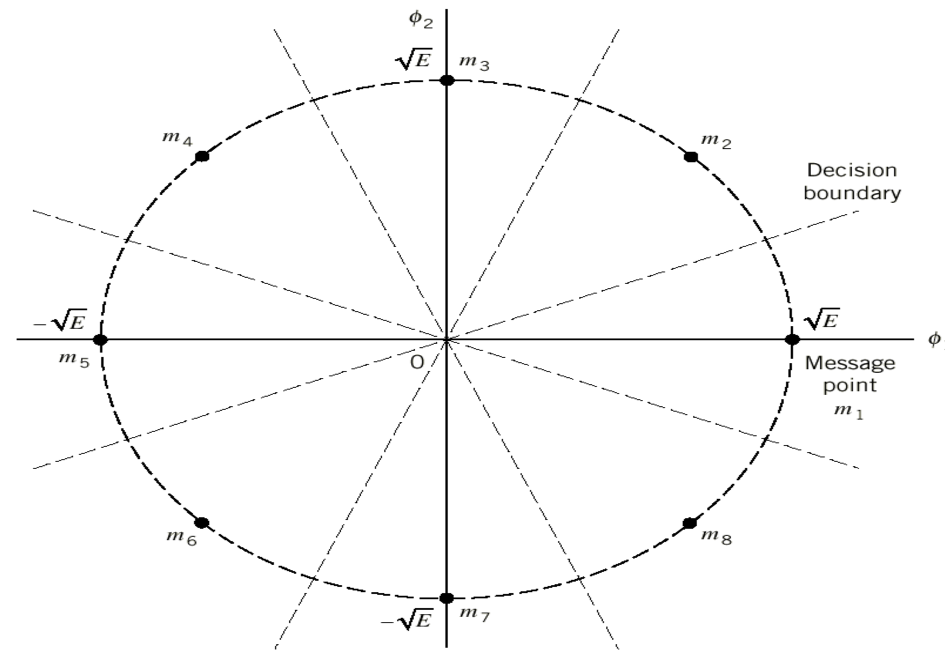
$$\theta_i = 2(i-1)\pi / M$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1, 2, \ldots, M$$

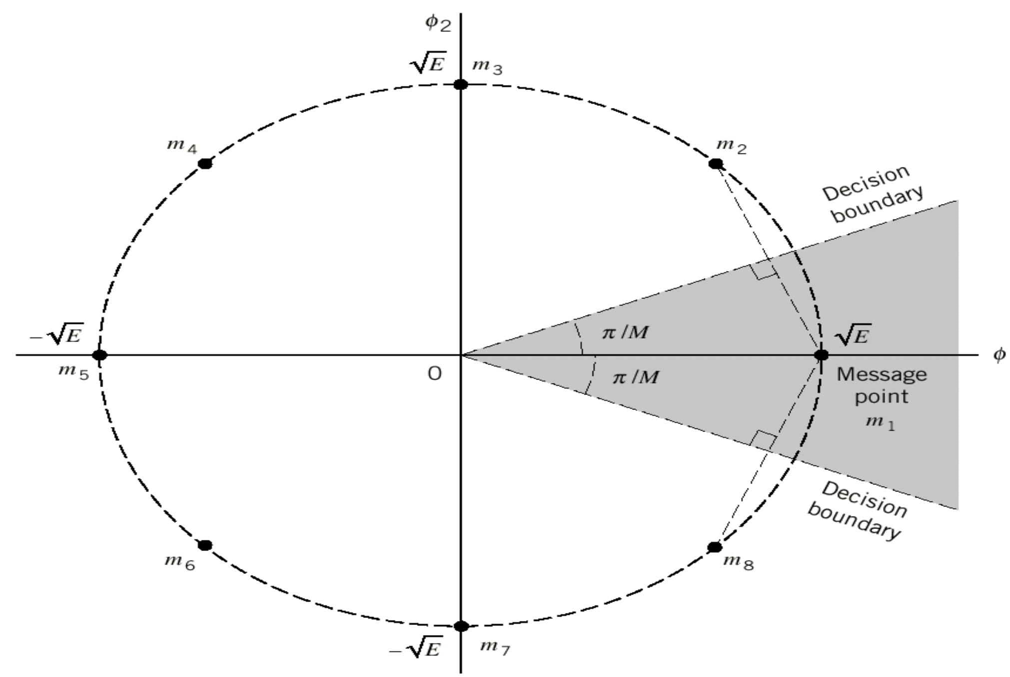
M-a

Signal-space diagram for octaphase-shift keying (i.e., $M = 8$). The decision boundaries are shown as dashed lines.

Signal-space diagram illustrating the application of the union bound for octaphase-shift keying.



(a)



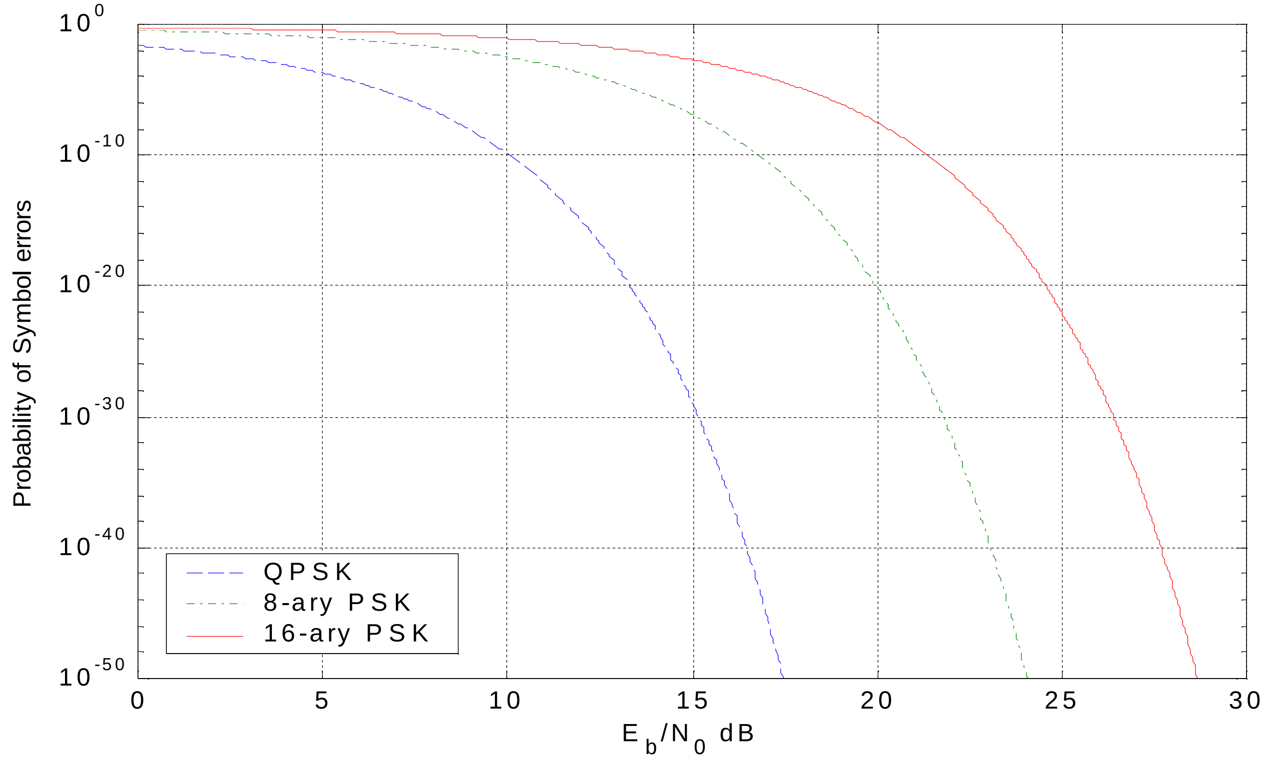
M-array PSK

Probability of errors

$$\therefore \; d_{12} = d_{18} = 2\sqrt{E}\sin\left(\pi / M\right)$$

$$P_e \approx \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\sin\left(\pi / M\right)\right); \qquad M \geq 4$$

M-ary PSK



10^{0}
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
Probability of Symbol errors
0
5
10
15
20
25
30
E_b/N_0 dB
QPSK
8-ary PSK
16-ary PSK

M-array PSK

Power Spectra (M-array)

$$S_{PSK}(f) = 2E\,\mathrm{sinc}^2(Tf)$$

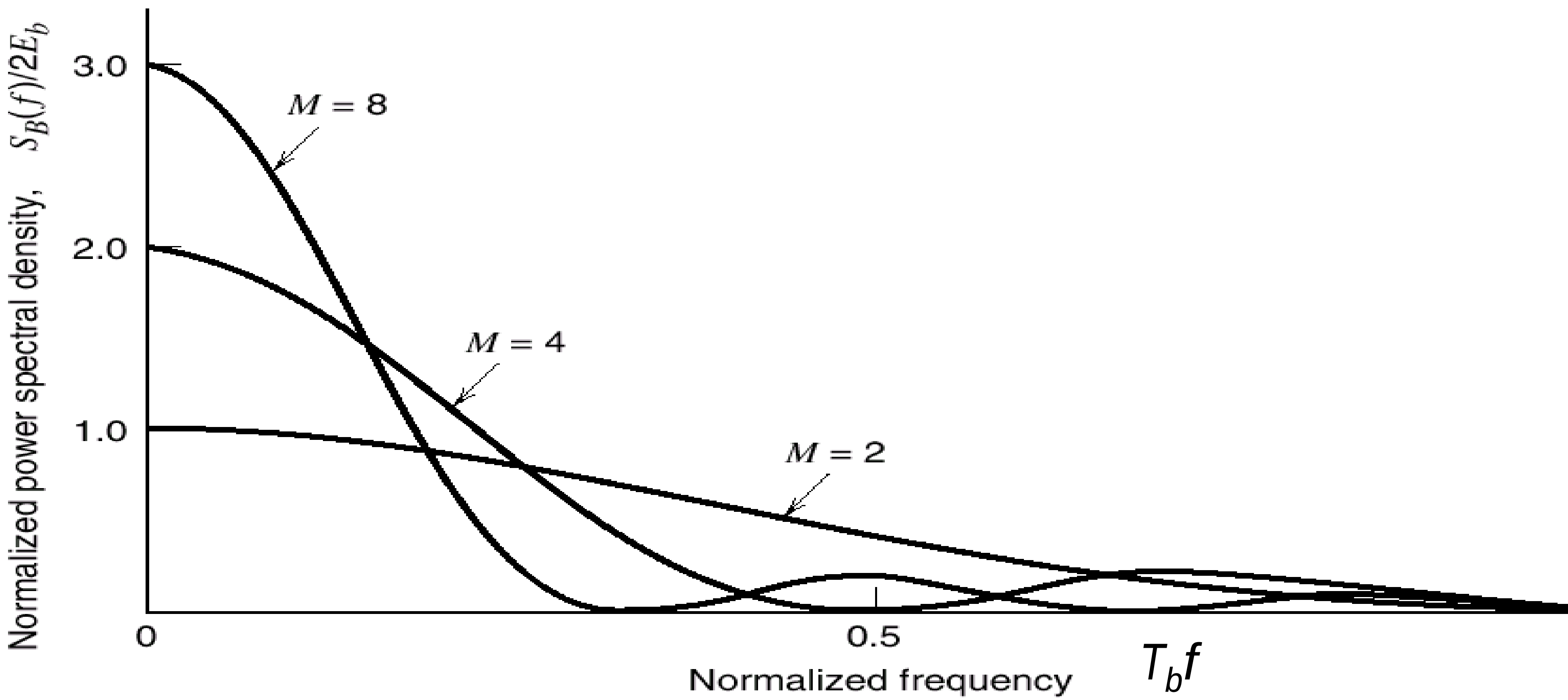
$$= 2E_b \log_2 M\,\mathrm{sinc}^2(T_b f \log_2 M)$$

M=2, we have

$$S_{BPSK}(f) = 2E_b\,\mathrm{sinc}^2(T_b f)$$

M-array PSK

Power spectra of *M*-ary PSK signals for M = 2, 4, 8.



M-array PSK

Bandwidth efficiency:

– We only consider the bandwidth of the main lobe (or null-to-null bandwidth)

$$B = \frac{2}{T} = \frac{2}{T_b \log_2 M} = \frac{2R_b}{\log_2 M}$$

– Bandwidth efficiency of M-ary PSK is given by

$$\rho = \frac{R_b}{B} = \frac{R_b}{2R_b} \log_2 M = 0.5 \log_2 M$$

M-ary QAM

QAM = Quadrature Amplitude Modulation

Both Amplitude and phase of carrier change according to the transmitted symbol, m_i.

$$s_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t); \qquad 0 < t \le T$$

where a_i and b_i are integers.

M-ary QAM

Again, we have

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos \; 2\pi f_c t \quad ;0 < t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \sin \; 2\pi f_c t \quad 0 < t \le T$$

as the basis functions

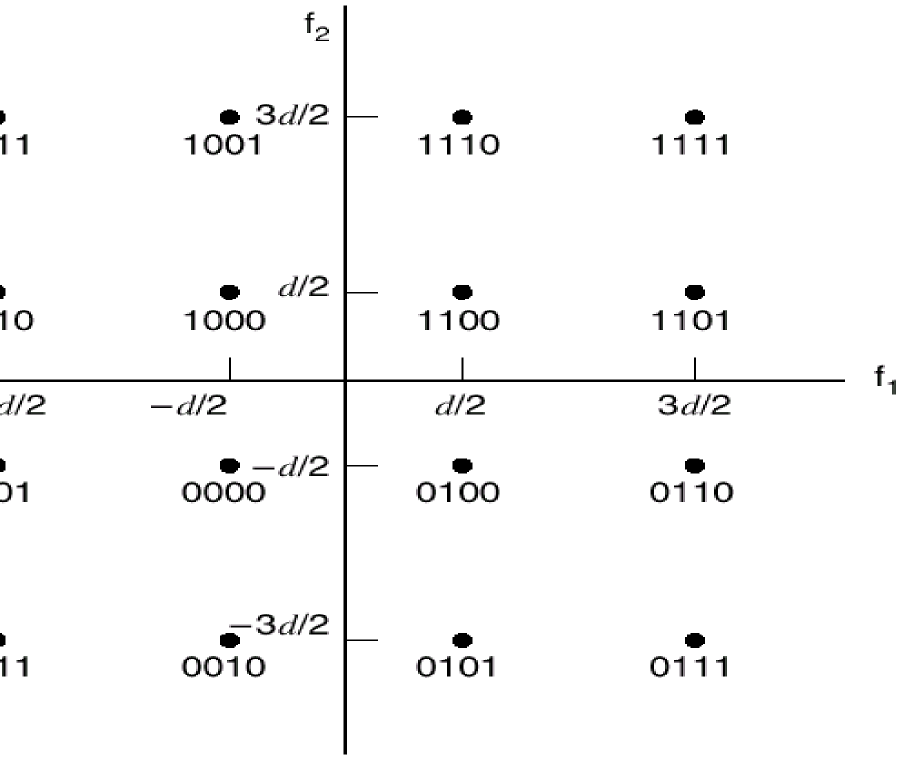
M-ary QAM

- QAM square Constellation
  - Having even number of bits per symbol, denoted by $2n$.
  - M=L x L possible values
  - Denoting

$$L = \sqrt{M}$$

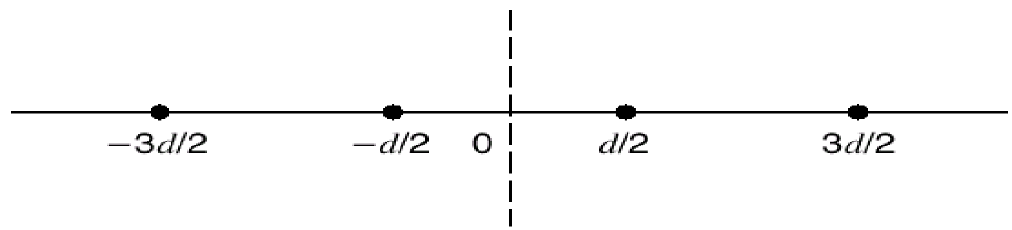
16-QAM

$$[a_i, b_i] = \begin{bmatrix} (-3,3) & (-1,3) & (1,3) & (3,3) \\ (-3,1) & (-1,1) & (1,1) & (3,1) \\ (-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\ (-3,-3) & (-1,-3) & (1,-3) & (3,-3) \end{bmatrix}$$



(a)

16-QAM



(b)

L-ary, 4-PAM

16-QAM

- Calculation of Probability of errors
  - Since both basis functions are orthogonal, we can treat the 16-QAM as combination of two 4-ary PAM systems.
  - For each system, the probability of error is given by

$$P_e' = \left(1 - \frac{1}{L}\right) erfc\left(\frac{d}{2\sqrt{N_0}}\right) = \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{E_0}{N_0}}\right)$$

16-QAM

– A symbol will be received correctly if data transmitted on both 4-ary PAM systems are received correctly. Hence, we have

$$P_c(symbol) = (1 - P_e')^2$$

– Probability of symbol error is given by

$$P_e(symbol) = 1 - P_c(symbol) = 1 - (1 - P_e')^2$$

$$= 1 - 1 + 2P_e' - (P_e')^2 \approx 2P_e'$$

16-QAM

– Hence, we have

$$P_e(symbol) = 2\left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{E_0}{N_0}}\right)$$

– But because average energy is given by

$$E_{av} = 2\left[\frac{2E_0}{L}\sum_{i=1}^{L/2}(2i-1)^2\right] = \frac{2(M-1)E_0}{3}$$

– We have

$$P_e(symbol) = 2\left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)$$

Coherent FSK

FSK = frequency shift keying

Coherent = receiver have information on where the zero phas of carrier.

We can treat it as non-linear modulation since information is put into the frequency.

Binary FSK

Transmitted signals are

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 < t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

where

$$f_i = \frac{n_c + i}{T_b}; \quad i = 1, 2$$

Binary FSK

$S_1(t)$ represented symbol “1”.
$S_2(t)$ represented symbol “0”.

This FSK is also known as Sunde’s FSK.

It is continuous phase frequency-shift keying (CPFSK).

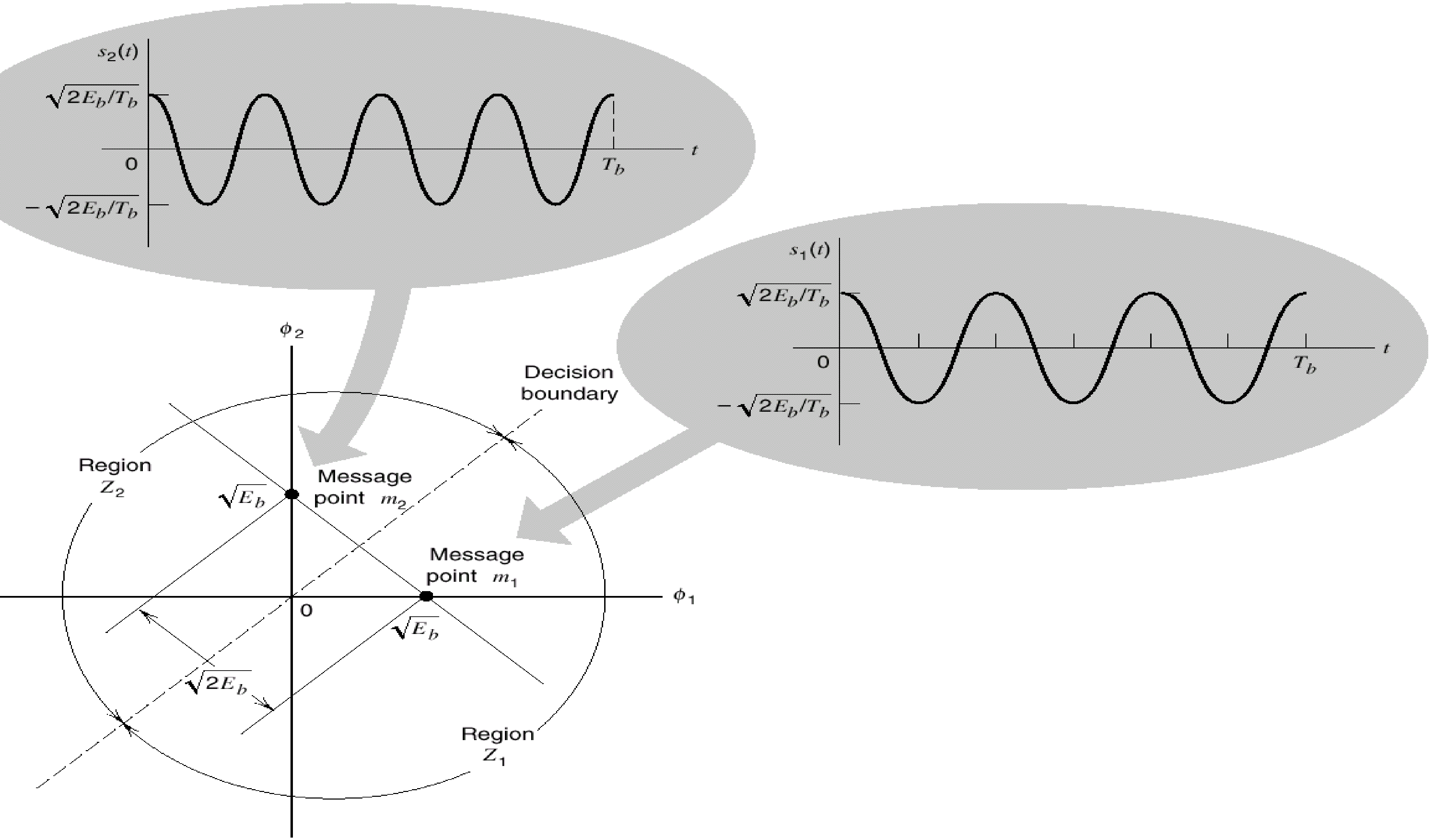
Binary FSK

There are two basis functions written as

$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t), & 0 < t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

As a result, the signal vectors are

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$



$s_2(t)$
$\sqrt{2E_b/T_b}$
0
$-\sqrt{2E_b/T_b}$
T_b
t
$s_1(t)$
$\sqrt{2E_b/T_b}$
0
$-\sqrt{2E_b/T_b}$
T_b
t
ϕ_2
Decision boundary
Region Z_2
$\sqrt{E_b}$
Message point m_2
Message point m_1
ϕ_1
0
$\sqrt{E_b}$
$\sqrt{2E_b}$
Region Z_1

BFSK

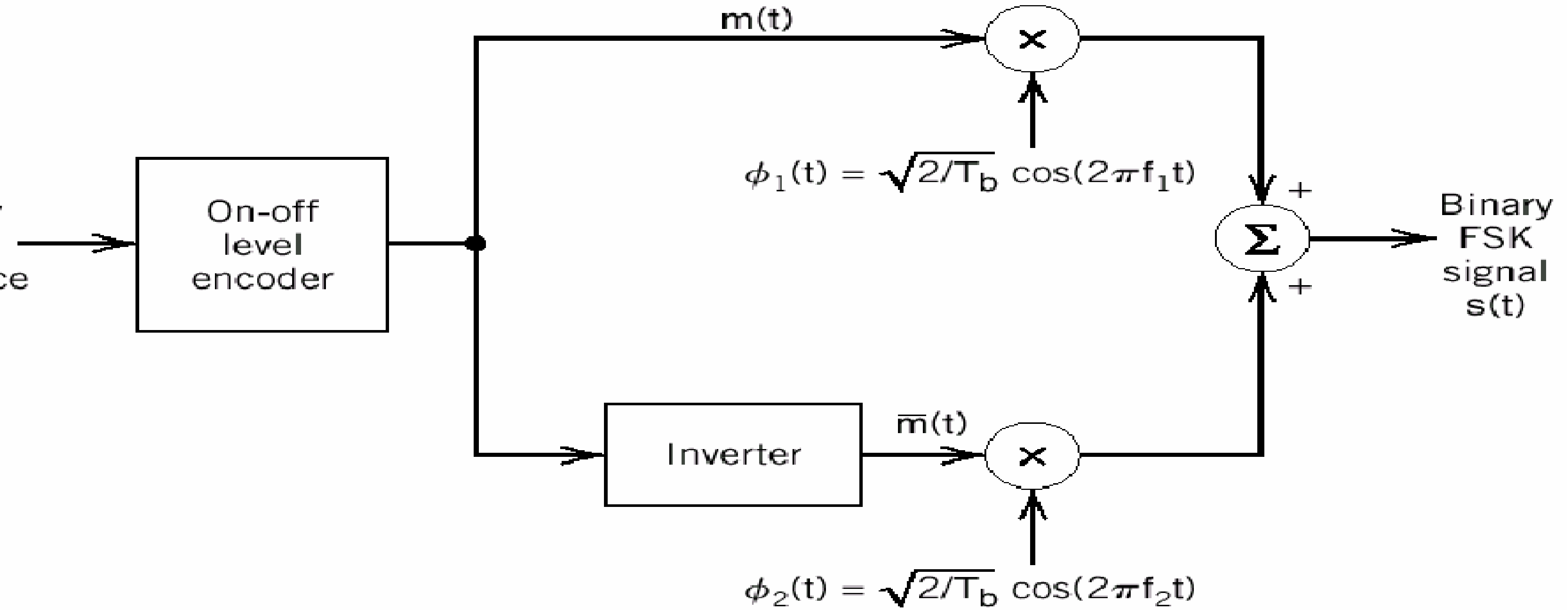
From the figure, we have $d_{12} = \sqrt{2E_b}$

In case of Pr(0)=Pr(1), the probability of error is given by

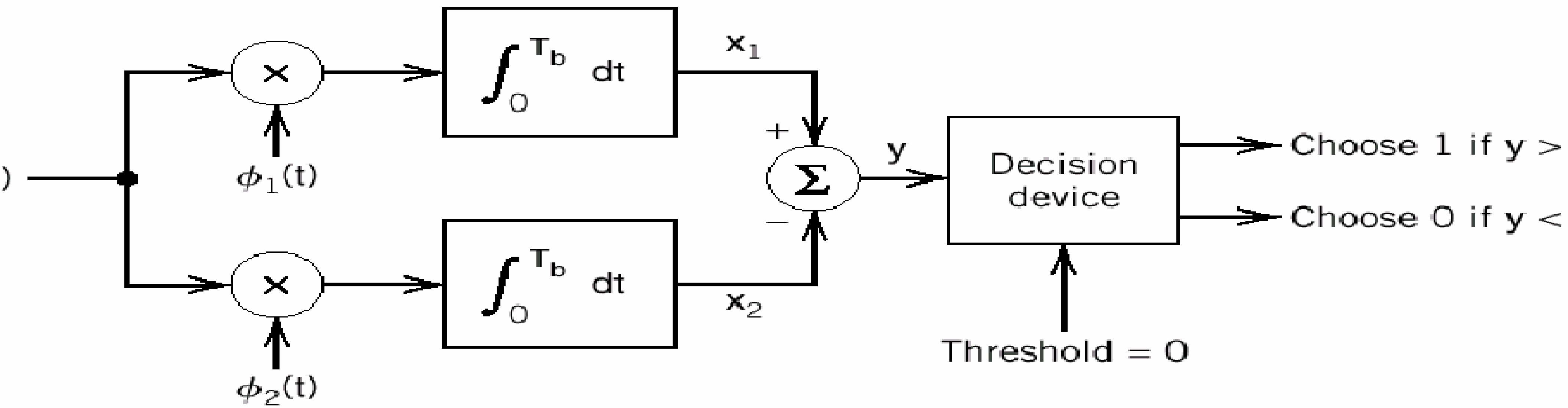
$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

We observe that at a given value of P_e, the BFSK system requires twice as much power as the BPSK system.

TRANSMITTER



(a)


)
×
×
$\phi_1(t)$
$\phi_2(t)$
$\int_0^{T_b} dt$
$\int_0^{T_b} dt$
x_1
x_2
+
−
Σ
y
Decision device
Threshold = 0
Choose 1 if y >
Choose 0 if y <

(b)

RECEIVER

Power Spectral density of BFSK

Consider the Sunde's FSK where f_1 and f_2 are different by $1/T_b$. We can write

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right)$$
$$= \sqrt{\frac{2E_b}{T_b}} \cos\left(\pm \frac{\pi t}{T_b}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin\left(\pm \frac{\pi t}{T_b}\right) \sin(2\pi f_c t)$$

We observe that in-phase component does not depend on m_i since

$$\sqrt{\frac{2E_b}{T_b}} \cos\left(\pm \frac{\pi t}{T_b}\right) = \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right)$$

Power Spectral density of BFSK

Half of the symbol power

We have

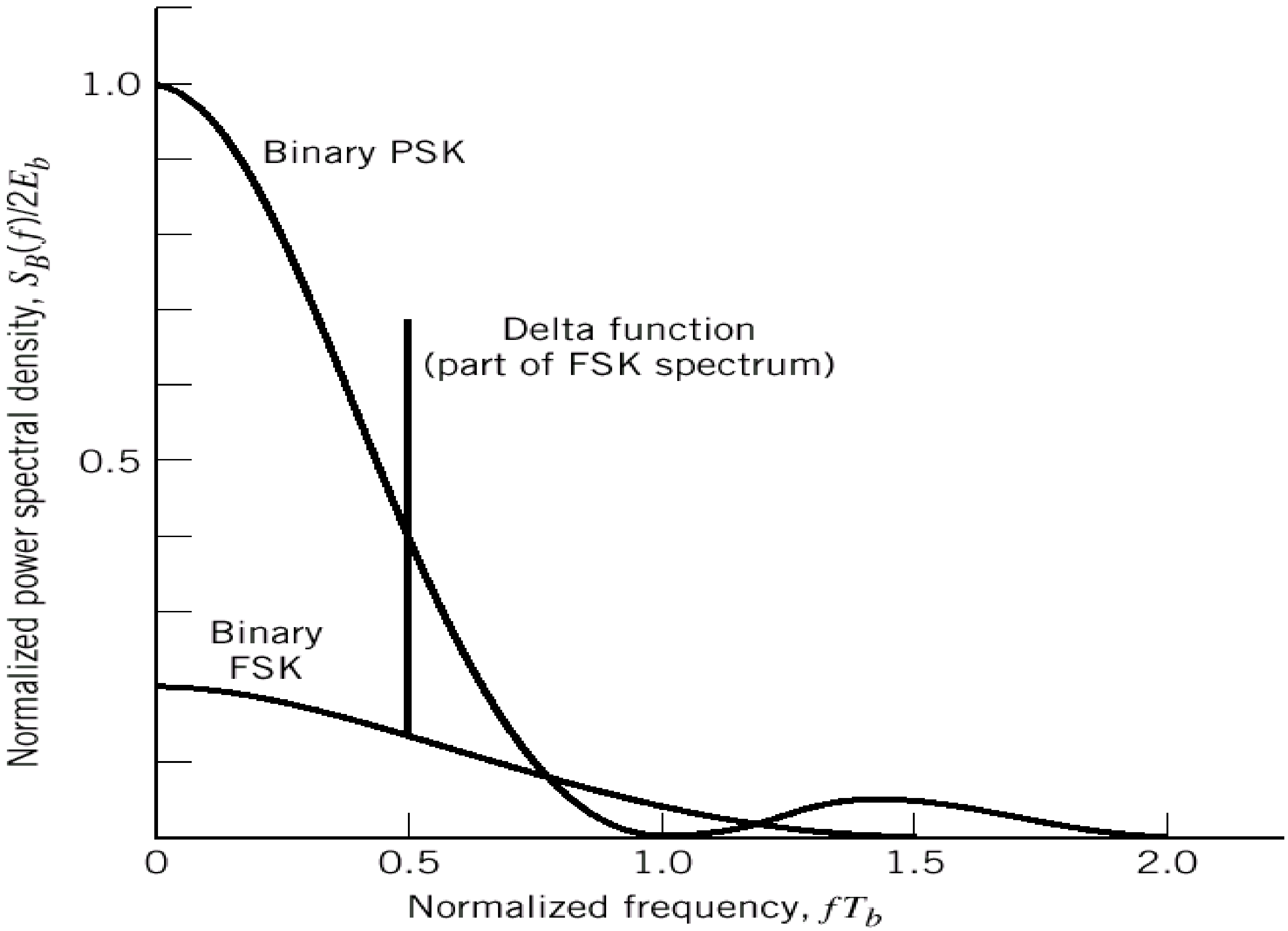
$$S_{BI}(f) = \left| F\left\{ \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right) \right\} \right|^2 = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b} \right) + \delta\left(f + \frac{1}{2T_b} \right) \right]$$

For the quadrature component

$$g(t) = \sqrt{\frac{2E_b}{T_b}} \sin\left(\pm \frac{\pi t}{T_b} \right) \quad \Longrightarrow \quad S_{BQ} = \frac{8E_b T_b \cos^2(\pi T_b f)}{\pi^2 \left(4T_b^2 f^2 - 1\right)^2}$$

Power Spectral density of BFSK

Finally, we obtain $S_B(f) = S_{BI}(f) + S_{BQ}(f)$



Phase Tree of BFSK

FSK signal is given by

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right)$$

At t = 0, we have

$$s(0) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c 0 \pm \frac{\pi 0}{T_b}\right) = \sqrt{\frac{2E_b}{T_b}} \cos(0)$$

The phase of Signal is zero.

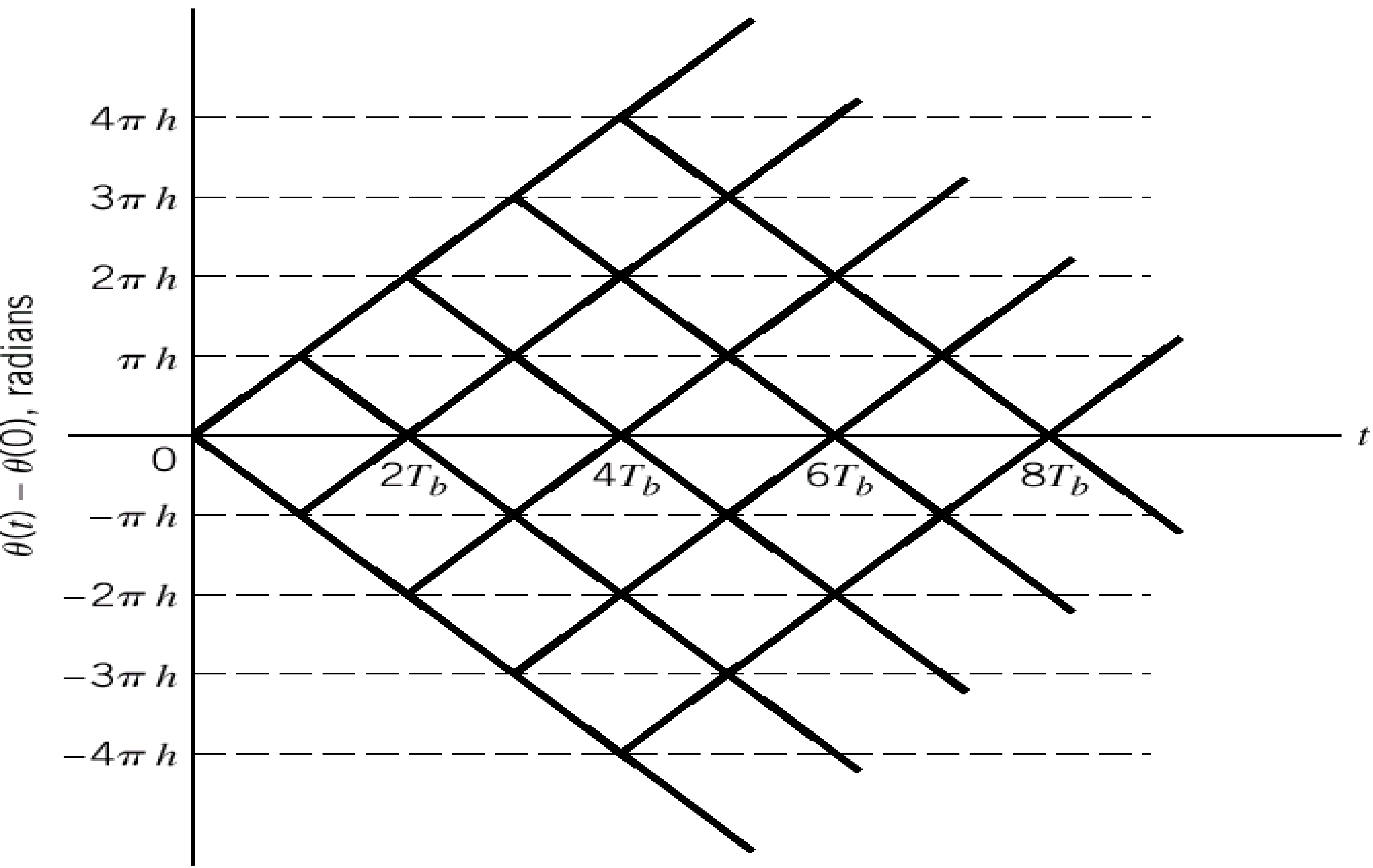
Phase Tree of BFSK

At t = T_b, we have

$$s(T_b) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c T_b \pm \frac{\pi T_b}{T_b}\right) = \sqrt{\frac{2E_b}{T_b}} \cos(\pm\pi)$$

We observe that phase changes by $\pm\pi$ after one symbol (T_b seconds). -
for symbol “1” and $+\pi$ for symbol “0”

We can draw the phase trellis as



$4\pi h$
$3\pi h$
$2\pi h$
πh
0
$-\pi h$
$-2\pi h$
$-3\pi h$
$-4\pi h$
$2T_b$
$4T_b$
$6T_b$
$8T_b$
t
$\theta(t) - \theta(0)$, radians

Minimum-Shift keying (MSK)

MSK tries to shift the phase after one symbol to just half of Sunde's FSK system. The transmitted signal is given by

$$) = \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_c t + \theta(t)\right] = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_1 t + \theta(0)\right] & \text{for "1"} \\ \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_2 t + \theta(0)\right] & \text{for "0"} \end{cases}$$

MSK

Where

$$\theta(t) = \theta(0) \pm \frac{\pi h}{T_b} t$$

Observe that

$$f_1 = f_c + \frac{h}{2T_b} \quad \text{and} \quad f_2 = f_c - \frac{h}{2T_b}$$

$$\Downarrow$$

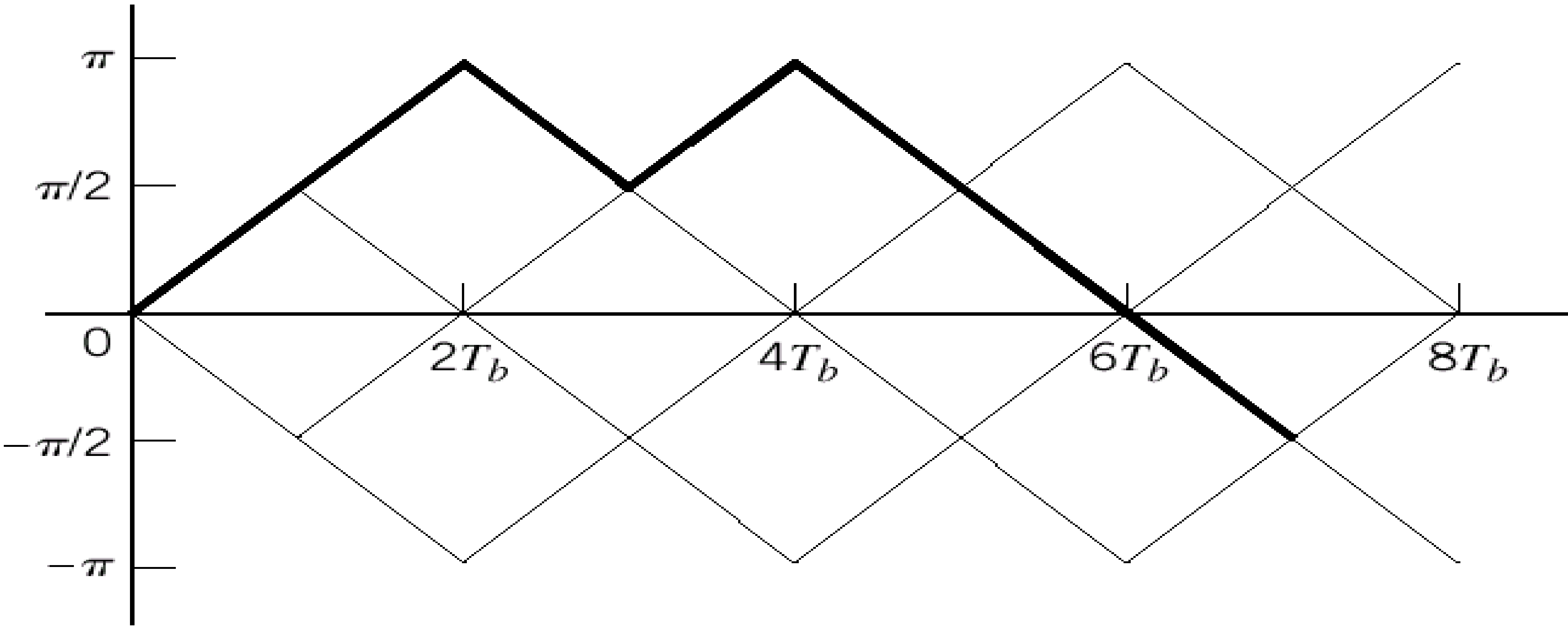
$$f_c = \frac{1}{2}(f_1 + f_2)$$

MSK

- $h = T_b(f_1 - f_2)$ is called "deviation ratio."
- For Sunde's FSK, $h = 1$.
- For MSK, $h = 0.5$.
- h cannot be any smaller because the orthogonality between $\cos(2\pi f_1 t)$ and $\cos(2\pi f_2 t)$ is still held for $h < 0.5$.
- Orthogonality guarantees that both signal will not interfere each other in detection process.

MSK

Phase trellis diagram for MSK signal 1101000



MSK

Signal *s*(t) of MSK can be decomposed into

$$
\begin{aligned}
s(t) &= \sqrt{\frac{2E_b}{T_b}}\cos\left[2\pi f_c t + \theta(t)\right] \\
&= \sqrt{\frac{2E_b}{T_b}}\cos\left[\theta(t)\right]\cos\left(2\pi f_c t\right) - \sqrt{\frac{2E_b}{T_b}}\sin\left[\theta(t)\right]\sin\left(2\pi f_c t\right) \\
&= s_I(t)\cos\left(2\pi f_c t\right) - s_Q(t)\sin\left(2\pi f_c t\right)
\end{aligned}
$$

where

$$
\theta(t) = \theta(0) \pm \frac{\pi}{2T_b}t \qquad ;0 < t \le T_b
$$

MSK

Symbol	$\theta(0)$	$\theta(T_b)$
1	0	$\pi/2$
	π	$-\pi/2$
0	0	$-\pi/2$
	π	$\pi/2$

MSK

For the interval $-T_b < t \le 0$, we have

$$\theta(t) = \theta(0) \pm \frac{\pi}{2T_b}t \quad ; -T_b < t \le 0$$

Let's note here that the $\pm$ for the interval $-T_b < t \le 0$ and $0 < t \le T_b$ ma
not be the same.

We know that

$$\cos\left[\theta(0) \pm \frac{\pi t}{2T_b}\right] = \cos[\theta(0)]\cos\left(\frac{\pi t}{2T_b}\right) \mp \sin[\theta(0)]\sin\left(\frac{\pi t}{2T_b}\right)$$

MSK

Since θ(0) can be either 0 or π depending on the past history. We have

$$\cos\left[\theta(0) \pm \frac{\pi t}{2T_b}\right] = \cos[\theta(0)]\cos\left(\frac{\pi t}{2T_b}\right) = \pm\cos\left(\frac{\pi t}{2T_b}\right)$$

"+" for θ(0) = 0 and "-" for θ(0) = π

Hence, we have

$$s_I(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi t}{2T_b}\right) \quad ; -T_b < t \le T_b$$

MSK

Similarly we can write

$$\theta(t) = \theta(T_b) \pm \frac{\pi}{2T_b}(t - T_b)$$

for 0< $t \le T_b$ and $T_b < t \le 2T_b$. Note the "+" and "-" may be different between these intervals.

Furthermore, we have that $\theta(T_b)$ can be $\pm\pi/2$ depending on the past history.

MSK

Hence, we have

$$\sin\left[\theta(T_b) \pm \frac{\pi(t-T_b)}{2T_b}\right] = \sin[\theta(T_b)]\cos\left(\frac{\pi(t-T_b)}{2T_b}\right) \pm \cos[\theta(T_b)]\sin\left(\frac{\pi(t-T_b)}{2T_b}\right)$$

$$= \sin[\theta(T_b)]\cos\left(\frac{\pi t}{2T_b} - \frac{\pi}{2}\right) \pm \cos[\theta(T_b)]\sin\left(\frac{\pi t}{2T_b} - \frac{\pi}{2}\right)$$

we have that $\theta(T_b)$ can be $\pm\pi/2$ depending on the past history.

$$\sin\left[\theta(T_b) \pm \frac{\pi(t-T_b)}{2T_b}\right] = \pm\cos\left(\frac{\pi t}{2T_b} - \frac{\pi}{2}\right) = \pm\sin\left(\frac{\pi t}{2T_b}\right)$$

MSK

Hence, we have

$$s_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi t}{2T_b}\right) \quad ;0 < t \le 2T_b$$

"+" for $\theta(T_b) = +\pi/2$ and "-" for $\theta(T_b) = -\pi/2$

The basis functions change to

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos\left(\frac{\pi t}{2T_b}\right)\cos\left(2\pi f_c t\right) \quad ;0 < t \le T_b$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}}\sin\left(\frac{\pi t}{2T_b}\right)\sin\left(2\pi f_c t\right) \quad ;0 < t \le T_b$$

MSK

We write MSK signal as

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\cos[\theta(t)]\cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}}\sin[\theta(t)]\sin(2\pi f_c t)$$

$$= \sqrt{\frac{2E_b}{T_b}}\cos[\theta(0)]\cos\left(\frac{2\pi t}{T_b}\right)\cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}}\sin[\theta(T_b)]\sin\left(\frac{2\pi t}{T_b}\right)\sin(2\pi f_c t)$$

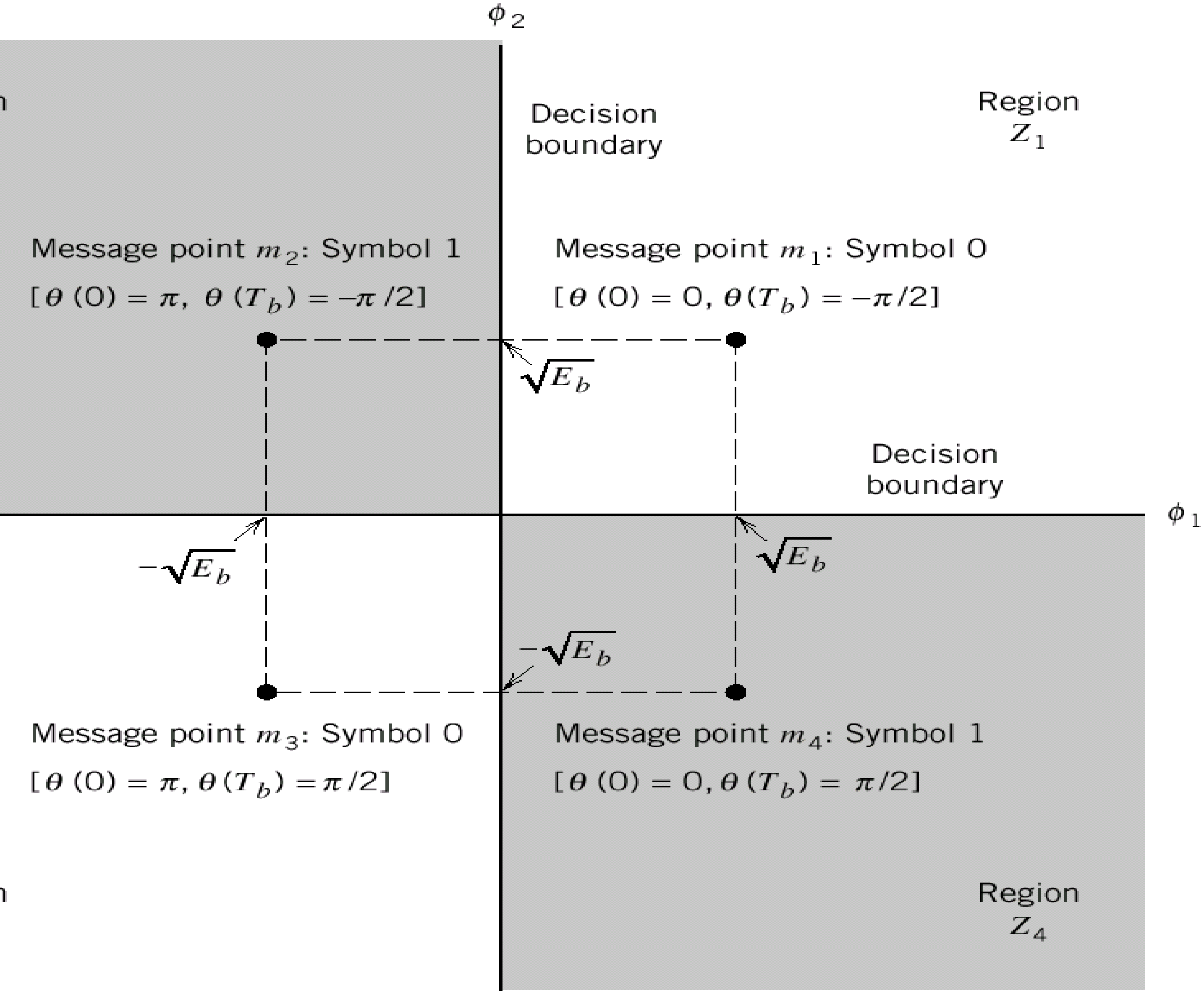
$$= \sqrt{\mathrm{E_b}}\cos[\theta(0)]\phi_1(t) - \sqrt{\mathrm{E_b}}\sin[\theta(T_b)]\phi_2(t)$$

$$= s_1\phi_1(t) + s_2\phi_2(t)$$

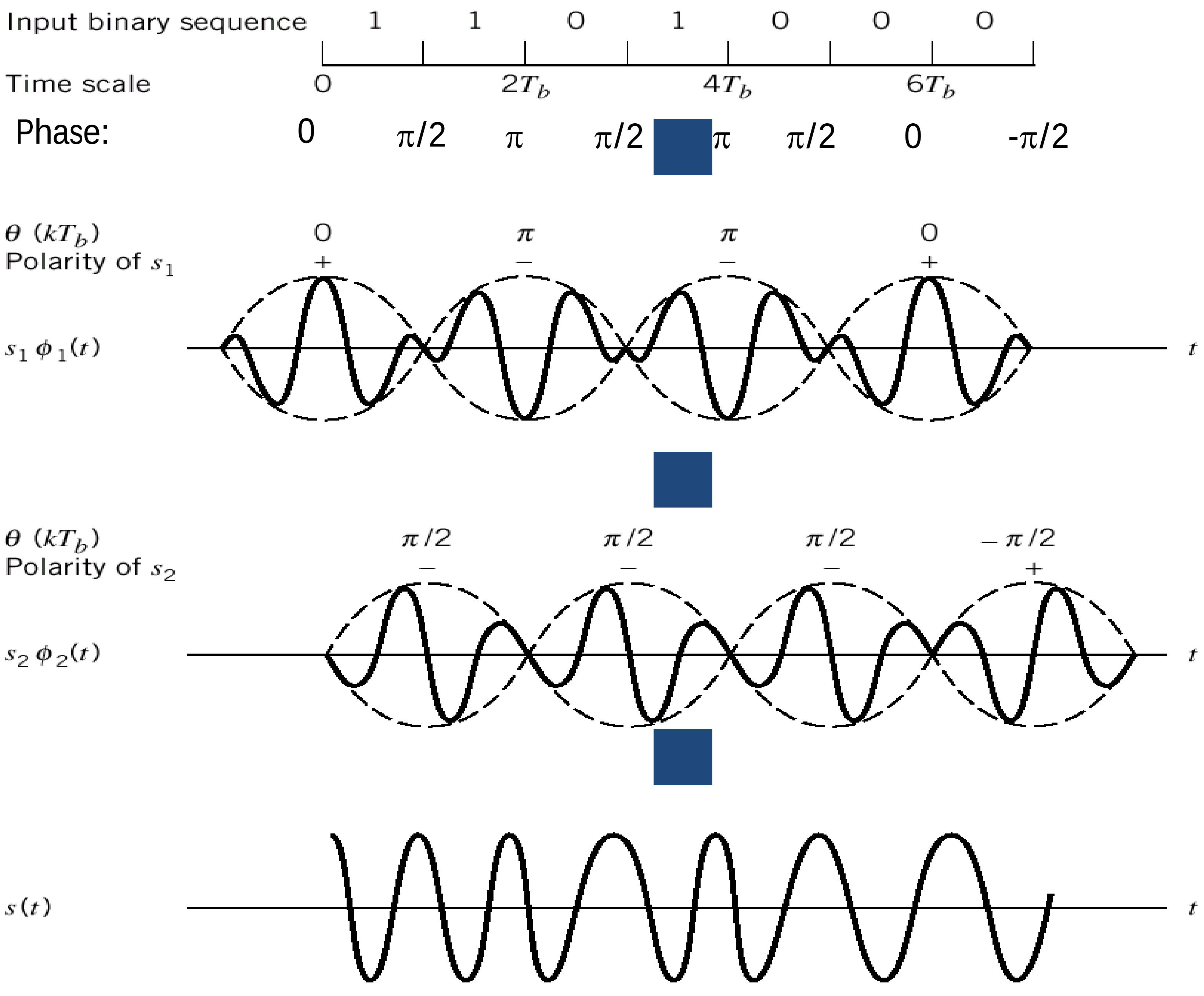
Where $s_1 = \sqrt{E_b}\cos[\theta(0)]$ and $s_2 = -\sqrt{E_b}\sin[\theta(T_b)]$

MSK

Symbol	$\theta(0)$	s_1	$\theta(T_b)$	s_2
1	0	$\sqrt{E_b}$	$\pi/2$	$-\sqrt{E_b}$
	π	$-\sqrt{E_b}$	$-\pi/2$	$\sqrt{E_b}$
0	0	$\sqrt{E_b}$	$-\pi/2$	$-\sqrt{E_b}$
	π	$-\sqrt{E_b}$	$\pi/2$	$\sqrt{E_b}$



$$P_e = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

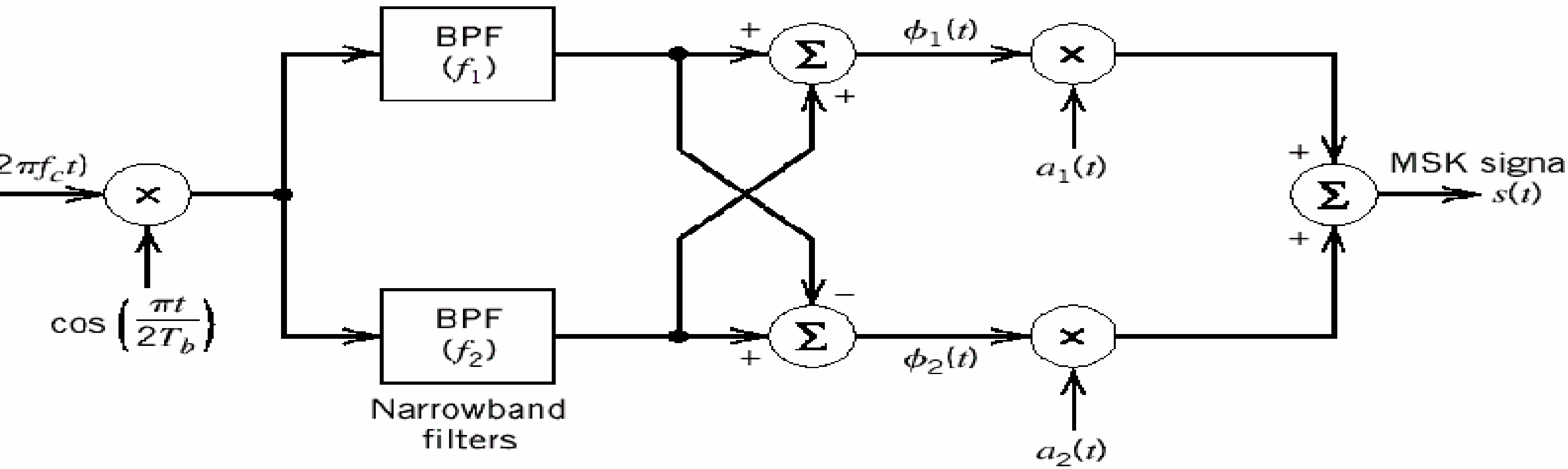

Input binary sequence
1 1 0 1 0 0 0
Time scale
0
2T_b
4T_b
6T_b
Phase:
0 π/2 π π/2 π π/2 0 -π/2
θ (kT_b)
Polarity of s_1
0 π π 0
+ – – +
$s_1 \phi_1(t)$
t
θ (kT_b)
Polarity of s_2
π/2 π/2 π/2 –π/2
– – – +
$s_2 \phi_2(t)$
t
$s(t)$
t

(d)

MSK

We observe that MSK is in fact the QPSK having the pulse shape $\cos\left(\frac{\pi t}{2T_b}\right)$

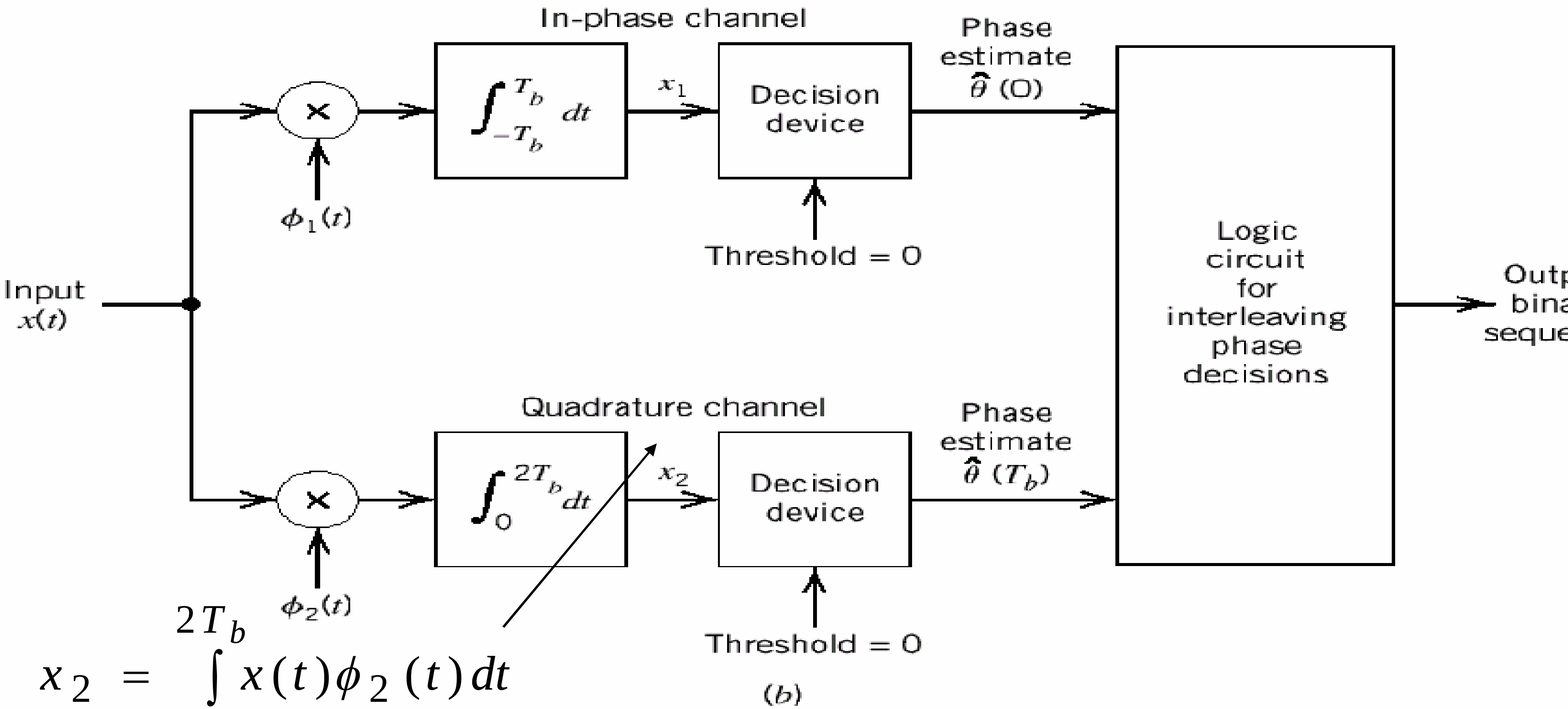
Block diagrams for transmitter and receiver are given in the next two slides.



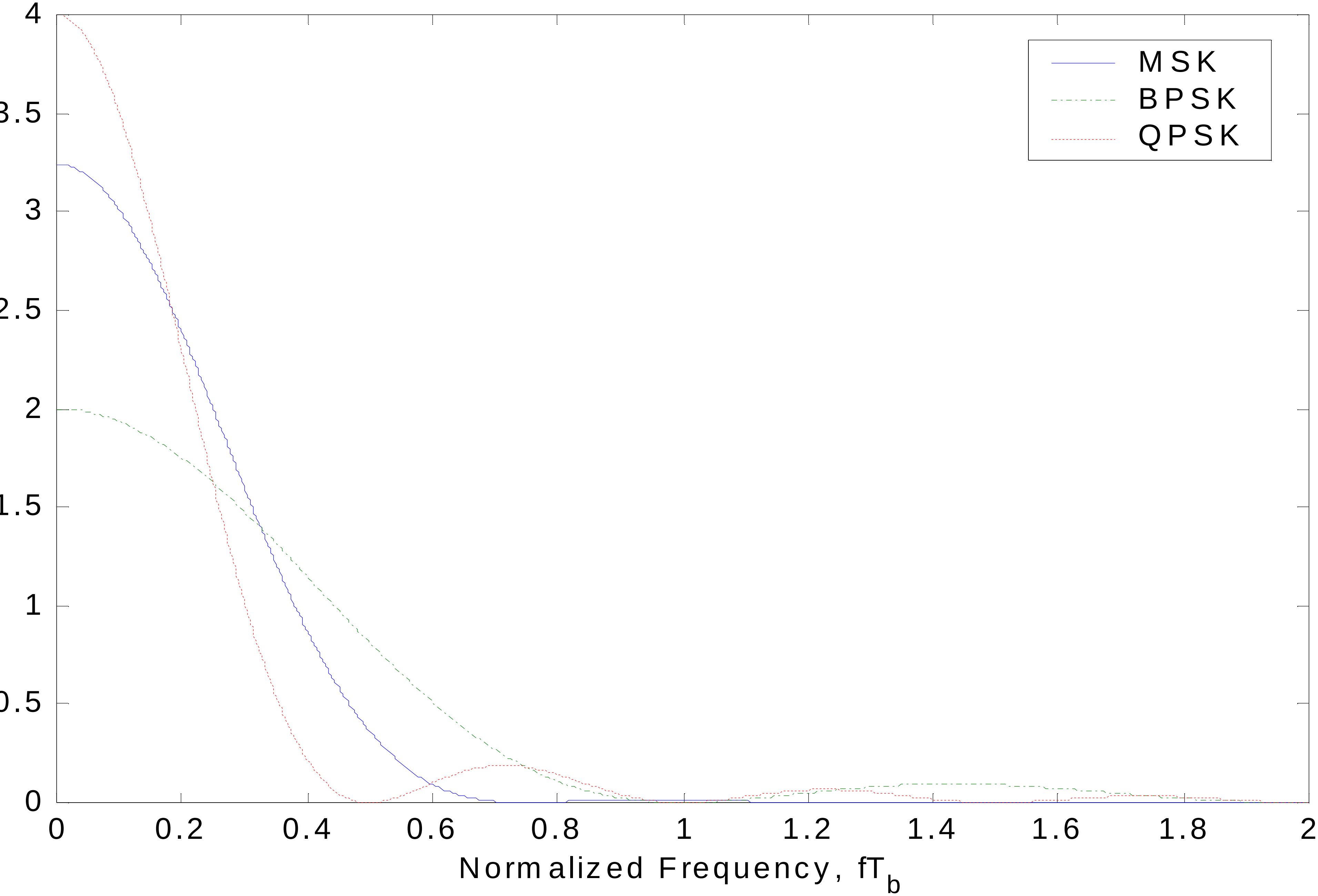
$2\pi f_c t)$
$\cos\left(\frac{\pi t}{2T_b}\right)$
BPF (f_1)
BPF (f_2)
Narrowband filters
$\phi_1(t)$
$\phi_2(t)$
$a_1(t)$
$a_2(t)$
MSK signal $s(t)$

(a)

$$x_1 = \int_{-T_b}^{T_b} x(t)\phi_1(t)\,dt$$



$$x_2 = \int_0^{2T_b} x(t)\phi_2(t)\,dt$$



MSK
BPSK
QPSK
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Normalized Frequency, fT_b

MSK

Probability of error of MSK system is equal to BPSK and QPSK

This due to the fact that MSK observes the signal for two symbol intervals whereas FSK only observes for single signal interval.

Bandwidth of MSK system is 50% larger than QPSK.

$$S_{MSK}(f) = \frac{32 E_b}{\pi^2} \left[\frac{\cos(2\pi T_b f)}{16 T_b^2 f^2 - 1} \right]^2$$

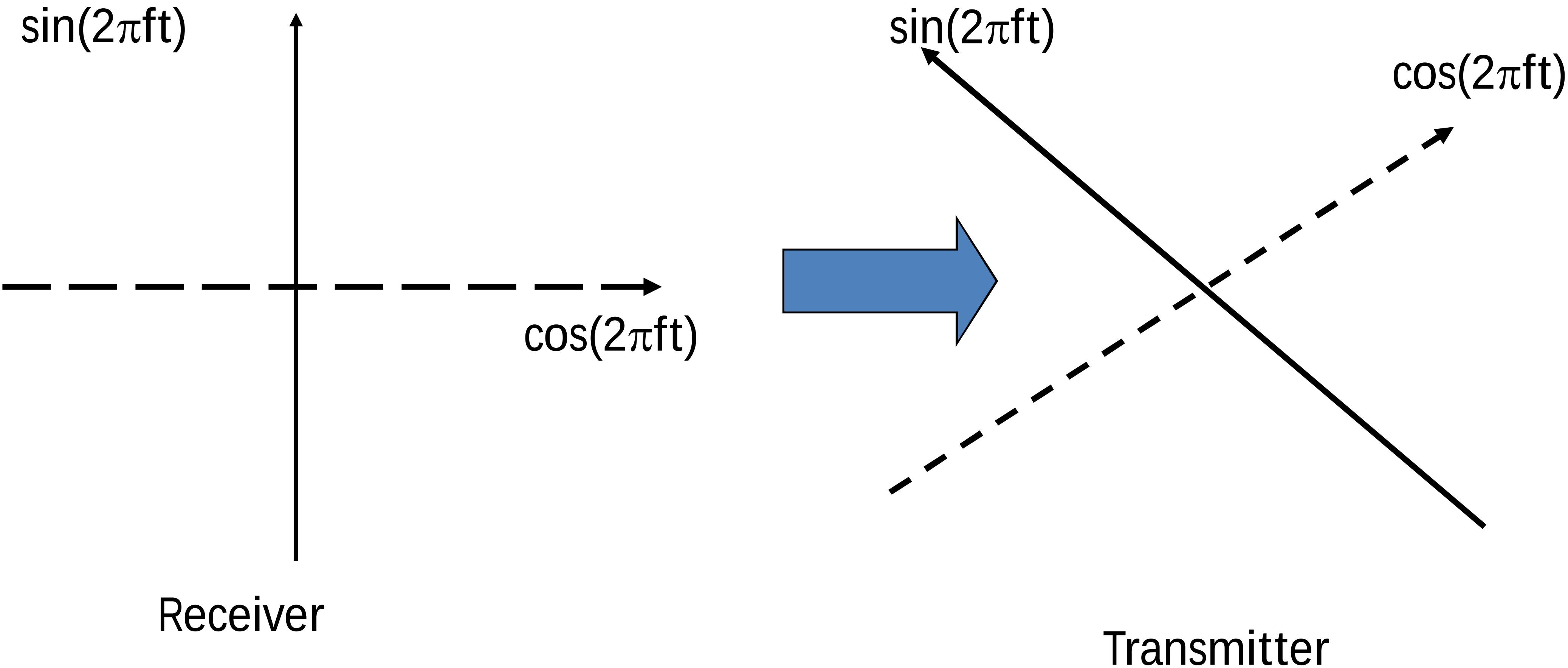
Noncoherent Orthogonal Modulation

Noncoherent implies that phase information is not available to the receiver.

As a result, zero phase of the receiver can mean any phase of the transmitter.

Any modulation techniques that transmits information through the phase cannot be used in noncoherent receivers.

Noncoherent Orthogonal Modulation



Noncoherent Orthogonal Modulation

- It is impossible to draw the signal constellation since we do not know where the axes are.
- However, we can still determine the distance of the each signal constellation from the origin.
- As a result, the modulation techniques that put information in the amplitude can be detected.
- FSK uses the amplitude of signals in two different frequencies. Hence non-coherent receivers can be employed.

Noncoherent Orthogonal Modulation

Consider the BFSK system where two frequencies f_1 and f_2 are used to represented two “1” and “0”.

The transmitted signal is given by

$$s(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_i t + \theta\right) \quad ; i = 1,2, 0 < t \le T_b$$

Problem is that θ is unknown to the receiver. For the coherent receiver θ is precisely known by receiver.

Noncoherent Orthogonal Modulation

Furthermore, we have

$$\begin{aligned} s(t) &= \sqrt{\frac{2E}{T}} \cos\left(2\pi f_i t + \theta\right) \\ &= \sqrt{\frac{2E}{T}} \cos(\theta) \cos\left(2\pi f_i t\right) + \sqrt{\frac{2E}{T}} \sin(\theta) \sin\left(2\pi f_i t\right) \\ &= s_{i1}\phi_1(t) + s_{i2}\phi_2(t) \end{aligned}$$

To get rid of the phase information (θ), we use the amplitude

$$\|s(t)\| = \sqrt{s_{i1}^2 + s_{i2}^2} = \sqrt{E\cos^2(\theta) + E\sin^2(\theta)} = E$$

Noncoherent Orthogonal Modulation

Where

$$s_{i1} = \int_0^T s(t)\phi_1(t)\,dt \;\Rightarrow\; x_1 = \int_0^T x(t)\phi_1(t)\,dt$$

$$s_{i2} = \int_0^T s(t)\phi_2(t)\,dt \;\Rightarrow\; x_2 = \int_0^T x(t)\phi_2(t)\,dt$$

The amplitude of the received signal

$$l_i = \left[\left(\int_0^T x(t)\cos\left(2\pi f_i t\right)dt\right)^2 + \left(\int_0^T x(t)\sin\left(2\pi f_i t\right)dt\right)^2\right]^{1/2}$$

Noncoherent Orthogonal Modulation

Probability of Errors

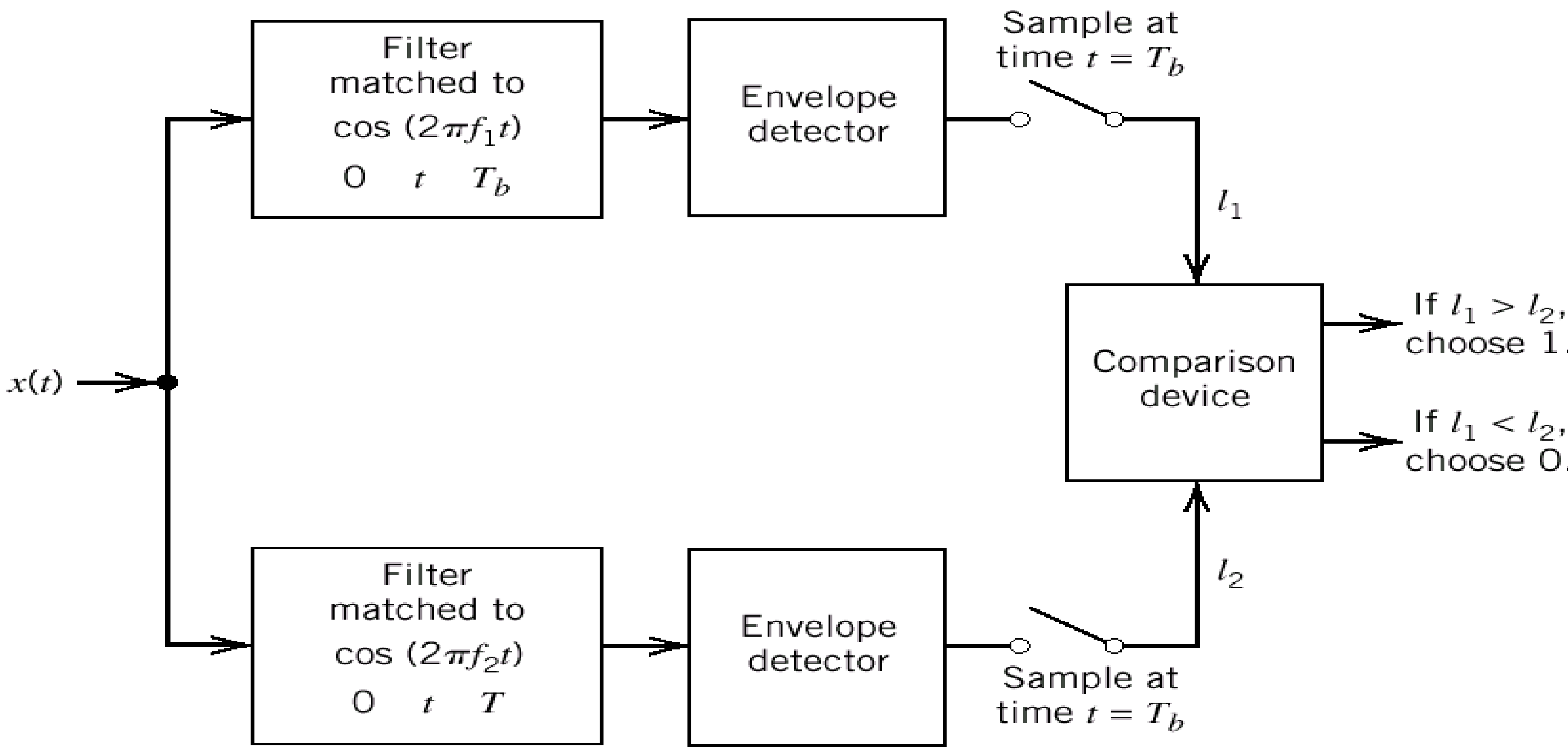
$$P_e = \frac{1}{2}\exp\left(-\frac{E}{2N_0}\right)$$

Noncoherent: BFSK

For BFSK, we have

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t); & 0 < t \le T_b \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Noncoherent: BFSK



Noncoherent: BFSK

Probability of Errors

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

DPSK

- Differential PSK
  - Instead of finding the phase of the signal on the interval $0<t\leq T_b$. Th receiver determines the phase difference between adjacent time intervals.
  - If “1” is sent, the phase will remain the same
  - If “0” is sent, the phase will change 180 degree.

DPSK

Or we have

$$s_1(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos\left(2\pi f_c t\right); & 0 < t \le 2T_b \\ \sqrt{\dfrac{E_b}{2T_b}} \cos\left(2\pi f_c t\right); & T_b < t \le 2T_b \end{cases}$$

and

$$s_2(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos\left(2\pi f_c t\right); & 0 < t \le 2T_b \\ \sqrt{\dfrac{E_b}{2T_b}} \cos\left(2\pi f_c t + \pi\right); & T_b < t \le 2T_b \end{cases}$$

DPSK

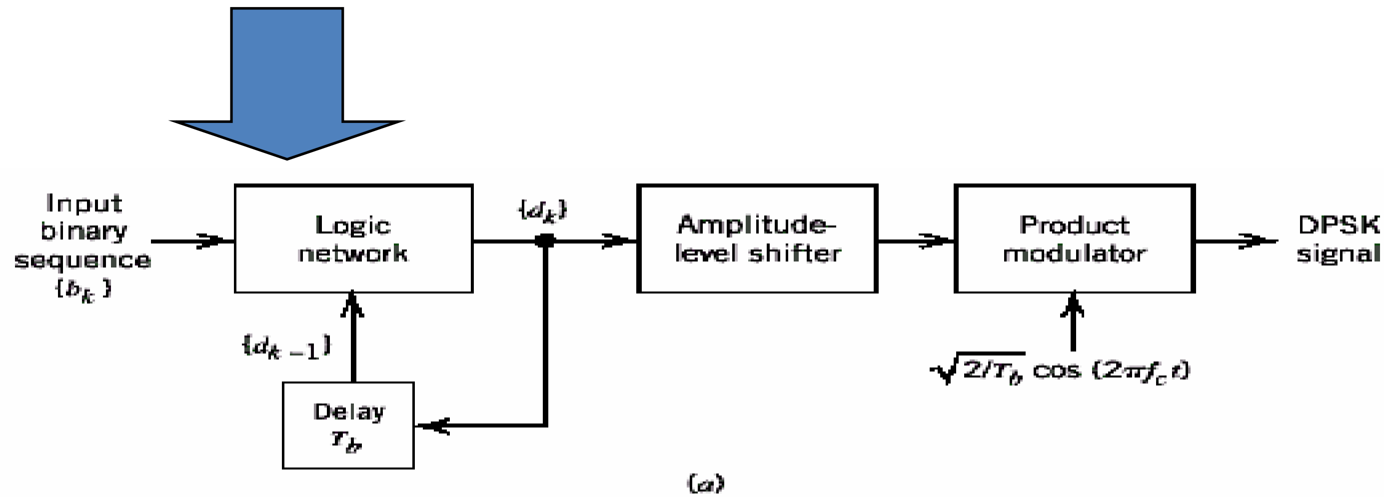
In this case, we have $T=2T_b$ and $E=2E_b$

Hence, the probability of error is given by

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

DPSK: Transmitter

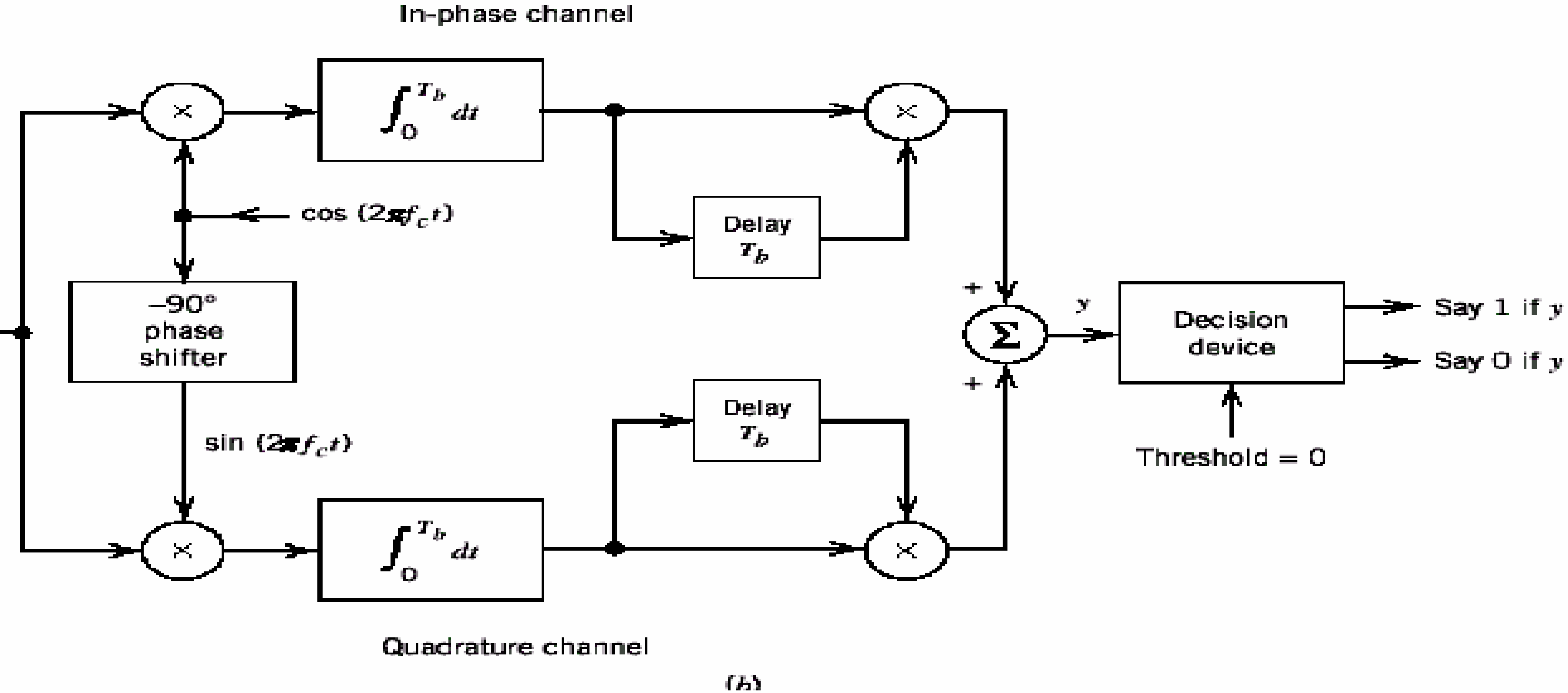
$$d_k = b_k d_{k-1} \oplus \overline{b_k}\,\overline{d}_{k-1}$$



DPSK

{b_k}		1	0	0	1	0	0	0	1	1
{$d_{k\text{-}1}$}		1								
Differential encoded {d_k}	1									
Transmitted Phase	0									

DPSK: Receiver



DPSK: Receiver

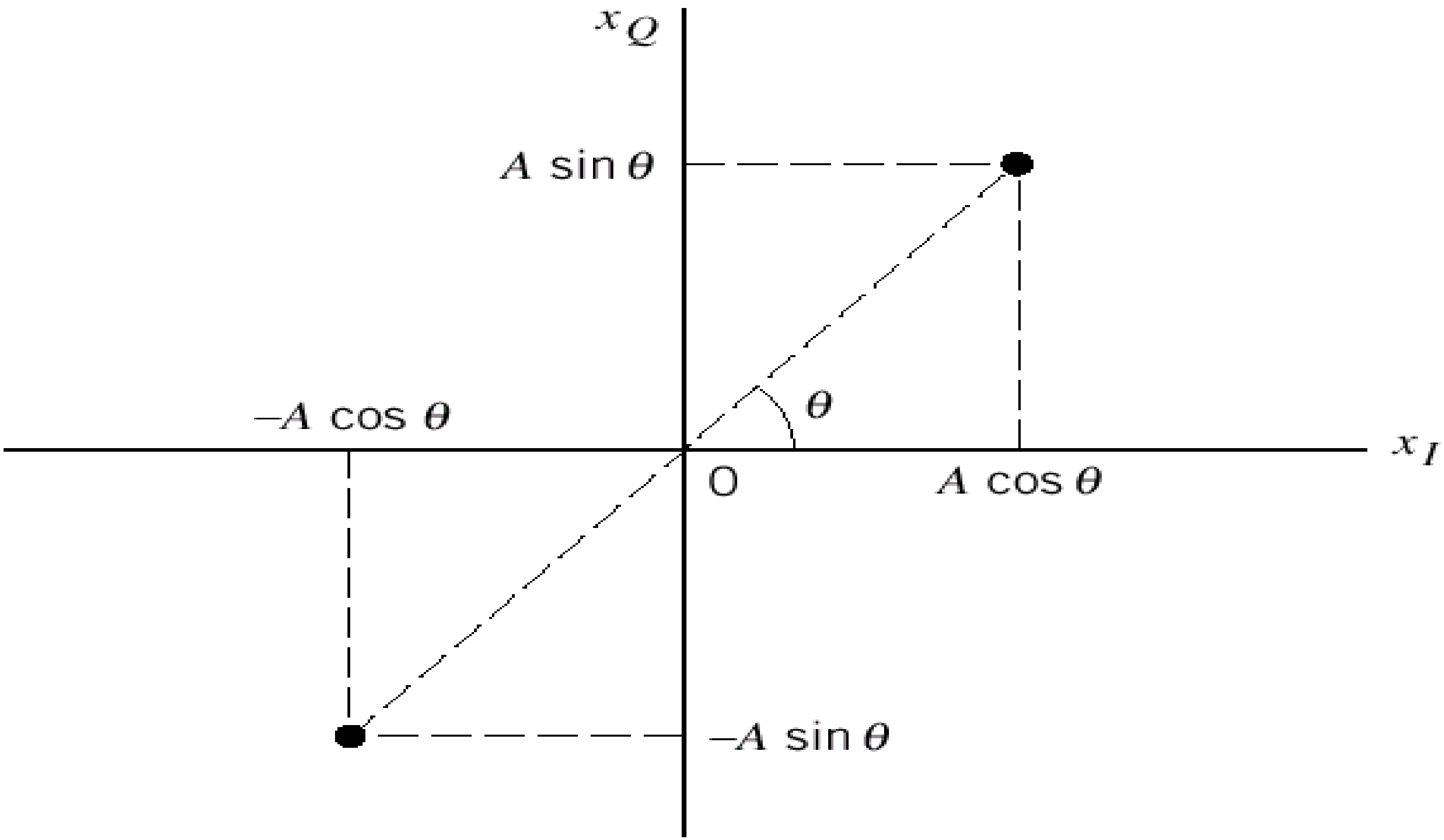
From the block diagram, we have that the decision rule as

$$l(\mathbf{x}) = x_{I0}x_{I1} + x_{Q0}x_{Q1} \underset{\text{say } 0}{\overset{\text{say } 1}{\gtrless}} 0$$

If the phase of signal is unchanged (send "1") the sign ("+" or "-") of both x_i and x_Q should <u>not</u> change. Hence, the $l(\mathbf{x})$ should be positive.

If the phase of signal is unchanged (send "0") the sign ("+" or "-1") of both x_i and x_Q should change. Hence, the $l(\mathbf{x})$ should be negative.

Signal-space diagram of received DPSK signal.



Unit-V –Introduction to Spread Spectrum Techniques

M-ary signaling scheme:

In this signaling scheme 2 or more bits are grouped together to form a symbol.

One of the M possible signals $s_1(t), s_2(t), s_3(t), \ldots\ldots s_M(t)$ is transmitted during each symbol period of duration T_s.

The number of possible signals = $M = 2^n$, where n is an integer.

The symbol values of M for a given value of n:

n	$M = 2^n$	Symbol
1	2	0, 1
2	4	00, 01, 10, 11
3	8	000, 001, 010,011,...
4	16	0000, 0001, 0010,0011,….
….	……	……….

- Depending on the variation of amplitude, phase or frequency of the carrier, the modulation scheme is calle M-ary ASK, M-ary PSK and M-ary FSK.

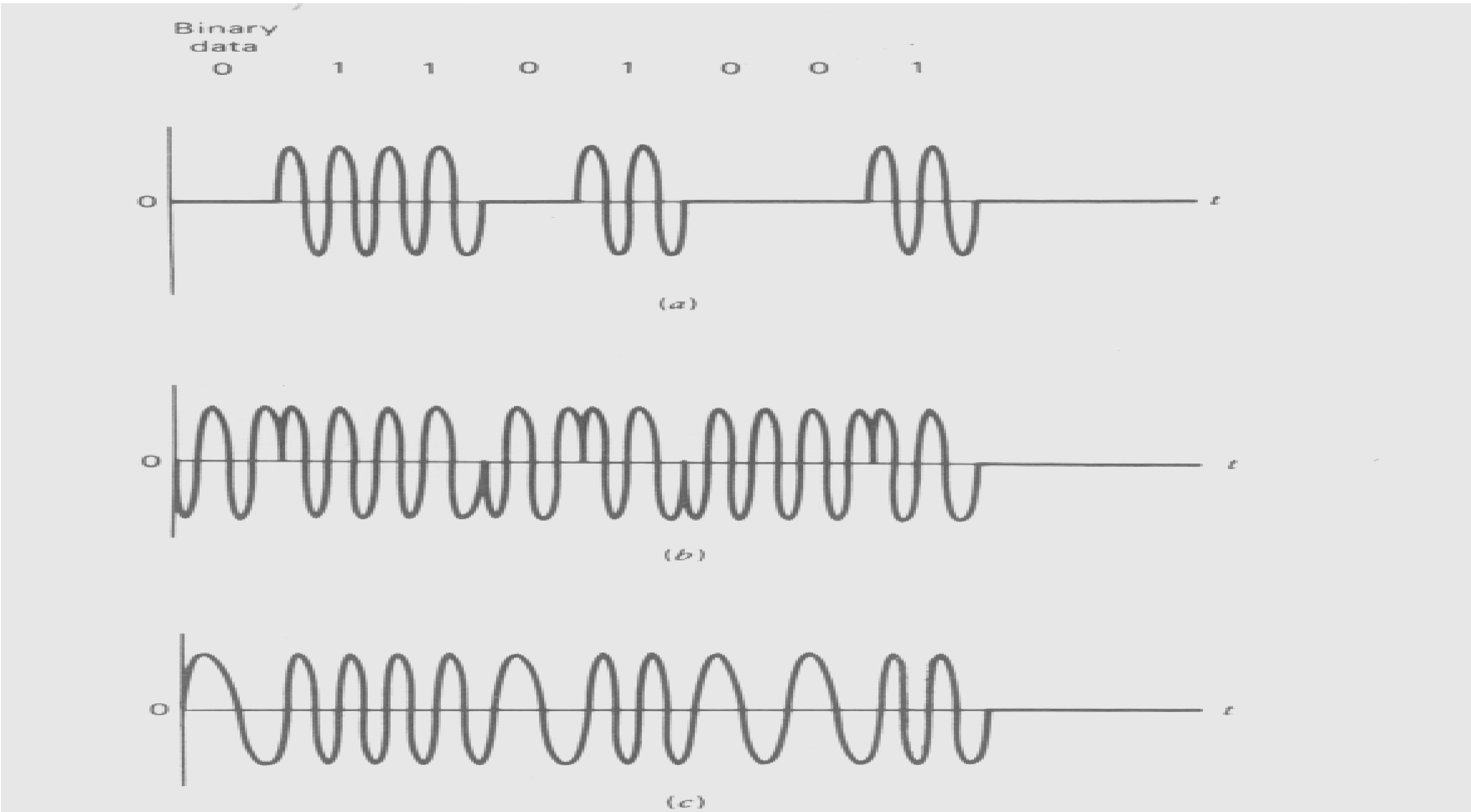


Fig: waveforms of (a) ASK (b) PSK (c)FSK

M-ary Phase Shift Keying(MPSK)

In M-ary PSK, the carrier phase takes on one of the M possible values, namely

$\theta_i = 2 * (i - 1)\pi / M$

where i = 1, 2, 3, …..M.

The modulated waveform can be expressed as

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left(2\pi f_c t + \boxed{\frac{2\pi}{M}(i-1)}\right), \quad 0 \le t \le T_s \qquad i = 1, 2, \ldots, M$$

where E_s is energy per symbol = $(\log_2 M)\, E_b$

T_s is symbol period = $(\log_2 M)\, T_b$.

e above equation in the Quadrature form is

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left[(i-1)\frac{2\pi}{M}\right]\cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T_s}}\sin\left[(i-1)\frac{2\pi}{M}\right]\sin(2\pi f_c t) \qquad i = 1, 2, \ldots, M$$

choosing orthogonal basis signals

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$$

efined over the interval $0 \le t \le T_s$

M-ary signal set can be expressed as

$$S_{\text{M-PSK}}(t) = \left\{ \sqrt{E_s}\cos\left[(i-1)\frac{\pi}{2}\right]\phi_1(t) - \sqrt{E_s}\sin\left[(i-1)\frac{\pi}{2}\right]\phi_2(t) \right\}$$
$$i = 1, 2, \ldots.., M$$

Since there are only two basis signals, the constellation of M-ary PSK is two dimensional.

The M-ary message points are equally spaced on a circle of radius $\sqrt{E_s}$, centered at the origin.

The constellation diagram of an 8-ary PSK signal set is shown in fig.

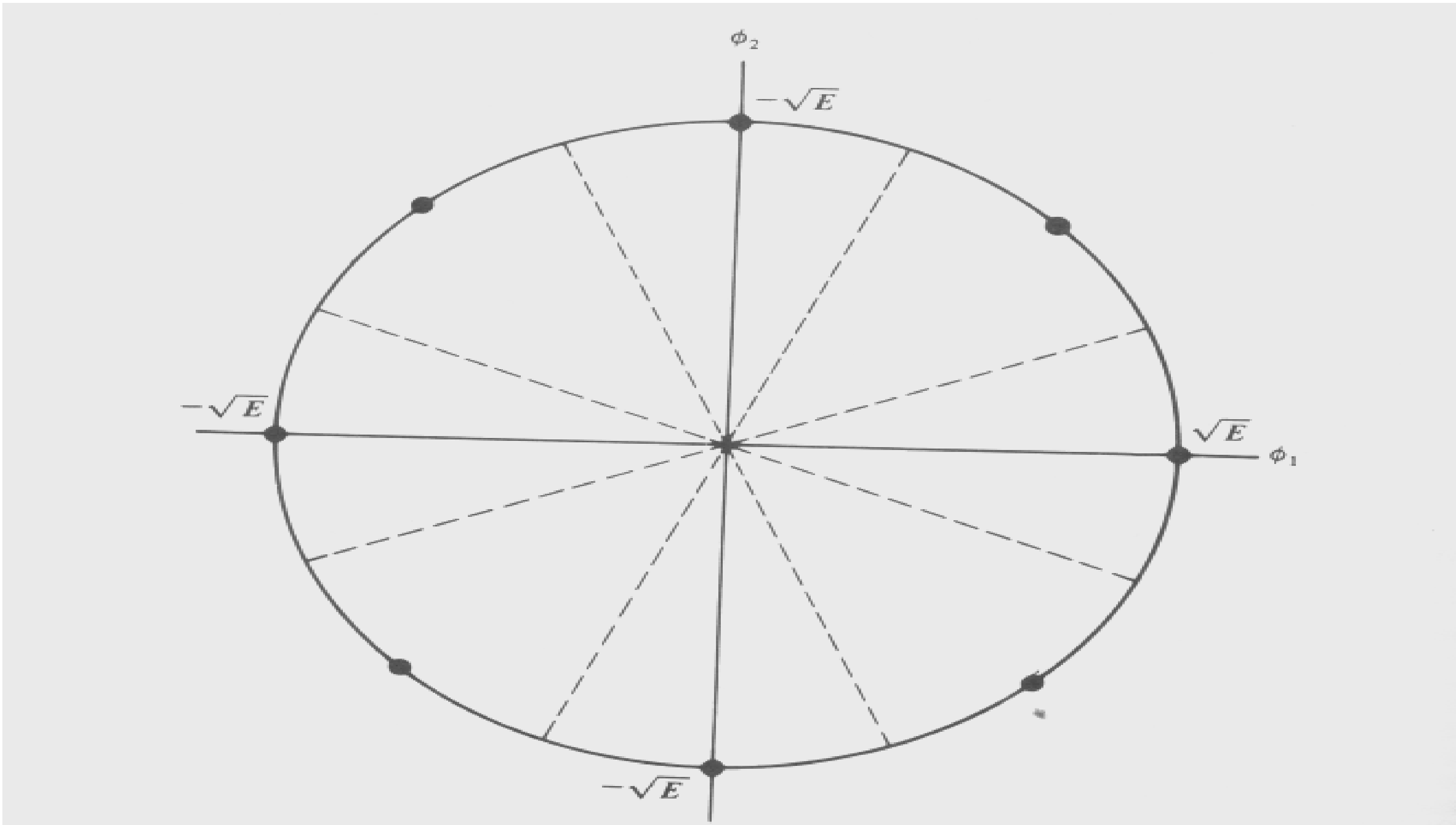


Fig: Constellation diagram of an M-ary PSK system(m=8)

Derivation of symbol error probability:

Decision Rule:

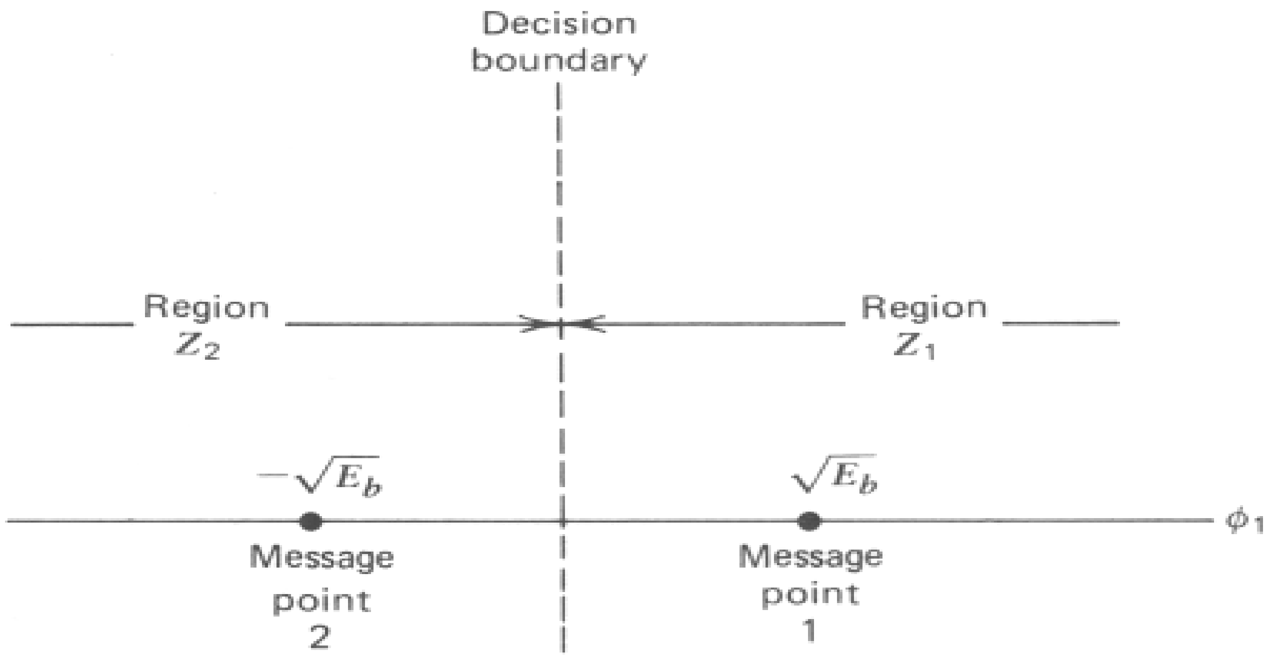


Fig: Constellation diagram for M=2 (Binary PSK)

If a symbol (0,0,0) is transmitted, it is clear that if an error occurs, the transmitted signal is most likely to be mistaken for (0,0,1) and (1,1,1) and the signal being mistaken for (1,1,0) is remote.

The decision pertaining to (0,0,0) is bounded by $\theta = -\pi/8$(below $\phi_1(t)$- axis) to $\theta = +\pi/8$ (above $\phi_2(t)$- axis)

The probability of correct reception is…

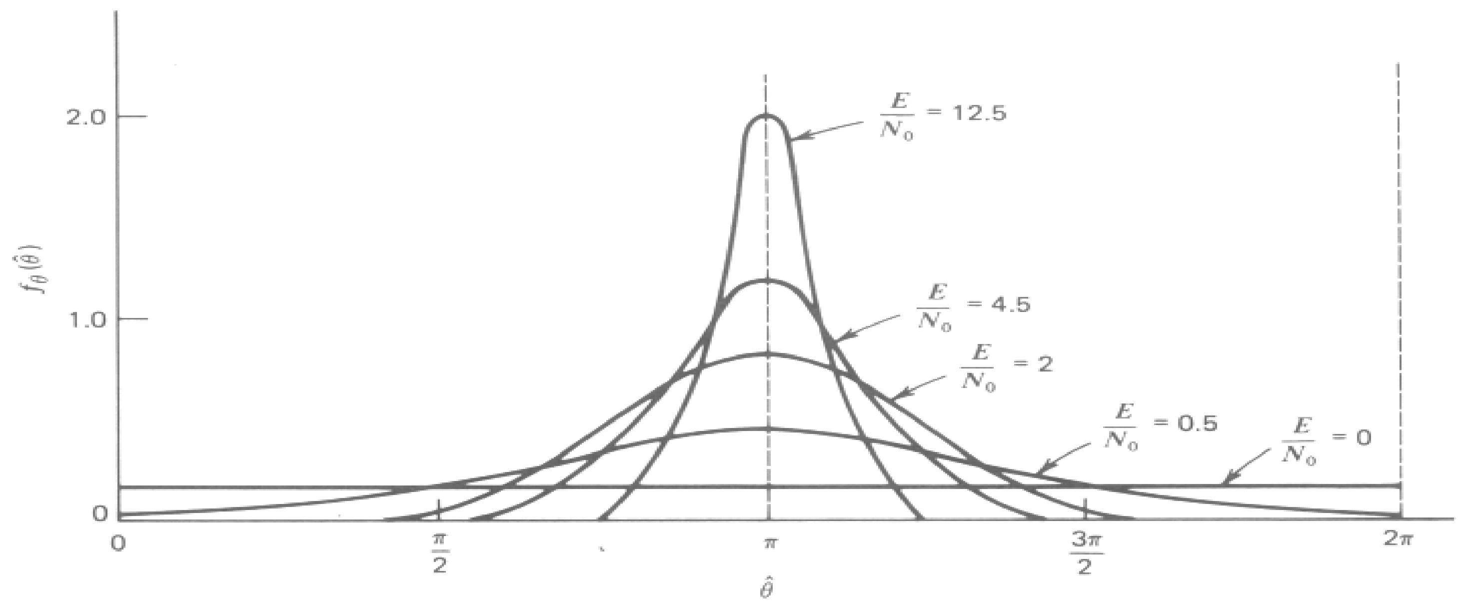


Fig: Probability density function of Phase θ.

The average symbol error probability of an coherent M-ary PSK system in AWGN channel is given by

$$P_e \le 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$

Similarly, The symbol error Probability of a differential M-ary PSK system in AWGN channel is given by

$$P_e \approx 2Q\left(\sqrt{\frac{4E_s}{N_0}} \sin\left(\frac{\pi}{2M}\right)\right)$$

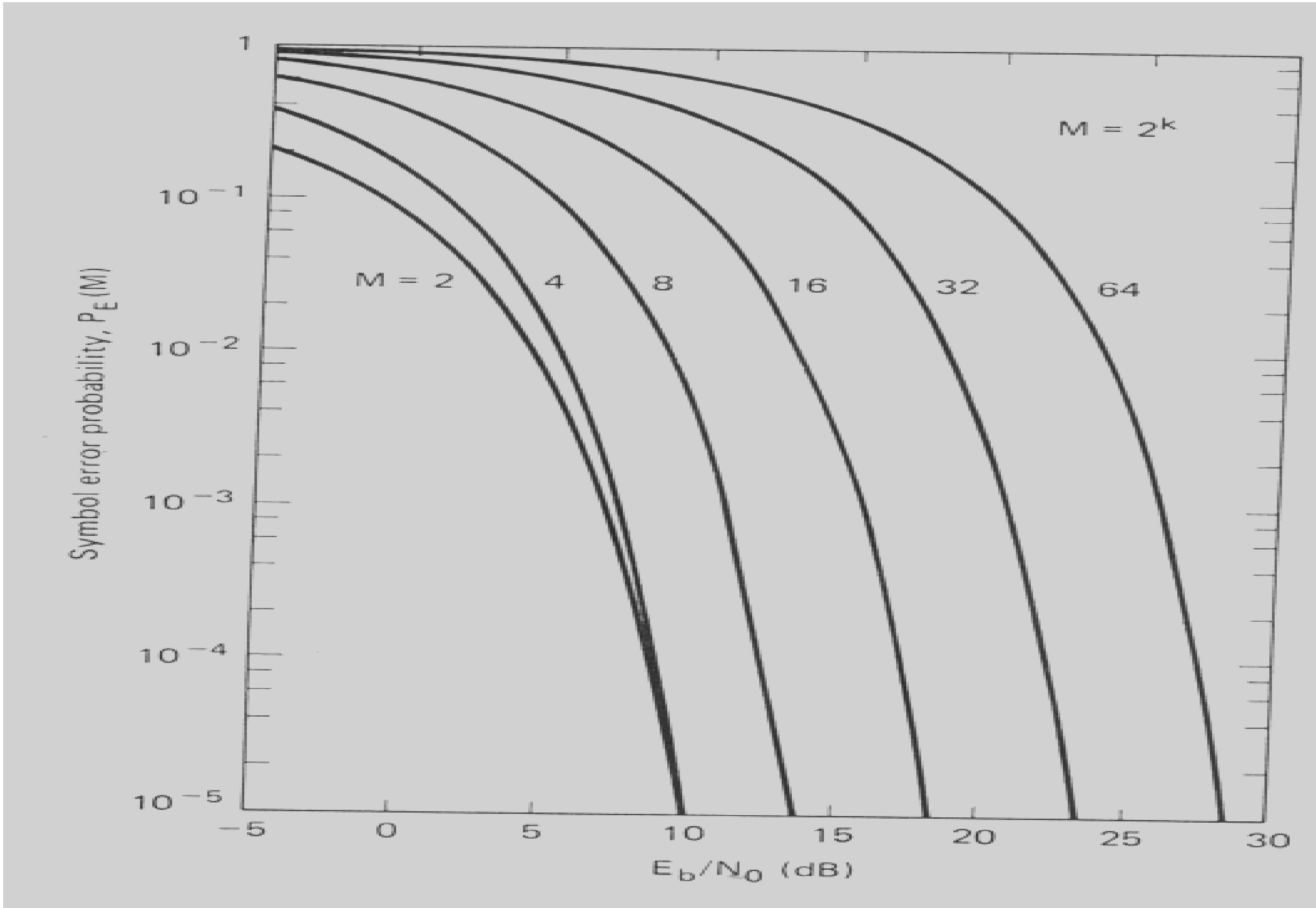


Fig: The performance of symbol error probability for

-different values of M

M-ary Quadrature Amplitude Modulation (QAM)

It's a Hybrid modulation

As we allow the amplitude to also vary with the phase, a new modulation scheme called quadrature amplitude modulation (QAM) is obtained.

The constellation diagram of 16-ary QAM consists of a square lattice of signal points.

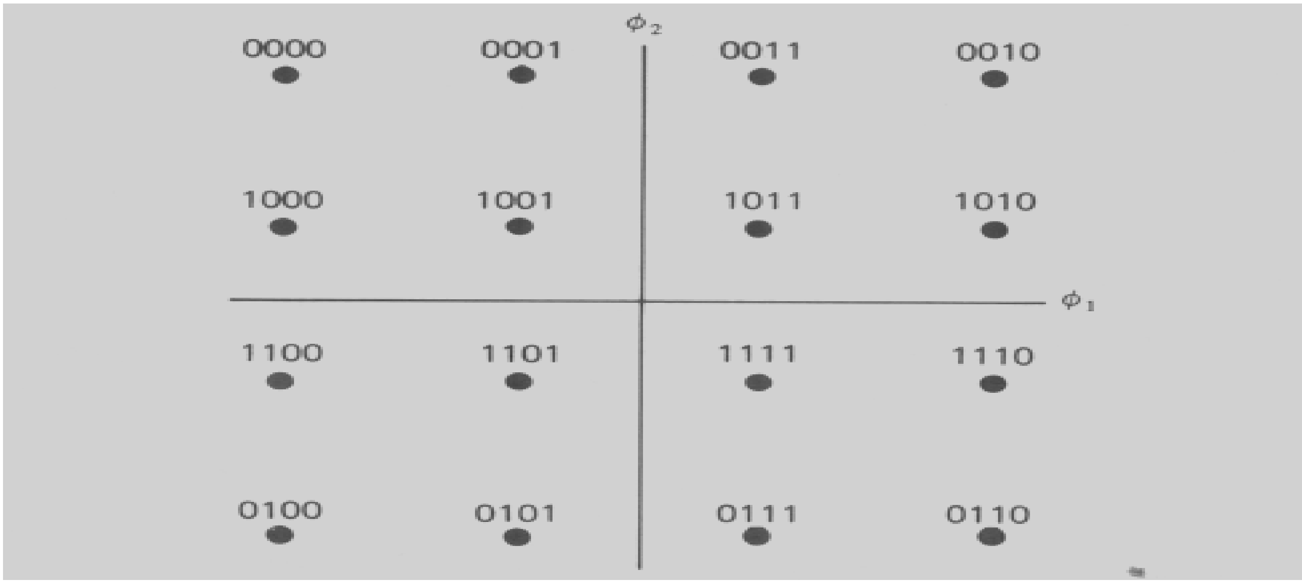


Fig: signal Constellation of M-ary QAM for M=16

M-ary Frequency Shift Keying(MFSK)

In M-ary FSK modulation the transmitted signals are defined by:

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[\frac{\pi}{T_s}(n_c + i)t\right] \quad 0 \le t \le T_s \quad i = 1, 2, \ldots\ldots, M$$

where $f_c = n_c/2T_s$, for some fixed integer n.

The M transmitted signals are of equal energy and equal duration, and the signal frequencies are separated by $1/2T_s$ Hertz, making the signals orthogonal to one another.

The average probability of error based on the union bound is given by

$$P_e \leq (M-1)\, Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

Using only the leading terms of the binomial expansion:

$$P_e = \sum_{k=1}^{M-1} \left(\frac{(-1)^{k+1}}{k+1}\right) \binom{M-1}{k} \exp\left(\frac{-kE_s}{(k+1)N_0}\right)$$

Power Efficiency and Bandwidth :

Bandwidth:

Table 5.6 Bandwidth and Power Efficiency of Coherent M-ary FSK [Zie92]

M	2	4	8	16	32	64
η_B	0.4	0.57	0.55	0.42	0.29	0.18
E_b/N_o for BER=10^{-6}	13.5	10.8	9.3	8.2	7.5	6.9

- The channel bandwidth of a M-ary FSK signal is :

$$B = \frac{R_b\,(M+3)}{2\log_2 M}$$

The channel bandwidth of a noncohorent MFSK is :

$$B = \frac{R_b M}{2\log_2 M}$$

This implies that the bandwidth efficiency of an M-ary FSK signal decreases with increasing M. Therefore, unlike M-PSK signals, M-FSK signals are bandwidth inefficient.

Introduction to Spread Spectrum

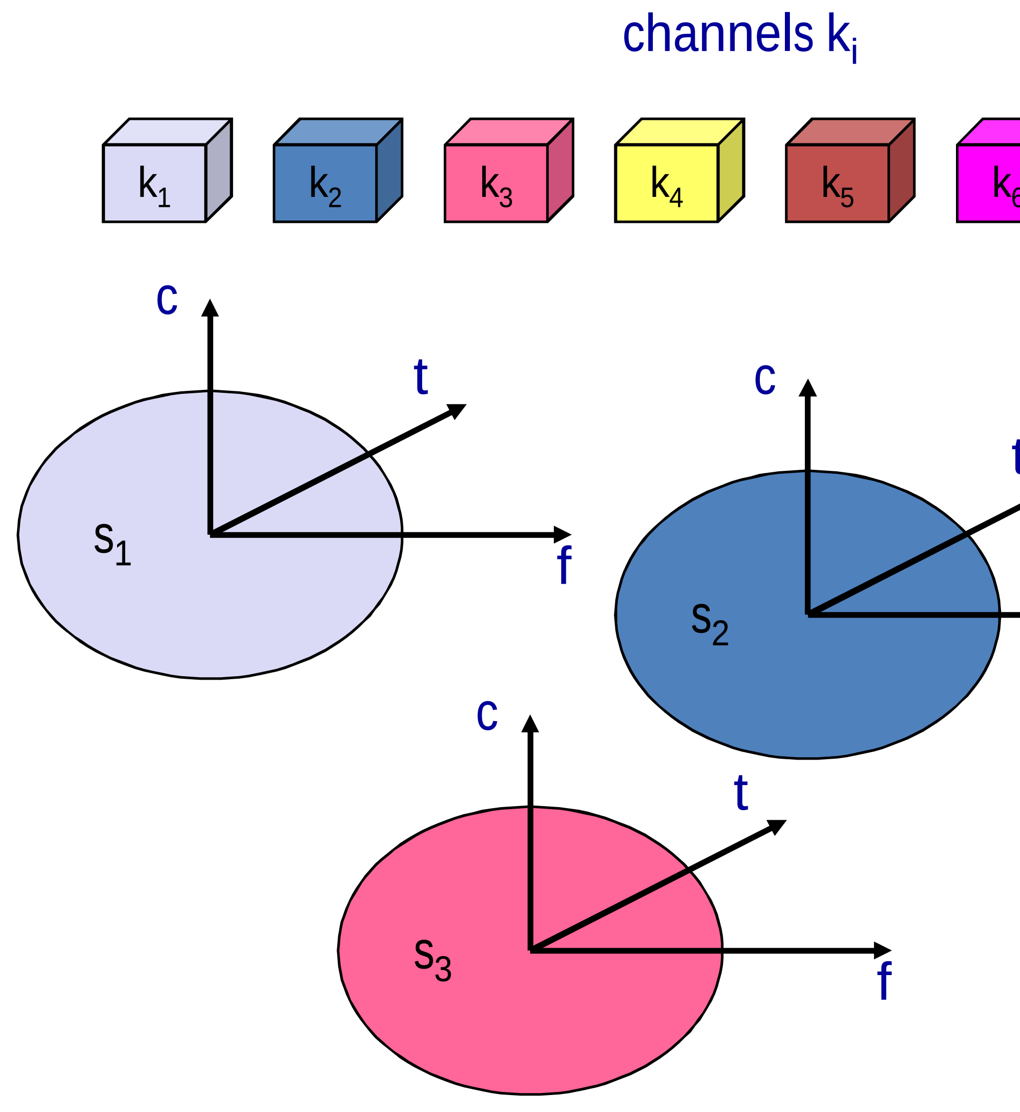
- Problems such as capacity limits, propagation effects, synchroniza
  occur with wireless systems
- Spread spectrum modulation spreads out the modulated signal bandwidth so it is much greater than the message bandwidth
- Independent code spreads signal at transmitter and despreads sig
  at receiver

Multiplexing

- Multiplexing in 4 dimensions
  - space (s_i)
  - time (t)
  - frequency (f)
  - code (c)

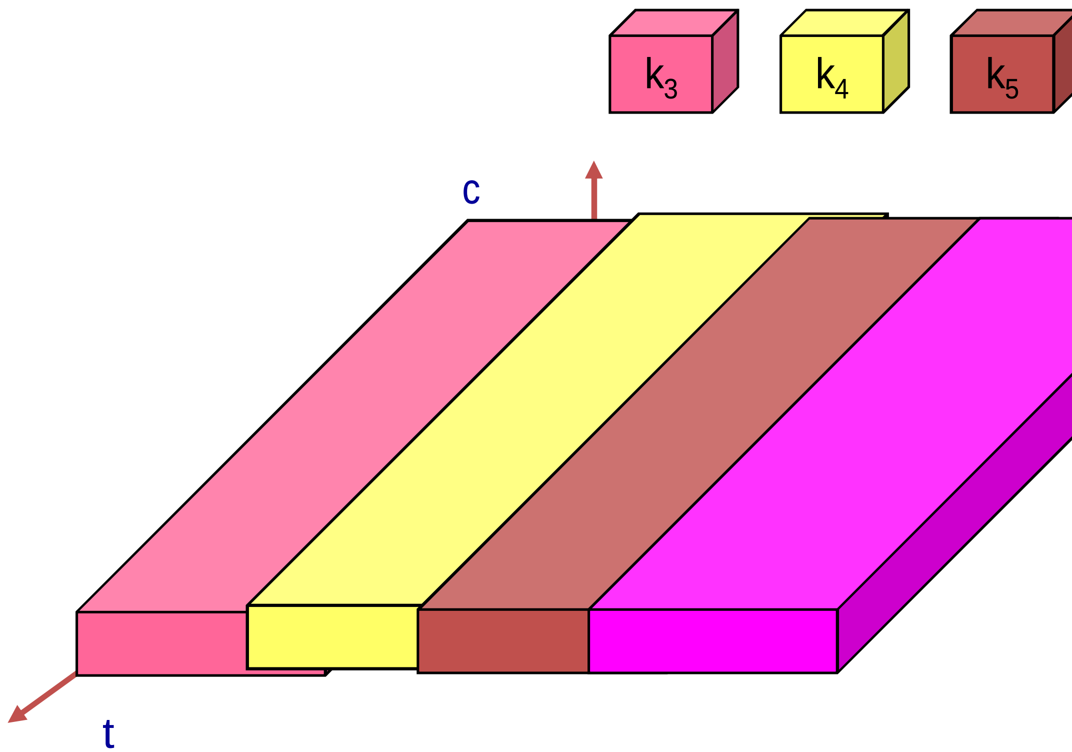
- Goal: multiple use of a shared medium

- Important: guard spaces needed!



Frequency multiplex

- Separation of spectrum into smaller frequency bands
- Channel gets band of the spectrum for the whole time
- Advantages:
  - no dynamic coordination needed
  - works also for analog signals
- Disadvantages:
  - waste of bandwidth
    if traffic distributed unevenly
  - inflexible
  - guard spaces



Time multiplex

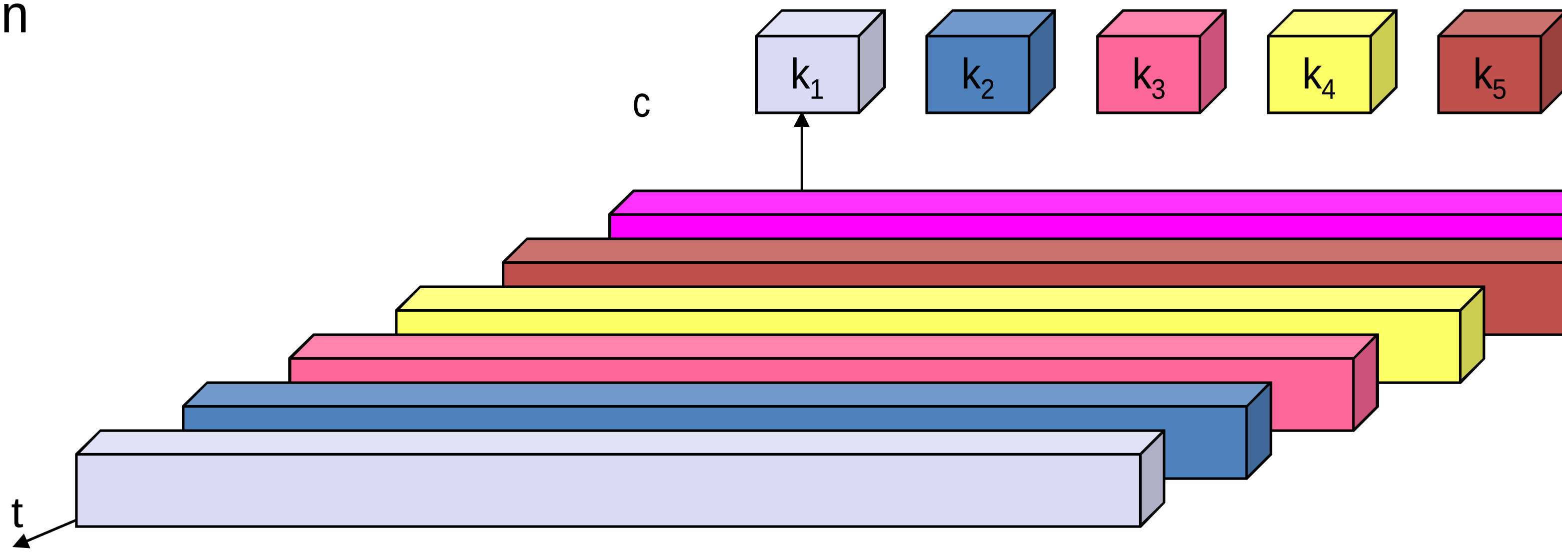
Channel gets the whole spectrum for a certain amount of time

Advantages:

- only one carrier in the medium at any time
- throughput high even for many users

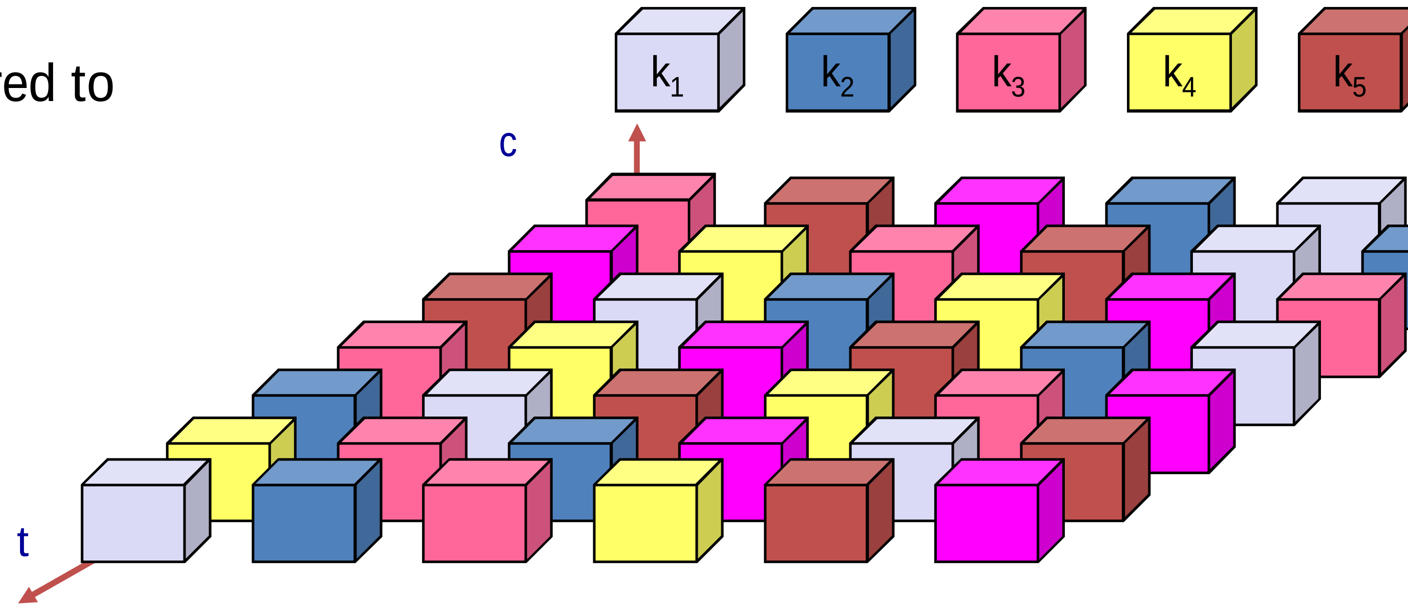
Disadvantages:

- precise synchronization necessary

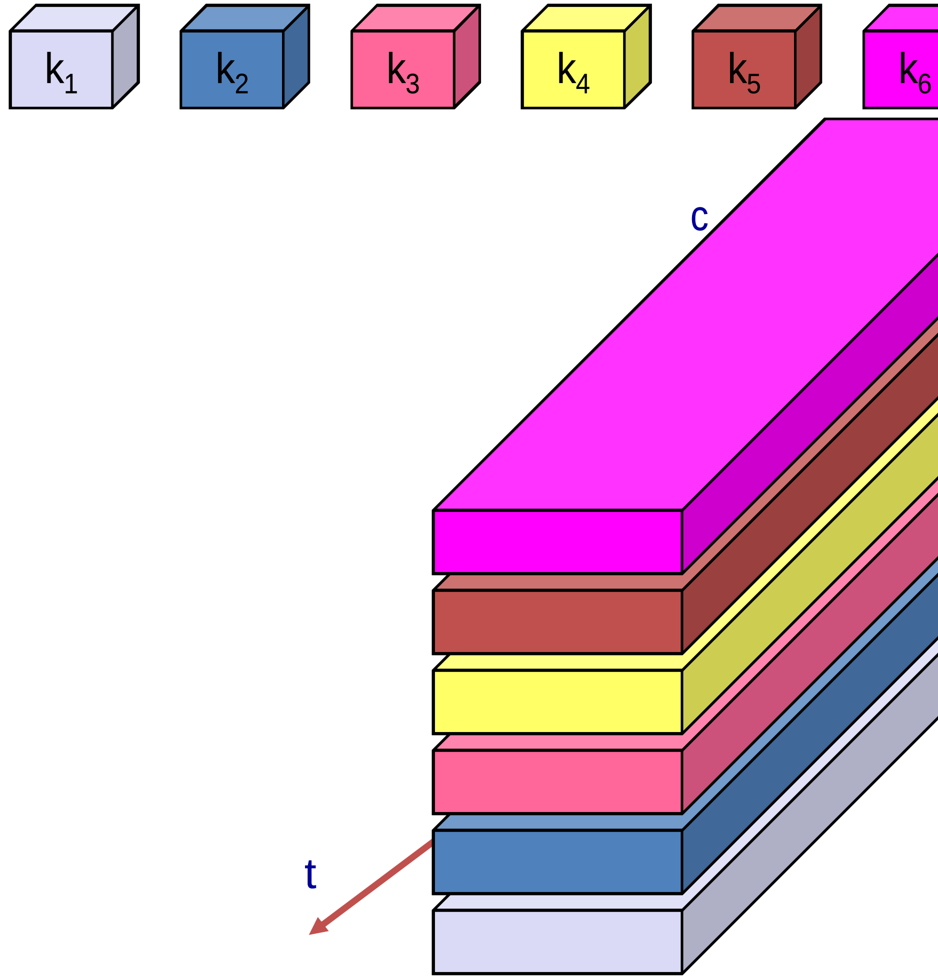


Time and frequency multiplex

- A channel gets a certain frequency band for a certain amount of time (e.g. GSM)
- Advantages:
  - better protection against tapping
  - protection against frequency selective interference
  - higher data rates compared to code multiplex
- Precise coordination required



Code multiplex



Each channel has unique code

All channels use same spectrum at same time

Advantages:

- bandwidth efficient
- no coordination and synchronization
- good protection against interference

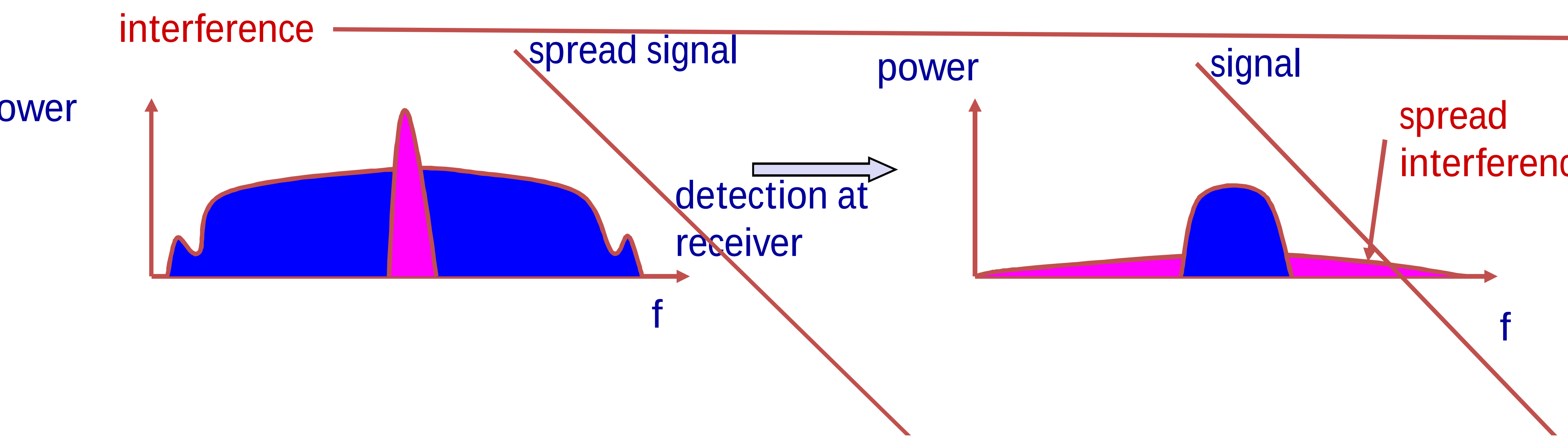
Disadvantages:

- lower user data rates
- more complex signal regeneration

Implemented using spread spectrum technology

Spread Spectrum Technology

- Problem of radio transmission: frequency dependent fading can wipe out narrow band signals for duration of the interference
- Solution: spread the narrow band signal into a broad band signal using a special code

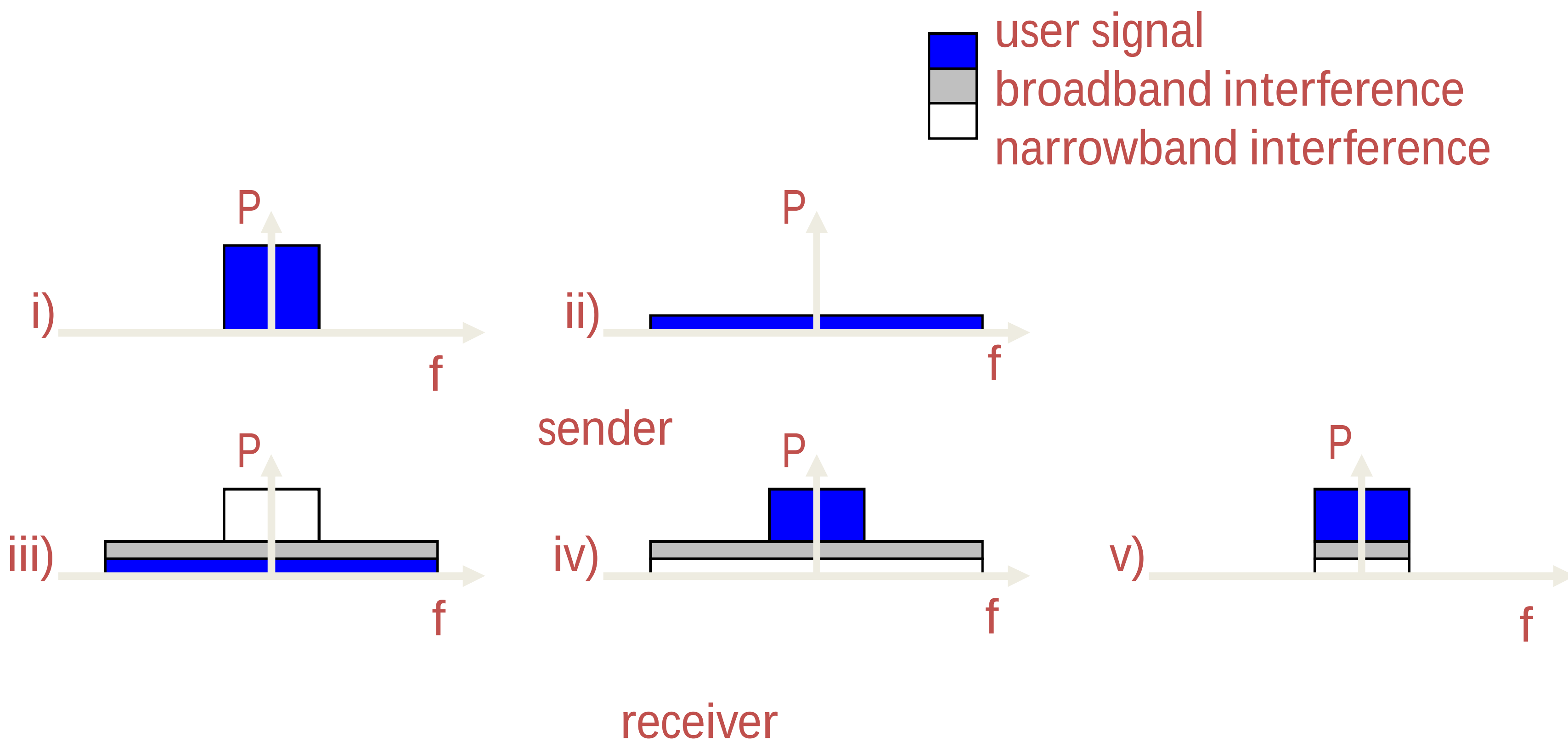


Spread Spectrum Technology

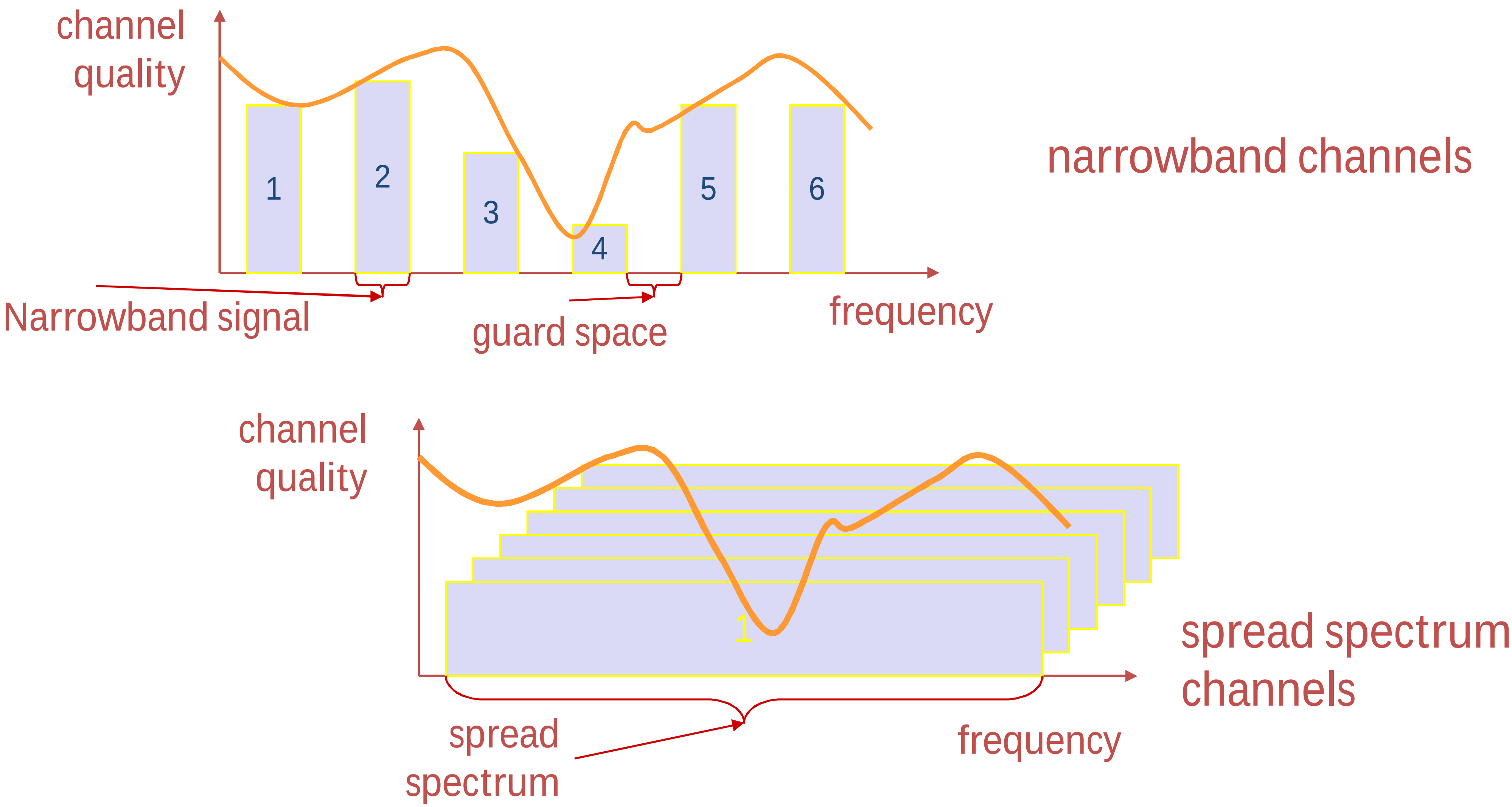
- Side effects:
  - coexistence of several signals without dynamic coordination
  - tap-proof
- Alternatives: Direct Sequence (DS/SS), Frequency Hopping (FH/SS)
- Spread spectrum increases BW of message signal by a factor *N*, Processing Gain

$$\text{Processing Gain } N = \frac{B_{ss}}{B} = 10\log_{10}\left(\frac{B_{ss}}{B}\right)$$

Effects of spreading and interference

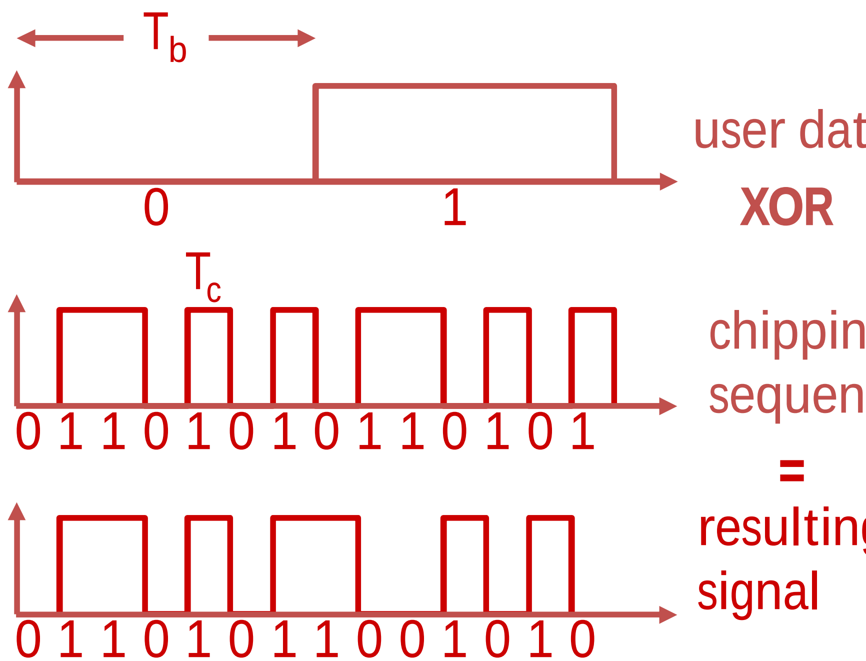


Spreading and frequency selective fading

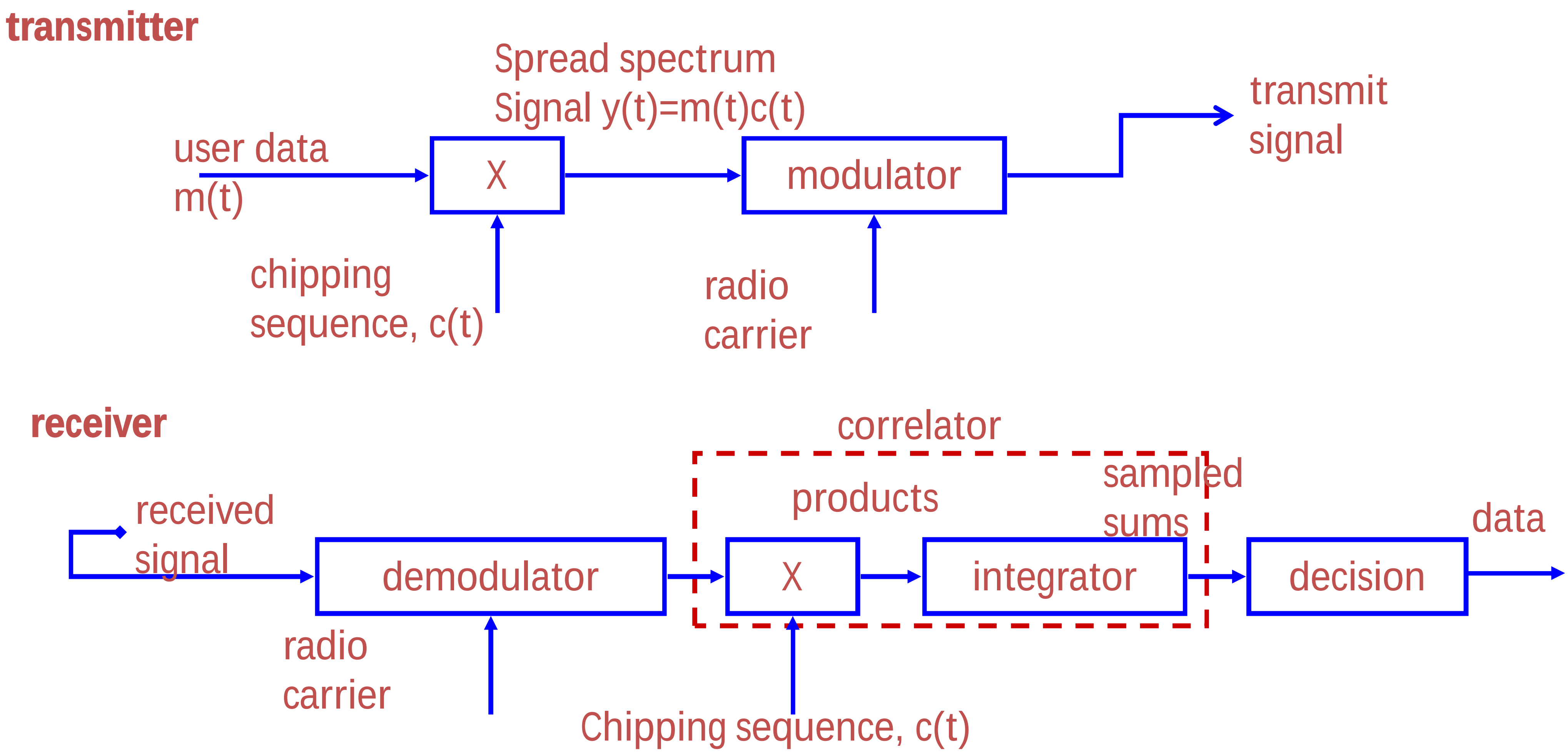


DSSS (Direct Sequence Spread Spectrum) I

- XOR the signal with pseudonoise (PN) sequence (chipping sequence)
- Advantages
  - reduces frequency selective fading
  - in cellular networks
    - base stations can use the same frequency range
    - several base stations can detect and recover the signal
- But, needs precise power control



DSSS (Direct Sequence Spread Spectrum) II



DS/SS Comments III

- Pseudonoise(PN) sequence chosen so that its autocorrelation is very narrow => PSD is very wide
  - Concentrated around $\tau \leq T_c$
  - Cross-correlation between two user's codes is very small

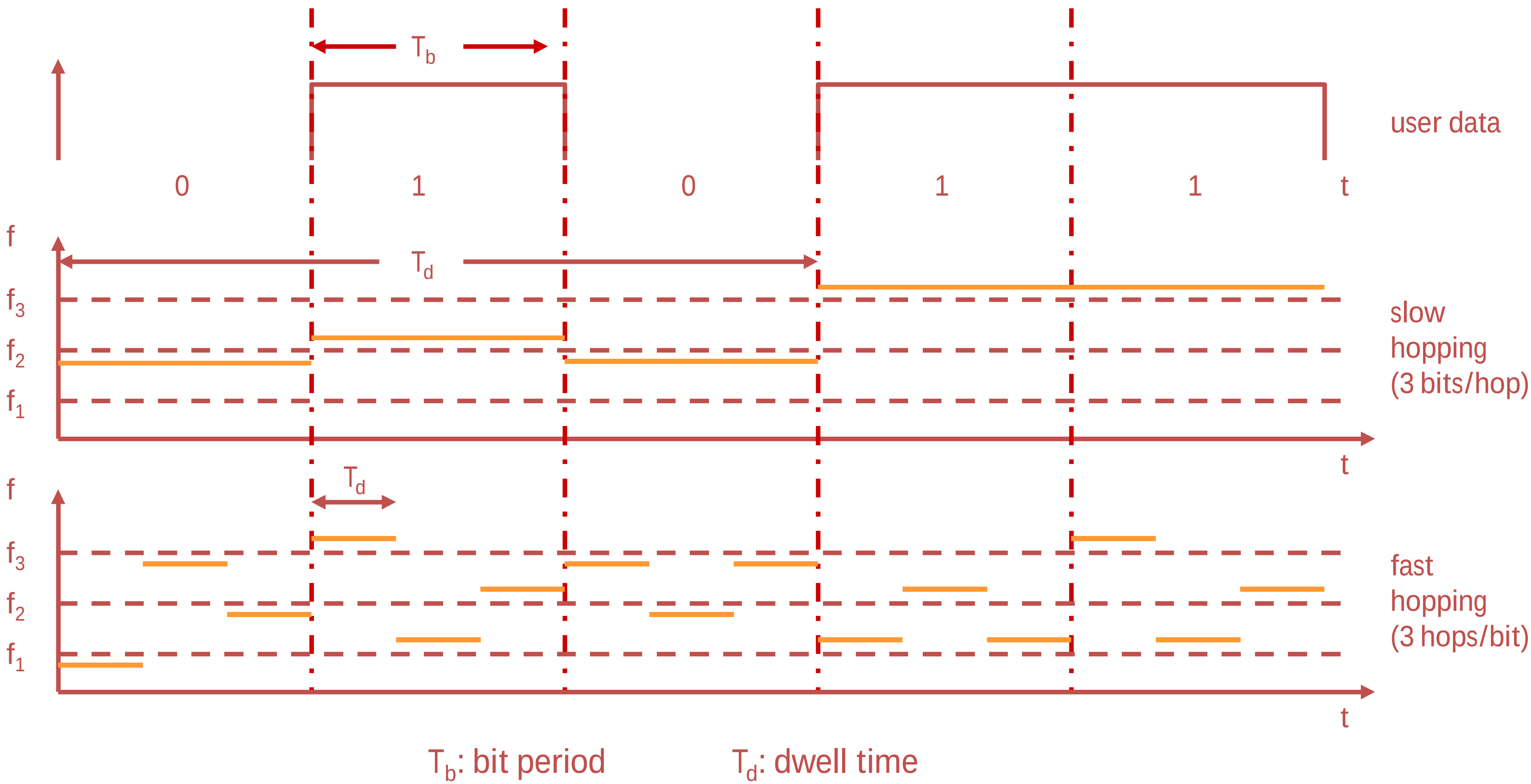
DS/SS Comments IV

- Secure and Jamming Resistant
  - Both receiver and transmitter must know c(t)
  - Since PSD is low, hard to tell if signal present
  - Since wide response, tough to jam everything
- Multiple access
  - If $c_i(t)$ is orthogonal to $c_j(t)$, then users do not interfere
- Near/Far problem
  - Users must be received with the same power

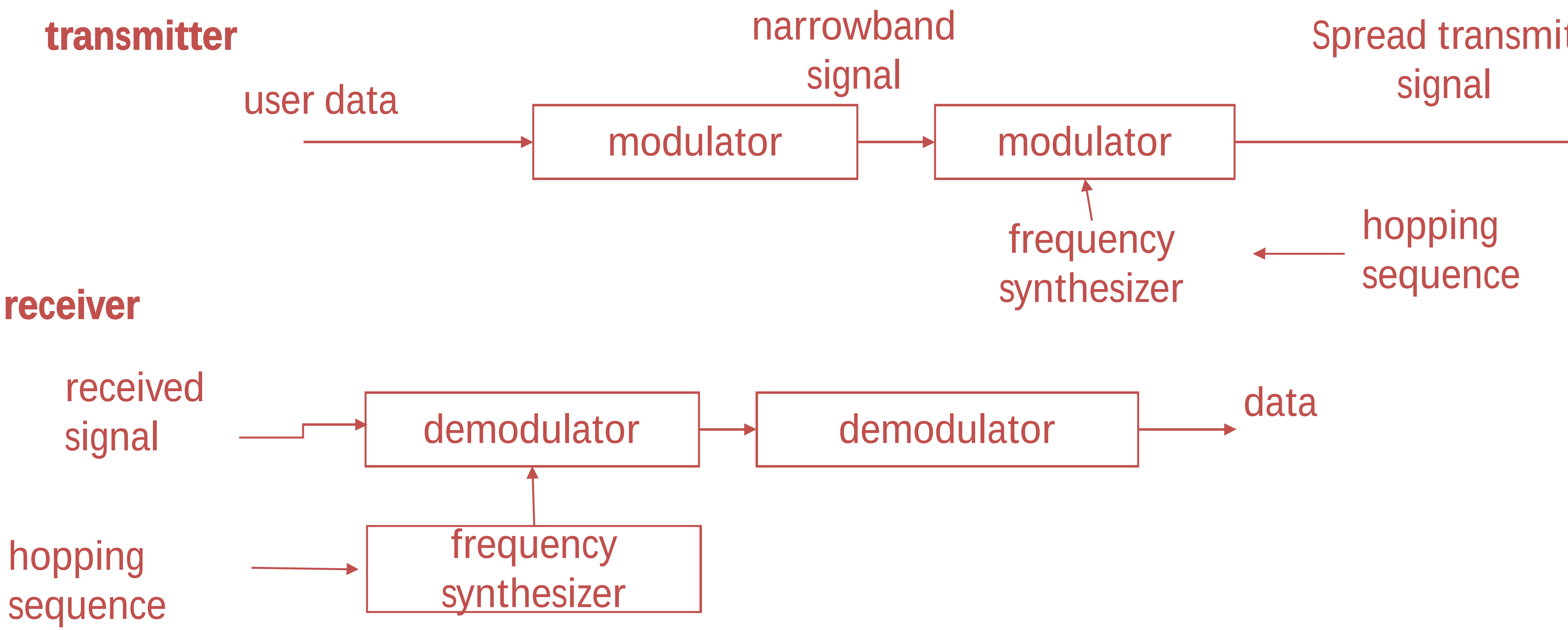
FH/SS (Frequency Hopping Spread Spectrum)

- Discrete changes of carrier frequency
  - sequence of frequency changes determined via PN sequence
- Two versions
  - **Fast Hopping**: several frequencies per user bit (FFH)
  - **Slow Hopping**: several user bits per frequency (SFH)
- Advantages
  - frequency selective fading and interference limited to short period
  - uses only small portion of spectrum at any time
- Disadvantages
  - not as robust as DS/SS
  - simpler to detect

FHSS (Frequency Hopping Spread Spectrum) I



FHSS (Frequency Hopping Spread Spectrum) III



Applications of Spread Spectrum

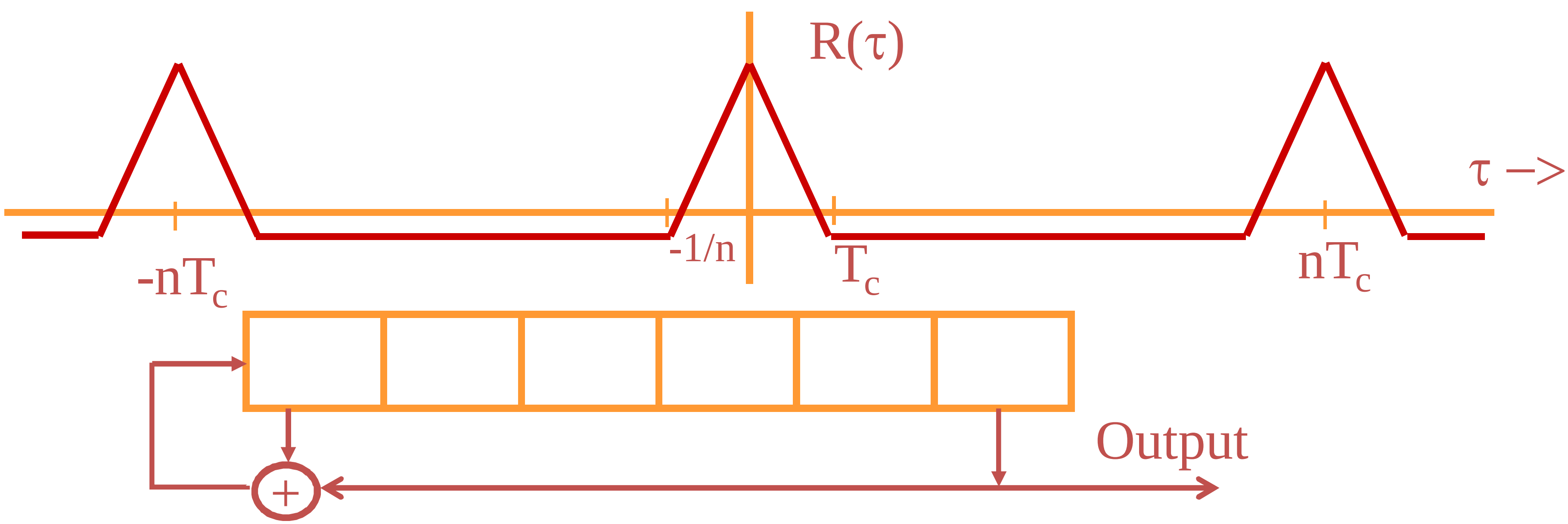
- Cell phones
  - IS-95 (DS/SS)
  - GSM
- Global Positioning System (GPS)
- Wireless LANs
  - 802.11b

Performance of DS/SS Systems

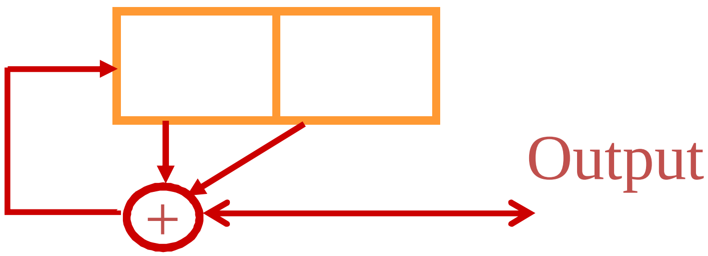
- Pseudonoise (PN) codes
  - Spread signal at the transmitter
  - Despread signal at the receiver
- Ideal PN sequences should be
  - Orthogonal (no interference)
  - Random (security)
  - Autocorrelation similar to white noise (high at τ=0 and low for τ not equal 0)

PN Sequence Generation

- Codes are periodic and generated by a shift register and XOR
- Maximum-length (ML) shift register sequences, *m*-stage shift register, length: $n = 2^m - 1$ bits



Generating PN Sequences



Take m=2 =>L=3

c_n=[1,1,0,1,1,0, . . .], usually written as bipolar c_n=[1,1,-1,1,1,-1, . . .]

$$(m) = \frac{1}{L}\sum_{n=1}^{L} c_n c_{n+m}$$

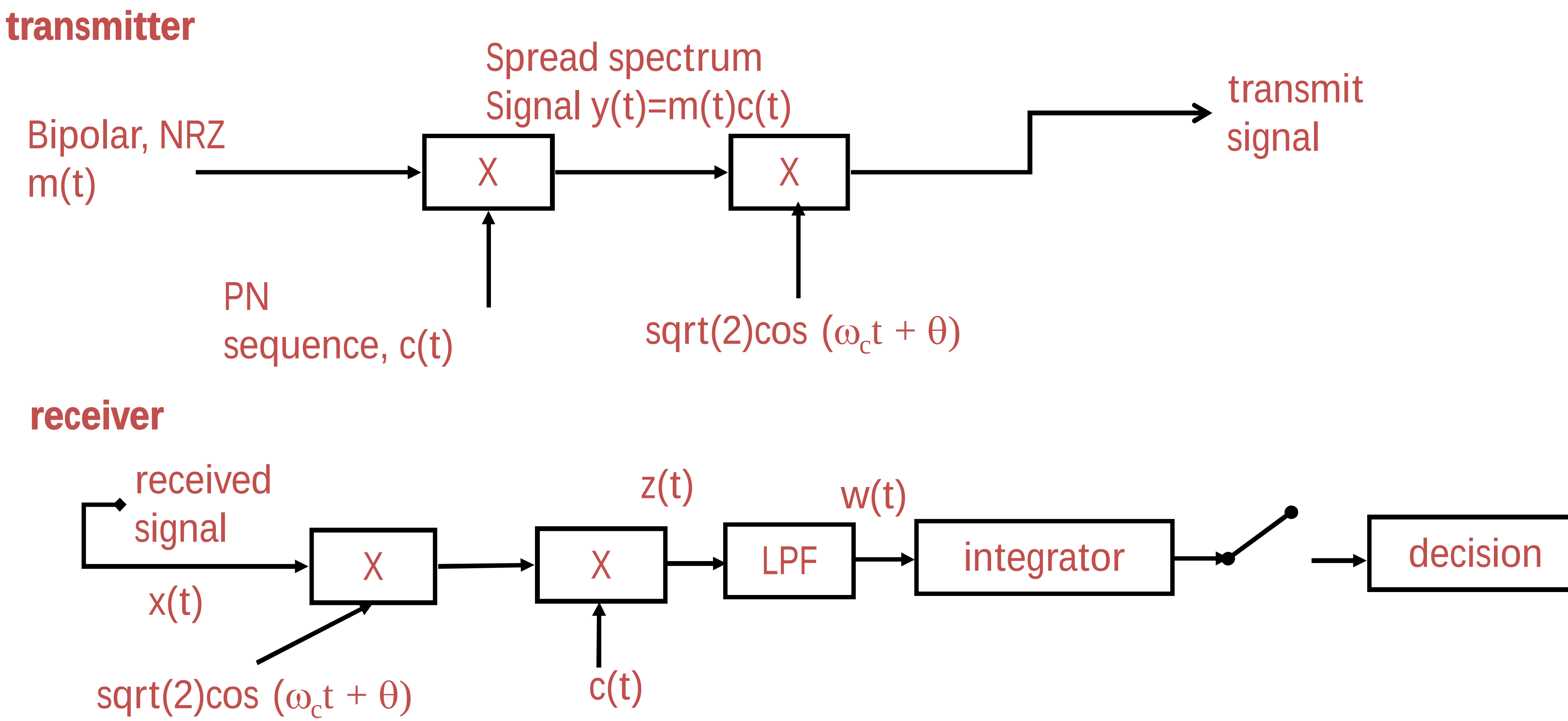
$$= \begin{cases} 1 & m = 0 \\ -1/L & 1 \le m \le L-1 \end{cases}$$

m	Stages connected to modulo-2 adder
2	1,2
3	1,3
4	1,4
5	1,4
6	1,6
8	1,5,6,7

Problems with *m*-sequences

- Cross-correlations with other *m*-sequences generated by different input sequences can be quite high
- Easy to guess connection setup in 2*m* samples so not too secure
- In practice, Gold codes or Kasami sequences which combine the output of m-sequences are used.

Detecting DS/SS PSK Signals



Optimum Detection of DS/SS PSK

Recall, bipolar signaling (PSK) and white noise give the optimum error probability

$$P_b = Q\left(\sqrt{\frac{2E_b}{\aleph}}\right)$$

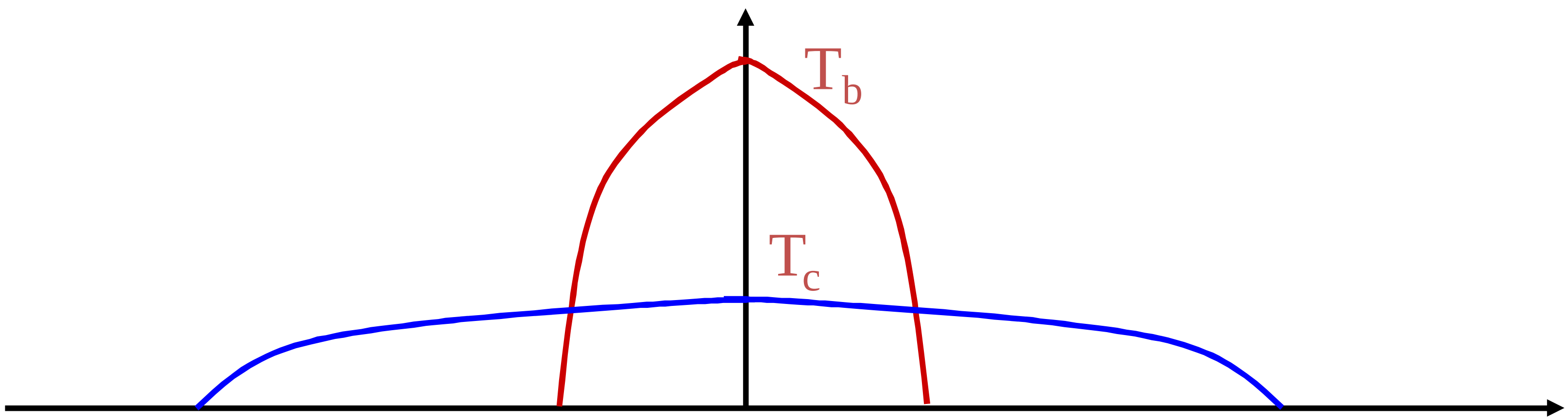
Not effected by spreading

- Wideband noise not affected by spreading
- Narrowband noise reduced by spreading

Signal Spectra

$$\text{Processing Gain } N = \frac{B_{ss}}{B} = 10\log_{10}\left(\frac{B_{ss}}{B}\right) = \frac{T_b}{T_c}$$

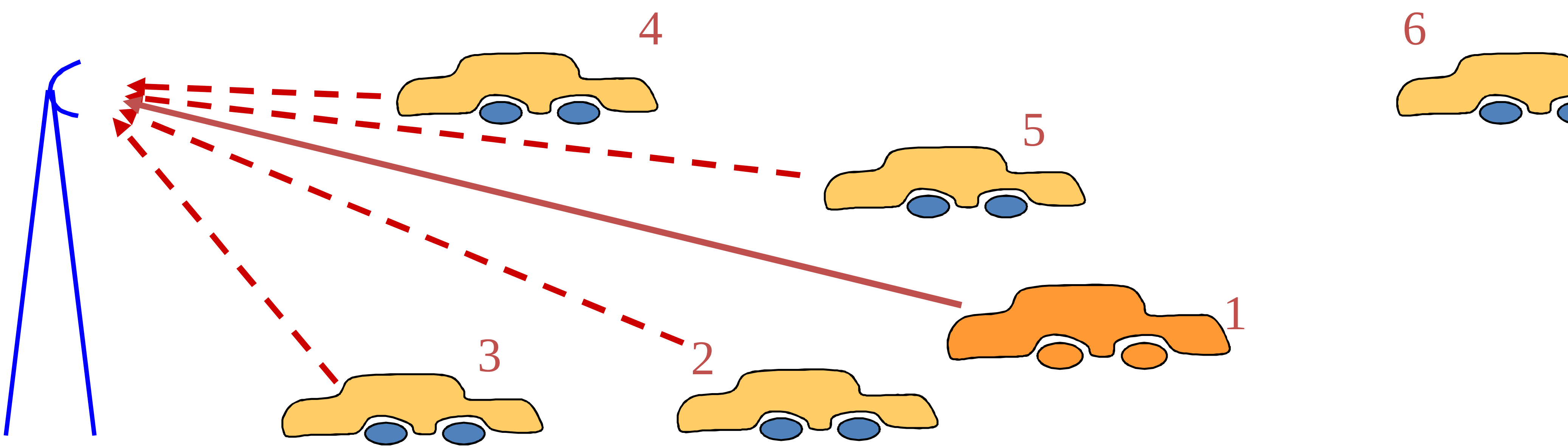
- Effective noise power is channel noise power plus jamming (NB) signal power divided by *N*



Multiple Access Performance

Assume K users in the same frequency band,

Interested in user 1, other users interfere



Signal Model

Interested in signal 1, but we also get signals from other *K-1* users:

$$
\begin{aligned}
x_k(t) &= \sqrt{2}\, m_k(t-\tau_k)\, c_k(t-\tau_k) \cos\left(\omega_c (t-\tau_k) + \theta_k\right) \\
&= \sqrt{2}\, m_k(t-\tau_k)\, c_k(t-\tau_k) \cos\left(\omega_c t + \phi_k\right) \qquad \phi_k = \theta_k -
\end{aligned}
$$

At receiver,

$$
x(t) = x_1(t) + \sum_{k=2}^{K} x_k(t)
$$

Interfering Signal

- After mixing and despreading (assume τ_1=0)

$$(t) = 2\, m_k(t-\tau_k)\, c_k(t-\tau_k) c_1(t) \cos(\omega_c t + \phi_k) \cos(\omega_c t$$

- After LPF

$$w_k(t) = m_k(t-\tau_k)\, c_k(t-\tau_k) c_1(t) \cos(\phi_k - \theta_1)$$

- After the integrator-sampler

$$I_k = \cos(\phi_k - \theta_1) \int_0^{T_b} m_k(t-\tau_k)\, c_k(t-\tau_k) c_1(t$$

At Receiver

(t) =+/-1 (PSK), bit duration Tb

terfering signal may change amplitude at τk

$$= \cos(\phi_k - \theta_1)\left[b_{-1}\int_0^{\tau_k} c_k(t-\tau_k)c_1(t)dt + b_0\int_{\tau_k}^{T_b} c_k(t-\tau_k)c_1\right.$$

User 1: $I_1 = \int_0^{T_b} m_1(t)\, c_1(t) c_1(t)\, dt$

eally, spreading codes are Orthogonal:

$$\int_0^{T_b} c_1(t)c_1(t)dt = A \qquad \int_0^{T_b} c_k(t-\tau_k)c_1(t)dt = 0$$